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Single Objective Functions in Location Routing Problems: A Comparative Case Study

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Abstract

This research explores the Location Routing Problem (LRP), a complex NP-hard problem that combines facility location and vehicle routing to minimize operational costs. The focus is on a variant of LRP with covering constraints. The primary objective is to investigate the impacts of different single-objective functions in LRPs, contrasting the traditional approach of minimizing combined opening and transportation costs with objectives prioritizing either the minimization of the number of facilities or the total distance. A Mixed-Integer Programming (MIP) model is developed to analyze the effects of these objectives on facility location and routing efficiency. The methodology involves running the MIP model under various objective functions and observing the changes in total distance and number of facilities, including the use of preemptive goal programming. Results from small and large problem instances reveal that focusing on minimizing the number of facilities significantly reduces their count but increases the total travel distance. On the other hand, prioritizing distance minimization shows minimal distance reduction but a slight increase in the number of facilities compared with the traditional approach of minimizing combined opening and transportation costs, but this may be due to the proneness to the scale of data. A Pareto analysis shows the trade-offs between these objectives. This research contributes to a deeper understanding of the importance of objective selection in LRPs, offering valuable insights for decision-makers in adapting LRP strategies to specific operational priorities.

Keywords
Location Routing Problem (LRP), Supply Chain Optimization, Mixed-Integer Programming (MIP), Facility Location, Vehicle Routing.

1. Introduction
The location routing problem (LRP) is a complex problem involving two important decision-making processes: facility location and vehicle routing. The primary goal of the LRP is to determine the optimal location for facilities, e.g. warehouses, and the best routes for vehicles leaving from these facilities to a set of customer/demand points in order to minimize overall costs, which can include transportation, facility opening, and other operational expenses. The LRP is a combination of two NP-hard problems, which makes the LRP itself an NP-hard problem, too (Lopes et al. 2013). Its complexity is due to its combinatorial nature and the interdependencies between location and routing decisions.

The LRP has a wide array of applications. Some of its traditional applications include healthcare, and telecommunications and network design. In healthcare logistics for example, LRP helps in locating medical facilities and routing medical supplies and services, ensuring accessibility and efficient distribution of healthcare resources. In telecommunications and network design, LRP is helps in determining optimal locations for servers, routers, or relay stations, and for efficient data routing. The LRP has seen some recent applications in the sharing economy as well.

One example is on-demand grocery delivery apps. Facilities in this example are dark stores, which are stores that serve as mini distribution hubs and are not open for customer walk-ins.

There are many variations of LRPs. In this research, a variant is considered where there are a set of demand points that must be served by a set of facilities with covering constraints. Covering constraints are determined based on some service threshold such as a maximum distance or travel time. For example, if the problem at hand requires a service within a maximum travel time T_{max}, then covering facilities for any demand point are those facilities from which the trip time to that demand point is less than or equal to T_{max}. The problem in this variant is twofold. First, the location and number of facilities must be determined. Second, the order of visits from each facility to its covered demand points must also be determined. Furthermore, demand points must be assigned to facilities because a demand point can be covered by more than one facility. The location problem in this variant of LRP is actually a set partitioning problem, while the routing problem is similar to a multi-depot vehicle routing problem. This is because the covering constraints usually lead to a situation where multiple facilities must be selected.

1.1 Objectives

One of the commonly used objective functions in LRP's is to minimize the total costs of opening facilities (fixed costs) and transportation (variable costs) without prioritizing one over the other. It is also common to use a single-objective function, be it minimizing the opening costs of facilities or the total distance/transportation costs. However, to the best of our knowledge, there is a lack of studies that compare the performance of using various single-objective functions. The main objective of this research is to deeply investigate the effect of using single-objective functions in LRP's. Using the common objective function of minimizing the sum of costs of opening facilities and total distance as a baseline, the goal is to compare between two single-objective functions, namely minimizing the number of facilities and minimizing the total distance. Specifically, the effect on the total distance and the location and number of facilities will be studied.

2. Literature Review

The LRP is one of the well-studied problems in the literature due to its critical role and applicability in many real-world supply chain applications. Other studies have provided comprehensive reviews of works related to it (Zajac and Huber 2021; Mara et al. 2021). The review presented here is by no means comprehensive and is focused mainly on the type of objectives typically used in LRP's with brief descriptions of the methods and applications.

One of the recent trends observed in LRP's research is the shift towards environmental-related objectives. Ahlaqqach et al. (2020) and Chen et al. (2018) delve into sustainability and reverse logistics. Ahlaqqach et al. (2020) addressed the end-of-life pharmaceutical products, optimizing reverse logistics processes to reduce environmental impacts and costs. They developed a mixed-integer linear programming (MIP) model with time windows constraints and inventory decisions in multi-echelon supply chains. Chen et al. (2018) focused on a low-carbon supply chain network, aiming to minimize carbon emissions and optimize transportation efficiency. They developed a MIP model and a Non-Sorting Genetic Algorithm (NSGA-II) and tested their approach in 36 problem instances. These studies, along with many others, highlight the growing emphasis on environmental sustainability in LRP's.

Another trend in the literature of LRP is integrating inventory management with location-routing decisions. Nekooghadirli et al. (2014) developed a model for determining the optimal locations for distribution centers while simultaneously planning routes for vehicles. The objective is to minimize the overall cost, which includes transportation, inventory holding, and facility operation costs. Rayat et al. (2017) developed an inventory LRP model considering factors such as demand variability, lead times, and service level requirements. Rafie-Majd et al. (2018) further advanced the inventory LRP by introducing more complex inventory policies. They explored different inventory replenishment strategies, such as just-in-time (JIT) and economic order quantity (EOQ) models, within the context of LRP. Their model allowed for an exploration of the trade-off between inventory-related costs (like holding and shortage costs) and transportation costs.

Other studies have applied the LRP in emergency and disaster situations. Adarang et al. (2020) developed a model for emergency medical services in urban disasters, optimizing facility locations and emergency vehicle routing to improve response times and service quality. Their approach is multi-phased. Facility location decision is optimized in the first phase; optimal routes for transferring patients are determined in the second phase. They used two objectives in their approach: total costs and relief time. Similarly, Chang et al. (2017) tackled a multiobjective problem in post-disaster scenarios, optimizing logistics planning under uncertain conditions such as variable road access and fluctuating demand at relief sites. They developed a non-linear MIP model that considers three objectives: minimizing total costs, maximizing the minimum material satisfaction rates, and maximizing transport capacities of the worst path. They chose Genetic Algorithm (GA) as the heuristic method for solving larger problem instances.

Toro et al. (2017a, 2017b) considered green capacitated LRPs, proposing models that explicitly minimize operational costs and fuel consumption. In Toro et al. (2017b), they introduced a bi-objective MIP model for green capacitated LRPs. The primary objectives were to minimize operational costs, including fuel consumption, and to optimize the environmental performance of logistics operations. They extended their work in Toro et al. (2017a) by introducing the Green Open tour LRP. Using the same objectives of minimizing operational costs and fuel consumption and a similar, they added an emission factor to their model to calculate fuel consumption, assuming a constant speed. This approach allowed for a more accurate estimation of the environmental impact of logistics operations.

Another type of objectives in LRPs involves profit maximization in revenue management applications. Rahim and Sepil (2014), Vidovic et al. (2016), and Ahmadi-Javid et al. (2018) integrated revenue management into LRPs and developed models that account for sales maximization, dynamic pricing strategies, customer segmentation, and service differentiation, among other factors.

A common observation in the reviewed works is that total cost minimization is often done by minimizing the sum of variable distribution costs and fixed cost of opening facilities in one function without prioritizing one over the other.

3. Methods

A MIP model is developed. To describe the MIP model, let N be the set of all nodes, C the set of demand nodes where $C \subset N$, F the set of facility nodes where $F \subset N$, and V the set of vehicles. Let d_{ij} represent the distance from node i to node j where $i, j \in N$, B represent a $|C| \times |F|$ covering matrix, and b_{cf} represent the elements of B. Covering matrix B is given and b_{cf} is a binary parameter where $b_{cf} = 1$ if demand node $c \in C$ is covered by facility node $f \in F$ and $b_{cf} = 0$ otherwise.

The decision variables are defined as follows: $x_{ijk} = 1$ if vehicle $k \in V$ travels from node i to node j $\forall\, i, j \in N$ and $x_{ijk} = 0$ otherwise, $y_f = 1$ if facility $f \in F$ is selected and $y_f = 0$ otherwise, $a_{cf} = 1$ if demand node $c \in C$ is assigned to facility $f \in F$ and $a_{cf} = 0$ otherwise, and finally z_{ck} is a continuous variable representing the arrival time of vehicle $k \in V$ at node $c \in C$. z_{ck} variables are mainly used for sub-tour elimination constraints but they also serve as scheduling variables. Below is a description of the full model.

$$\sum_{k}\sum_{i}\sum_{j} x_{ijk} \,.\, d_{ij} + \sum_{f} y_f \tag{1}$$

$$\sum_{k}\sum_{i}\sum_{j} x_{ijk} \,.\, d_{ij} \tag{2}$$

$$\sum_{f} y_f \tag{3}$$

Equations (1) through (3) are the objective functions. Equation (1) represents the sum of total distance and the number of facilities; equation (2) represents the total distance only; and equation (3) represents the number of facilities to open. Below is a description of the constraints.

$$\sum_{i \in N}\sum_{k \in V} x_{ick} = 1 \qquad \forall\, c \in C, i \neq c \tag{4}$$

$$\sum_{c \in C} x_{fck} = y_f \qquad \forall\, k \in V, \forall\, f \in F \tag{5}$$

$$\sum_{c \in C} x_{cfk} = y_f \qquad \forall\, k \in V, \forall\, f \in F \tag{6}$$

$$\sum_{i \in N} x_{ijk} = \sum_{i \in N} x_{jik} \qquad \forall\, k \in V, \forall\, j \in N, i \neq j \tag{7}$$

$$z_{ik} + d_{ij} \leq z_{jk} + M\left(1 - \sum_{k} x_{ijk}\right) \qquad \forall\, i, j \in C, i \neq j \tag{8}$$

$$\sum_{f \in F} b_{cf} \,.\, y_f \geq 1 \qquad \forall\, c \in C \tag{9}$$

$$\sum_{f \in F} a_{cf} = 1 \qquad \forall\, c \in C \tag{10}$$

$$a_{cf} \leq b_{cf} \qquad \forall\, c \in C, \forall\, f \in F \tag{11}$$

$$x_{fck} \leq b_{cf} \qquad \forall\, k \in V, \forall\, f \in F, \forall\, c \in C \tag{12}$$

$$x_{cfk} \leq b_{cf} \qquad \forall\, k \in V, \forall\, f \in F, \forall\, c \in C \tag{13}$$

$$x_{ijk} \leq a_{if} \qquad \forall\, f \in F, \forall\, i, j \in C\ i \neq j, \forall\, k \in V \tag{14}$$

$$x_{ijk} \leq a_{jf} \qquad \forall\, f \in F, \forall\, i, j \in C\ i \neq j, \forall\, k \in V \tag{15}$$

Constraints (4) ensure that each demand point is visited once. Constraints (5) and (6) ensure that all vehicles depart from and return to a depot, respectively, if the depot is selected. Constraints (7) are flow-conservation constraints. Constraints (8) are sub-tour elimination constraints. Constraints (9) are covering constraints ensuring that each demand point is covered by at least one facility. Constraints (10) assign each demand point to a single facility and constraints (11) ensure that the assignment is to a covering facility. Constraints (12) ensure that a vehicle leaves a facility for a demand node covered by that facility, while constraints (13) ensure that a vehicle returns to a facility from a demand node covered by that facility. Finally, constraints (14) and (15) ensure that only demand nodes with the same assigned facility can share a route.

3.1 Methodology

In order to investigate the effect of the various objective functions described above, the following steps were followed:

1. Run the model using objective function (1); observe the total distance and number of selected facilities
2. Run the model using objective function (2); observe the total distance and number of selected facilities
3. Since objective function (3) does not necessarily minimize the total distance, preemptive goal programming was used in order to minimize both the number of facilities and the total distance. However, the goal of minimizing the number of facilities preempts the goal of minimizing the total distance. Preemptive goal programming in this step is described in the following sub-steps:
    a. Run the model using objective function (3); observe the selected facilities
    b. Save the values of facility selection decision variables, i.e. $y_f, f \in F$
    c. Run the model using objective function (2) fixing the values from step (b) as constraints
    d. Observe the total distance
4. Compare the total distance and selected facilities obtained from steps 1, 2 and 3

For the sake of consistency, the number of vehicles was set to be 1 in all experiments. Setting the number of vehicles to be 1 reduces the formulation to be double-indexed. This will still result in multiple routes because of the covering constraints. Covering constraints along with demand nodes assignment variables force the routes to include only demand points that are assigned to the same facility. Having only one vehicle should not be confused with having multiple routes. A single vehicle can operate all the routes; or if multiple vehicles exist, they can be assigned to the different routes based on some criterion such as drivers' proximity to facility nodes. In fact, it is common in practice that drivers do not have to report to/start from a central facility as seen in some sharing economy applications. This variant of LRP can be used in such situations.

4. Data Collection

Problem instances were generated randomly to carry out the methodology described in section 3. The instances were generated according to the following process:

1. Generate points with uniformly random (x, y) coordinates within a 100×100 box
2. Create a square distance matrix containing pair-wise Euclidean distances
3. Randomly assign 33% to 40% of the points to be candidate facilities
4. Create a $0/1$ covering matrix. The number of rows of the matrix is equal to the number of demand nodes while the number of columns is equal to the number of candidate facilities. Each demand node is covered by the n closest candidate facilities where n ranges randomly from 20% to 33% of the candidate facilities

One hundred instances were generated. The total number of nodes (demand nodes + candidate facility nodes) ranges from 33 to 44 nodes in the generated instances.

5. Results and Discussion

First, a comparison between the three objectives in terms of the total distance and number of facilities is presented. Table 1 shows the average changes caused by objective functions 2 and 3 when compared with objective function 1. Full results for all 100 instances are included in appendix A.

Focusing on objective function 2, some minor decreases by 1 to 2 distance units were observed in 8 out of 100 instances, but overall there is no significant difference in terms of the total distance when compared with objective function 1. As for the number of facilities, there is a slight increase by 0.32 facilities on average. The number of facilities increased by 1 in 15 out of 100 instances; increased by 2 in 4 out of 100 instances; and increased by 3 in 3 out of 100 instances.

The noticeable differences were observed in objective function 3. When using objective function 3, the number of facilities decreased, on average, by 3 (- 45% difference) when compared with objective function 1. The number of facilities decreased by 4 in 17 out of 100 instances; decreased by 5 in 9 out of 100 instances; and decreased by 6 in 6 out of 100 instances. The maximum decrease was 8 and it occurred in 1 instance. The total distance, on the other hand, increased by 13.12%. The increase in the total distance was 16.5% or lower in 75 out of 100 instances. The maximum increase in the total distance was 37.5%.

Table 1. Average changes caused by objective functions 2 and 3

	Objective Function (1)	Objective Function (2)	Objective Function (3)
Avg. Change in Total Distance (%)	Baseline	≈ 0	+ 13.12
Avg. Change in Number of Facilities	Baseline	+ 0.32	- 3 (- 45%)

5.1 Pareto Front for Total Distance vs. Number of Facilities

In one of the instances, namely instance ‘n36_f14_8’, the number of facilities decreased by 8 when using objective function 3. The significant reduction in the number of facilities can be deemed positive due to the potential reduction in facility-related costs. However, this reduction in the number of facilities caused a 25.45% increase in total distance. This tradeoff between the number of facilities and total distance calls for further analysis. The results of instance ‘n36_f14_8’ will be used since the tradeoff is pronounced in this particular instance.

Figure 1 shows the tradeoff for instance ‘n36_f14_8’. The total number of nodes in this instance is 36, 14 of which are candidate facility nodes. To obtain the results for this figure, the model was run using objective function (2) multiple times. In each time the number of facilities was fixed as a constraint starting from 3, which is the minimum possible number of facilities in this instance, all the way to 14 facilities. The total distance at 3 facilities is 414 units. On the other hand, the lowest possible total distance is 330 units. Notice how the total distance significantly reduces from 414 to 348 units (within ≈ 5% of optimal minimum distance at 11 facilities) when the number of facilities increases from 3 to 4 (by only 1 facility). This result shows that it is possible to get the benefits of both worlds in LRP's, reduced number of facilities and reduced total distance.

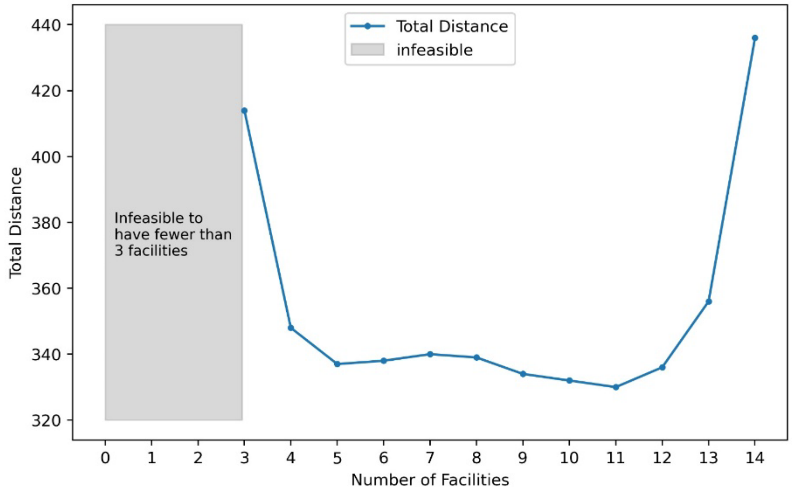

Figure 1. Trade-off between the objective of minimizing total distance and minimizing the number of facilities.

5.2 Large Instance Results

Another set of instances were generated with larger instance sizes. Larger instances were generated for two reasons, for further validation of the results presented above and for deeper investigation of the causes of performance changes, especially changes attributed to objective function (3). The instances were generated according to the same process described in section 4 with two minor changes. The points were generated within a 200×200 box; and the percentage of candidate facility nodes out of the total nodes ranges from 20% to 33%. The total number of nodes (demand nodes + candidate facility nodes) ranges from 100 to 104 nodes in the generated instances. 50 instances were generated.

A single instance of this size can be solved to optimality within a reasonable amount of time, a few minutes in some cases. However, the methodology of this research requires that a single instance is solved 4 times, each time with a different objective function and slight changes in the constraints (see steps 1, 2, 3-a, and 3-c in section 3.1). This adds a significant amount of time to the runtime of one iteration of the methodology if solving to optimality. A single iteration involves completing steps 1 to 3 of the methodology. Therefore, a decision was made to terminate the solver once a solution with 10% optimality gap was found. The average runtime of one iteration was 161.24 seconds. All instances were solved with Gurobi using a computer with Intel Core i5-4300U CPU, 1.9 GHz processor and 4 GB of RAM. Full results for all 50 instances are included in appendix B.

First, a comparison between the three objectives in terms of the total distance and number of facilities is presented. Table 2 shows the average changes caused by objective functions 2 and 3 when compared with objective function 1.

Table 2. Average changes caused by objective functions 2 and 3 (large instances)

	Objective Function (1)	Objective Function (2)	Objective Function (3)
Avg. Change in Total Distance (%)	Baseline	≈ 0	+ 8.47
Avg. Change in Number of Facilities	Baseline	+ 1	- 7.82 (- 66%)

The results of large instances are similar to those of small instances. Minimal changes were observed in the results of objective function (2); pronounced changes were observed in the results of objective function (3).

Minimal changes in the results of objective function (2) is due to the fact that the total distance part in objective function (1) overshadows the sum of facilities part. The large difference in scale of these two sums allowed the total distance part to dominate the objective function, leaving the sum of facilities with no effect on the model. This may not be the case in real life because usually the cost of facilities, which is typically high, is factored into the objective function, tilting the balance towards facility costs. Decision makers should be aware of proneness of objective function (1) to scale of the data.

5.3 Changes in Total Distance and Number of Facilities vs. Number of Candidate Facilities

Results of large instances allowed for deeper analysis of the relationship between the changes observed in the results of objective function (3) and the number of candidate facility nodes. This is because of the higher variation in the number of candidate facilities among large instances.

A relationship between the number of candidate facility nodes in a problem and increase in the total distance and decrease in number of facilities was found to be significant. Figures 2 and 3 show this relationship.

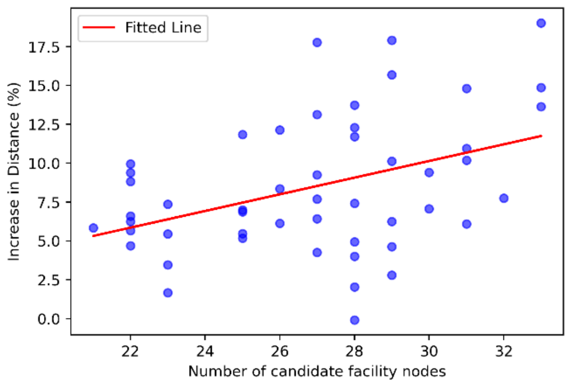

Figure 2. Relationship between the increases in total distance caused by objective function (3) and the number of candidate facility nodes

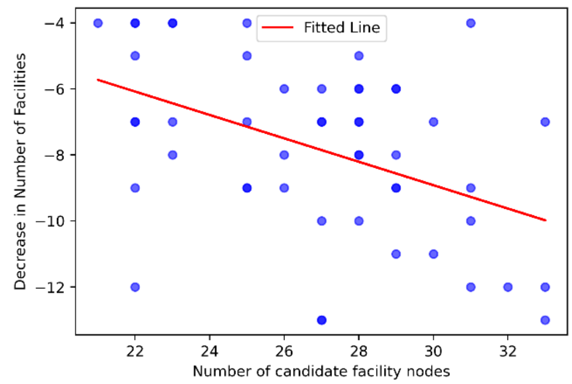

Figure 3. Relationship between the decreases in facilities caused by objective function (3) and the number of candidate facility nodes

As seen in figures 2 and 3, the changes in total distance and number of selected facilities intensify as the number of candidate facility nodes in a problem increases. To validate these results, two hypothesis tests were performed whose null hypotheses were that the slopes of the fitted lines are zeros. Using Wald Test with t-distribution of the test statistic, p-values were found to be 0.003 and 0.001 for fitted lines in Figures 2 and 3, respectively.

6. Conclusion

This study delved into the Location Routing Problem (LRP), focusing on how different single-objective functions can influence outcomes, particularly in facility location and vehicle routing. The aim was to see if moving away from the traditional approach of minimizing the combined facility opening costs and distribution cost would lead to significant changes in outcomes in LRP.

This study contributed, theoretically, by developing a Mixed-Integer Programming (MIP) model for an LRP variant with covering constraints. The developed model brought some interesting findings to light. A key discovery was that when the focus was on minimizing the number of facilities only, it did lead to fewer facilities, but this came with longer travel distances. In some cases, the increase in travel distance is significant. However, using Pareto analysis for investigating this trade-off, it was found that a balance between savings on facilities and transport could be achieved. On the other hand, when the goal shifted to minimizing total travel distance only, the impact was small. This might be because the baseline function, which combines facility and transportation costs, is quite sensitive to the scale of the cost data, total distance potentially overshadowing the number of facilities. These insights highlight the importance of carefully choosing objectives in LRP. The study shows that the selection can significantly change outcomes.

The study did have some limitations, including the inability to solve large problem instances to optimality due to the problem's complexity. While a 10% optimality gap is considered sufficiently small for the results to be reliable, achieving full optimality or a narrower gap might slightly alter the findings. Therefore, one direction for future research should focus on creating an efficient heuristic for the LRP variant examined in this study.

Another research direction involves incorporating cost data to examine how the scale of this data impacts the performance of the objective functions discussed, particularly objective function (2), which showed minimal variation from the baseline case.

References

Adarang, H., Bozorgi-Amiri, A., Khalili-Damghani, K. and Tavakkoli-Moghaddam, R., A robust bi-objective location-routing model for providing emergency medical services. *Journal of Humanitarian Logistics and Supply Chain Management*, *10*(3), pp.285-319, 2020.

Ahlaqqach, M., Benhra, J., Mouatassim, S. and Lamrani, S., Closed loop location routing supply chain network design in the end of life pharmaceutical products. In *Supply Chain Forum: An International Journal* (Vol. 21, No. 2, pp. 79-92), 2020. Taylor & Francis.

Ahmadi-Javid, A., Amiri, E. and Meskar, M., A profit-maximization location-routing-pricing problem: A branch-and-price algorithm. European Journal of Operational Research, 271(3), pp.866-881, 2018.

Chang, K., Zhou, H., Chen, G. and Chen, H., Multiobjective location routing problem considering uncertain data after disasters. *Discrete Dynamics in Nature and Society*, *2017*.

Chen, C., Qiu, R. and Hu, X., The location-routing problem with full truckloads in low-carbon supply chain network designing. *Mathematical Problems in Engineering*, *2018*.

Lopes, R.B., Ferreira, C., Santos, B.S. and Barreto, S., A taxonomical analysis, current methods and objectives on location-routing problems. International Transactions in Operational Research, 20(6), pp.795-822, 2013.

Mara, S.T.W., Kuo, R.J. and Asih, A.M.S., Location-routing problem: a classification of recent research. International Transactions in Operational Research, 28(6), pp.2941-2983, 2021.

Nekooghadirli, N., Tavakkoli-Moghaddam, R., Ghezavati, V.R. and Javanmard, A.S., Solving a new bi-objective location-routing-inventory problem in a distribution network by meta-heuristics. *Computers & Industrial Engineering*, *76*, pp.204-221, 2014.

Rafie-Majd, Z., Pasandideh, S.H.R. and Naderi, B., Modelling and solving the integrated inventory-location-routing problem in a multi-period and multi-perishable product supply chain with uncertainty: Lagrangian relaxation algorithm. *Computers & chemical engineering*, *109*, pp.9-22, 2018.

Rahim, F. and Sepil, C., A location-routing problem in glass recycling. Annals of Operations Research, 223, pp.329-353, 2014.

Rayat, F., Musavi, M. and Bozorgi-Amiri, A., Bi-objective reliable location-inventory-routing problem with partial backordering under disruption risks: A modified AMOSA approach. *Applied Soft Computing*, *59*, pp.622-643, 2017.

Toro, E.M., Franco, J.F., Echeverri, M.G. and Guimarães, F.G., A multi-objective model for the green capacitated location-routing problem considering environmental impact. *Computers & Industrial Engineering*, *110*, pp.114-125, 2017b.

Toro, E., Franco, J., Echeverri, M., Guimarães, F. and Rendón, R., Green open location-routing problem considering economic and environmental costs. *International Journal of Industrial Engineering Computations*, *8*(2), pp.203-216, 2017a.

Vidović, M., Ratković, B., Bjelić, N. and Popović, D., A two-echelon location-routing model for designing recycling logistics networks with profit: MILP and heuristic approach. Expert Systems with Applications, 51, pp.34-48, 2016.

Zajac, S. and Huber, S., Objectives and methods in multi-objective routing problems: a survey and classification scheme. *European journal of operational research*, *290*(1), pp.1-25, 2021.

Appendix A

(Full results of small instance problems)

Instance name format:

n = total nodes, f = number of candidate facility nodes (out of total)

Example: instance 'n33_f12_46' has 33 nodes in total, 12 of which are candidate facility nodes.

Last two numbers in instance name is just a serial number

	Objective Function (1)		Objective Function (2)		Objective Function (3)			Objective Function (1)		Objective Function (2)		Objective Function (3)			Objective Function (1)		Objective Function (2)		Objective Function (3)	
Instance	Total Distance	Number of Facilities	Total Distance	Number of Facilities	Total Distance	Number of Facilities	Instance	Total Distance	Number of Facilities	Total Distance	Number of Facilities	Total Distance	Number of Facilities	Instance	Total Distance	Number of Facilities	Total Distance	Number of Facilities	Total Distance	Number of Facilities
n33_f12_46	360	7	359	8	380	4	n36_f15_20	369	4	369	5	372	3	n40_f15_79	328	6	328	6	377	4
n33_f12_50	343	7	343	7	384	4	n36_f15_56	317	7	317	7	374	4	n40_f15_93	417	7	417	7	461	5
n33_f13_22	383	4	383	4	413	4	n37_f13_39	393	6	393	6	421	4	n40_f16_33	318	10	318	10	357	4
n33_f13_82	307	6	307	6	377	3	n37_f13_52	376	5	376	5	410	3	n40_f16_77	363	5	363	6	396	3
n33_f13_90	329	7	329	7	383	4	n37_f14_30	318	7	318	7	371	4	n40_f16_81	353	8	353	8	427	3
n33_f14_35	349	6	349	6	408	4	n37_f14_37	362	5	362	5	386	4	n40_f16_91	381	6	379	9	418	3
n33_f14_60	330	6	330	6	406	3	n37_f15_4	342	5	342	5	368	4	n40_f16_95	322	10	322	11	389	3
n34_f12_2	394	5	394	6	441	4	n38_f13_41	356	9	356	9	410	4	n41_f14_38	407	6	407	6	440	4
n34_f12_57	388	6	388	6	432	3	n38_f14_53	389	7	388	8	437	3	n41_f14_76	434	5	434	5	456	5
n34_f13_26	305	7	305	9	386	3	n38_f14_66	361	9	361	9	394	4	n41_f15_23	362	7	362	7	397	4
n34_f13_27	388	5	388	5	423	3	n38_f14_83	379	6	379	9	398	4	n41_f15_86	387	6	387	6	419	4
n34_f13_5	389	6	389	6	424	4	n38_f15_16	321	5	321	5	338	4	n41_f16_1	391	9	391	9	426	3
n34_f13_58	380	3	380	3	392	3	n38_f15_80	366	6	366	6	411	4	n41_f16_75	300	8	300	8	385	3
n34_f13_65	367	5	367	7	395	3	n38_f15_88	322	9	322	9	436	4	n41_f16_89	332	6	332	6	391	4
n34_f13_78	323	3	323	3	350	3	n38_f15_9	424	5	424	6	441	4	n41_f16_98	324	8	324	8	367	3
n34_f14_67	332	6	332	6	375	4	n38_f15_99	352	9	352	9	405	4	n41_f17_70	364	5	364	5	395	3
n35_f13_29	374	5	374	5	425	3	n38_f16_64	348	8	348	8	397	4	n41_f17_84	370	4	369	7	429	3
n35_f13_36	340	9	340	9	410	3	n39_f13_51	358	6	358	6	377	3	n41_f17_85	294	6	294	6	340	2
n35_f13_40	384	6	384	6	428	4	n39_f14_17	406	8	406	8	432	4	n41_f17_94	368	5	368	5	390	4
n35_f13_42	322	8	322	8	415	4	n39_f14_3	363	7	362	8	423	3	n42_f14_71	418	5	418	5	445	4
n35_f13_73	340	5	340	5	366	4	n39_f15_45	352	8	352	8	386	4	n42_f15_0	362	6	362	6	410	4
n35_f14_13	312	9	312	9	407	4	n39_f15_54	338	7	338	7	419	4	n42_f15_34	426	11	426	11	487	5
n35_f14_28	363	5	363	5	435	3	n39_f15_63	326	7	326	7	360	5	n42_f15_43	389	9	389	9	430	3
n35_f14_48	362	6	362	7	381	4	n39_f16_69	358	8	358	8	416	3	n42_f15_55	444	8	444	8	479	4
n35_f14_61	384	4	384	4	403	3	n39_f16_96	331	6	331	6	386	3	n42_f16_18	343	7	343	7	399	4
n35_f14_87	297	7	297	7	361	3	n40_f14_10	411	5	411	5	427	4	n42_f16_44	362	7	362	7	448	3
n35_f14_92	327	6	327	6	358	3	n40_f14_11	389	5	389	6	410	3	n42_f16_68	346	5	346	7	374	3
n35_f15_24	368	8	368	8	441	4	n40_f14_12	380	8	380	8	413	4	n42_f17_32	360	6	360	6	419	3
n36_f13_14	389	5	389	6	398	4	n40_f14_31	360	4	360	5	414	4	n42_f17_47	335	9	335	9	418	3
n36_f13_19	370	7	370	7	396	4	n40_f14_49	405	6	405	6	450	4	n42_f17_72	353	6	353	6	380	3
n36_f13_21	341	5	341	5	372	3	n40_f14_62	332	7	332	7	418	4	n42_f17_97	427	7	427	7	450	4
n36_f13_59	396	7	396	8	430	3	n40_f14_74	351	4	350	6	367	3	n42_f18_15	385	7	385	7	415	3
n36_f14_6	314	7	314	8	432	3	n40_f15_25	329	8	329	8	429	4							
n36_f14_8	330	11	330	11	414	3	n40_f15_7	394	7	394	8	449	4							

Appendix B

(Full results of large instance problems)

Instance name format:

n = total nodes, f = number of candidate facility nodes (out of total)

Example: instance 'n100_f22_50' has 100 nodes in total, 22 of which are candidate facility nodes.

Last two numbers in instance name is just a serial number

	Objective Function (1)		Objective Function (2)		Objective Function (3)			Objective Function (1)		Objective Function (2)		Objective Function (3)	
instance	Total Distance	Number of Facilities	Total Distance	Number of Facilities	Total Distance	Number of Facilities	instance	Total Distance	Number of Facilities	Total Distance	Number of Facilities	Total Distance	Number of Facilities
n100_f22_50	1314	11	1286	14	1430	4	n102_f27_91	1230	10	1210	9	1309	4
n100_f22_78	1411	15	1407	13	1504	3	n102_f28_43	1320	10	1273	10	1373	4
n100_f25_24	1237	13	1235	12	1301	4	n102_f28_46	1469	12	1398	13	1499	4
n100_f25_9	1373	9	1391	12	1469	4	n102_f28_55	1231	14	1238	17	1400	4
n100_f26_92	1329	11	1347	11	1440	3	n102_f28_81	1317	11	1315	9	1382	4
n100_f27_99	1233	17	1232	15	1347	4	n102_f28_95	1256	9	1271	15	1403	4
n100_f29_18	1195	14	1191	13	1409	5	n102_f29_90	1255	12	1247	17	1382	4
n100_f32_83	1292	16	1302	16	1392	4	n102_f30_60	1301	12	1276	16	1393	5
n100_f33_73	1150	11	1145	13	1321	4	n102_f31_44	1141	14	1150	19	1310	4
n101_f22_5	1387	13	1377	13	1525	4	n102_f31_70	1307	12	1306	15	1440	3
n101_f22_52	1343	11	1347	12	1427	4	n102_f33_37	1209	16	1220	12	1439	4
n101_f25_85	1308	11	1321	18	1398	4	n103_f22_61	1386	8	1421	11	1451	4
n101_f26_64	1273	10	1281	11	1351	4	n103_f23_19	1338	12	1330	11	1411	4
n101_f27_10	1338	11	1341	12	1395	4	n103_f23_23	1466	8	1467	11	1574	4
n101_f27_34	1211	11	1210	13	1370	4	n103_f25_57	1266	14	1278	12	1416	5
n101_f28_62	1282	10	1269	10	1281	4	n103_f26_42	1336	13	1336	13	1498	4
n101_f28_71	1282	11	1281	10	1377	4	n103_f27_40	1238	14	1247	13	1458	4
n101_f29_28	1288	10	1282	10	1324	4	n103_f29_77	1297	13	1305	15	1357	4
n101_f29_54	1173	15	1180	13	1357	4	n103_f31_53	1298	16	1347	14	1440	4
n101_f29_82	1391	10	1374	15	1478	4	n104_f22_35	1386	9	1399	9	1516	4
n101_f33_14	1181	17	1149	19	1342	4	n104_f23_74	1324	9	1334	8	1346	5
n102_f21_8	1352	8	1408	7	1431	4	n104_f28_7	1319	12	1336	14	1481	4
n102_f22_87	1415	8	1408	10	1495	4	n104_f30_97	1233	15	1246	16	1349	4
n102_f23_33	1355	11	1356	11	1402	4	n104_f31_58	1262	8	1257	14	1339	4
n102_f25_49	1245	8	1225	10	1313	4							
n102_f27_0	1302	17	1315	16	1402	4							