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# A Variable Neighborhood Search Approach to a Multi Order Fulfillment and Consolidation Problem

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## Abstract

This study focuses on the problem of multi-order fulfillment and consolidation in e-Commerce retail (MOFCP). We examine a retailer that operates an online channel and a network of stores or warehouses. In this context, the term" store" is broadly used to refer to an individual physical store or a local warehouse. Customers can place orders containing multiple items through on-line platforms. The goal is to find the locations that fulfill each order and a consolidation point of each order such that the total transportation costs are minimized. To capture the economy of scale of the transportation costs, we model the transportation costs through piece-wise linear cost functions. We propose an integer programming IP formulation enhanced by valid inequalities, as well as a nested Variable Neighborhood Search heuristic. Through preliminary numerical experiments, we show that the heuristic has 1.58% and 2.98% overall average increase in costs compared to the MILP for the case of tight inventory to demand ratio ($k = 1$) and suplus inventory to demand ($k = 1.5$) respectively. The overall average running time is about 2 times faster for tight inventory and about 5 times slower than the IP when inventory is surplus.

## Keywords
Order consolidation, e-commerce, Nested VNS, multi order, heuristic.

## 1. Introduction
The emergence of mobile Internet and e-Commerce has had a major impact on consumer behaviour, leading to a preference for online shopping and providing online retailers with the opportunity to increase sales (Nguyen et al., 2018). This shift has resulted in the global e-Commerce market reaching an estimated 5 trillion US dollars in 2022 and is projected to continue to grow in the coming years (Statista, 2023). This increase in online sales requires eCommerce retailers (e-tailers) to respond effectively to customer demands. To meet this demand, online retailers are expanding their logistic network (Hubner et al., 2016), which has led to more complex fulfilment operations and an¨ increase in logistic costs (Handfield et al., 2013). To reduce these costs, e-tailers are implementing strategies such as reducing the frequency of delivery by the last mile and employing omnichannel approaches.

E-tailers are challenged to maintain high service quality while reducing fulfillment expenses. When customers place orders online, their goal is to design the most cost-effective fulfilment plans, considering operational constraints and customer expectations (Acimovic / Graves, 2017). The time between order placement and delivery in online settings, known as the window of decision opportunity (Torabi et al., 2015), allows e-tailers to optimize the fulfillment process and make informed decisions through different fulfilment strategies such as order consolidation. Consolidation-based fulfilment is a logistics strategy that aims to minimize transportation costs by aggregating multiple orders and their corresponding items into a single, larger shipment, thereby achieving economies of scale in transport (Dror / Hartman, 2007). This approach holds substantial promise for cost reduction and improved service levels, particularly for orders comprising multiple items.

In this study, we focus on the problem of multi-order fulfilment and consolidation in e-Commerce retail (MOFCP). We examine a retailer that operates an online channel and a network of stores or warehouses. In this context, the term" store" is broadly used to refer to an individual physical store, a local warehouse, or a cluster of such facilities in proximity. Customers can place orders containing multiple items through on-line platforms, such as a website. The goal is to find the locations that fulfil each order and a consolidation point of each order such that the total transportation costs are minimized. To capture the economy of scale of the transportation costs, we model the transportation costs through piece-wise linear cost functions. MOFCP generalizes the one order fulfilment and consolidation problem, which was proven to be NP-Hard in Akyuz et al. (2022).

Contribution For MOFCP, we propose a mixed integer programming formulation enhanced by valid inequalities, as well as a nested Variable Neighbourhood Search heuristic. Via numerical experiments, we show that the heuristic gives close to optimal solutions with an average cost increase of 2.28 % and the overall average running time is about 3 times slower.

## 2. Literature Review

This work is closely related to two research streams in the literature: online order fulfilment, which studies from which store to fulfil orders, and order consolidation, which studies how to combine several orders.

An efficient policy for fulfilling orders is crucial in e-commerce retail to reduce costs and satisfy customer needs. Various optimization strategies for fulfilment have been explored. Acimovic / Graves (2015) created a heuristic based on linear programming (LP) aimed at single item orders. This heuristic aims to minimize outbound shipping costs by considering not only immediate costs, but also future expected shipping expenses. They found that their approach could cut outbound shipping costs by 1% compared to a short-sighted heuristic. The complexity of the fulfilment optimization increases when multi-item orders are allowed due to availability of items across locations. Torabi et al. (2015) studied the effects of combined decisions on both fulfilment and transshipment for multi-item orders, proposing a Mixed Integer Programming (MIP) and a Bender decomposition to tackle this problem. Hubner¨ et al. (2016) described various fulfilment and distribution options available to fulfil online orders. Ishfaq / Bajwa (2019) argue that utilizing retail stores for order fulfilment could be more advantageous than relying solely on direct warehouse shipments. Additional fulfilment strategies for order fulfilment include the simultaneous optimization of order allocation and routing, as demonstrated by Li et al. (2019). They addressed this problem using an adaptive large neighborhood search combined with a greedy heuristic. Furthermore, Jiang et al. (2022) developed an integrated approach for order fulfillment and vehicle routing for online retailers, considering specific delivery time windows and synchronization constraints, and solved it using an adaptive large neighbourhood search heuristic.

The subject of shipment consolidation and its potential for cost reduction has been a significant focus of logistics research. Studies in this area have focused predominantly on coordinating replenishment and delivery decisions (Nguyen et al., 2014), encompassing the management of material flows from one or multiple vendors to single or multiple retailers. Such consolidation typically involves a small number of customers with large freight per order, and they use long-distance vehicles for transportation (Capar, 2013). The logistics sector has thoroughly investigated the cost benefits of shipment consolidation, focusing on the trade-off between fixed shipping costs and the costs associated with inventory storage. Zhang et al. (2019) have empirically demonstrated that consolidating orders can lead to cost savings, as opposed to splitting them. Wei et al. (2021) found that shipment consolidation is beneficial for retailers operating in multiple channels. Shan / Tian (2022) tackled a combined optimization challenge involving both order splitting and consolidation in the context of online supermarket retail, using an enhanced Bender decomposition method.

In our study, we build upon the work of Akyuz et al. (2022), expanding their model to address the issue of multi-item¨ orders while incorporating a capacity limit for inventory available at each store. This addition introduces a new level of complexity to the problem. To address this, we propose a nested variable neighborhood search (VNS) strategy that utilizes linear programming (LP) relaxation solution to find an initial solution.

# 3. Method

## 3.1. Mathematical model

The Multi-Order Fulfilment Consolidation Problem (MOFCP) can be formulated as follows. Consider a graph $G = (R,A)$, where the set $R$ of nodes represents the stores, and the set $A$ represents the transshipment arcs between the stores in $R$. The retailer has a set of $I$ items stored at stores in $R$. Each item $i \in I$ have a weight $a_i$. At each store $r \in R$, an inventory $C_i^r$ of item $i$ is held. Note that some stores may not hold any inventory ($C_i^r = 0$) and only serve as possible consolidation points. An online order that contains multiple items can be fulfilled from different stores. The items in each order should be consolidated in one of the stores, not necessarily in a store that fulfils those items in the order. Given a set $O$ of orders containing a set of distinct items in $I$, one has to find a fulfilment plan and a consolidation point for each order such that the total transportation and consolidation costs are minimized. We assume that the transportation costs between two stores $r$ and $q$ depend on the weight transported on the arc $(r,q)$ and are given by a piecewise-linear, non-decreasing concave transportation function $f_{rq}(.)$. The assumption that the cost function is concave and non-decreasing models that the incremental costs are lower for higher weights. Moreover, for each order $o$, $c_{ro}$ is the cost to deliver order $o$ from store $r \in R$ to the customer.

Next, we present an integer program formulation for MOFCP that extends the formulation for one order presented in Akyuz et al. (2022). To formulate the problem as an MIP, we introduce the following variables:

$$x_r^{oi} = \begin{cases} 1 \text{ if item } i \text{ of order } o \text{ is consolidated at store } r \in R_i \text{ from inventory at } r \\ 0 \; otherwise \end{cases}$$

$$z_{rq}^{oi} = \begin{cases} 1 \text{ if item } i \text{ of order } o \text{ is transshipped on arc } (r,q) \\ 0 \; otherwise \end{cases}$$

$$y_{qo} = \begin{cases} 1 \text{ if store } r \text{ is chosen as the consolidation point for an order} \\ 0 \; otherwise \end{cases}$$

The MOFCP can be formulated as an Integer Program as follows:

$$(P) \; minimize \; f_{rq}\Big(\sum_{o \in O} \sum_{i \in I} a_i x_{rq}^{oi}\Big) + \sum_{r \in R} c_{qo} y_{qo} \qquad (1)$$

$$s.t. \sum_{r \in R} y_{ro} = 1 \qquad o \in O \qquad (2)$$

$$\sum_{(q,r) \in A: q \in R_i} z_{rq}^{oi} = y_{ro} \qquad o \in O, i \in I, r \notin R_i \qquad (3)$$

$$x_r^{oi} + \sum_{(q,r) \in A: q \in R_i} z_{rq}^{oi} = y_{ro} \qquad o \in O, i \in I, r \in R_i \qquad (4)$$

$$\sum_{o \in O} x_r^{oi} + \sum_{o \in O} \sum_{q:(r,q) \in A} z_{rq}^{oi} \le C_i^r \qquad i \in I, r \in R_i \qquad (5)$$

$$x_r^{oi} \in \{0,1\} \qquad o \in O, i \in I, r \in R_i \qquad (6)$$

$$z_r^{oi} \in \{0,1\} \qquad o \in O, i \in I, r \in R_i, q \in R \qquad (7)$$

$$y_{ro} \in \{0,1\} \qquad r \in R, o \in O \qquad (8)$$

The objective function consists of the cost of transhipping items between stores and the cost of delivery from the consolidation points. Recall that the transshipment costs on each arc $(r, q)$ are given by the function $f_{rq}(w_{rq})$, where $w_{rq}$ is the weight transhipped on arc $(r, q)$. In our problem, $w_{rq} = \sum_{o\in O}\sum_{i\in I} a_i z_{rq}^{oi}$. Constraints (2) ensure that a single store is selected as consolidation for each order. Constraints (3) guarantee that if *r* is a consolidation point for order *o*, an item $i \in o$ must be fulfilled from another store $q$: $(q, r) \in A$. At the same time, if *r* is not a consolidation point for *o*, i.e., $y_{ro} = 0$, no items in *o* are transhipped to $r$ $\left(z_{rq}^{oi} = 0, \forall\, (r, q) \in A\right)$ $and$ $i \in I)$. Constraints (4) ensure that if $r \in R_i$ for some item $i \in I$, and *r* is the consolidation point for *o*, i.e., $y_{ro} = 1$, then an item *i* may be chosen either from the inventory at *r* or from trans-shipments to *r*. If *r* is not the consolidation point for *o*, $y_{ro} = 0$, no item *i* is chosen for consolidation from the inventory at *r*, and no item is transhipped to *r*. Note, however, that this does not restrict items *i* to be transhipped from store *r*, on arcs $z_{rq}^{oi}$. Finally, constraint (5) ensures that for each item *i*, the amount consolidated at *r* and shipped from *r* does not exceed the inventory.

### 3.2 Linearizing the cost function

In order to linearize the objective function, we follow the approach employed in Akyuz et al. (2022). Assume that for each arc $(r, q) \in A$, the weight levels are given by $b_{rq}^0 < b_{rq}^1 < \cdots < b_{rq}^L$. We assume that $b_{rq}^0 = 0$ and $b_{rq}^L$ is equal to the maximum possible flow on arc $(r, q)$ (e.g., the total weight of all items that can traverse arc $(r, q)$. The cost of sending a unit flow on arc $(r, q)$ at level $l$ is $c_{rq}^l$. Clearly, when the weight $w_{rq}^l \in [b_{rq}^{l-1}, b_{rq}^l]$, the total cost will accumulate over the lower weight levels. The fixed cost of sending the flow over the arc $(r, q)$ at the level *l* will be denoted by $K_{rq}^l$ and defined by

$$K_{rq}^l = \begin{cases} K_{rq}^{l-1} + \left(b_{rq}^l - b_{rq}^{l-2}\right)c_{rq}^{l-1} & l > 1 \\ \eta & otherwise \end{cases} \tag{9}$$

where *η* is the fixed cost of sending for the first weight level.

To model the function $f_{rq}$, we introduce the binary variable $v_{rq}^l$ to indicate that the weight level *l* is chosen for the arc $(r, q)$ and the continuous variable $w_{rq}^l$ to indicate the flow on the arc $(r, q)$.

For each arc $(r, q) \in A$, the function $f_{rq}$ is now given by:

$$f_{rq}\left(\sum_{l=0}^{L} w_{rq}\right) = \sum_{l=0}^{L}(K_{rq}^l v_{rq}^l + c_{rq}^l w_{rq}^l) \tag{10}$$

The objective function of $(P)$ becomes.

$$min \sum_{(r,q)\in A}\sum_{l\in L}\left(K_{rq}^l v_{rq}^l + c_{rq}^l w_{rq}^l\right) + \sum_{q\in R}\sum_{o\in O} c_{qo} y_{qo} \tag{11}$$

In order to model the dependence of the cost on the weight transhipped in each arc and the choice of a weight level, the following constraints are added to (*P*):

$$\sum_{o\in O}\sum_{i\in I} a_i z_{rq}^{oi} = w_{rq} \qquad (r, q) \in A \tag{12}$$

$$w_{rq} \le b_{rq}^l v_{rq}^l \qquad l \in L, (r, q) \in A \tag{13}$$

$$w_{rq} \ge b_{rq}^{l-1} v_{rq}^l \qquad l \in L, (r, q) \in A \tag{14}$$

$$\sum_{l} v_{rq}^l \le 1 \qquad (r, q) \in A \tag{15}$$

$$w_{rq} \ge 0 \qquad (r, q) \in A \tag{16}$$

$$v_{rq}^l \in \{0,1\} \qquad l \in L, (r, q) \in A \tag{17}$$

Constraints (12) set the total weight carried on an arc equal to the continuous auxiliary variable $w_{rq}$. Constraints (13) -( 14) ensure that $w_{rq}$ is placed in the corresponding weight level. To ensure that at most one range is selected for each arc, constraints (15) are used. If no range is selected for the arc $(r, q)$, this corresponds to the situation in which no items are carried on this arc. Finally, constraints (16) and (17) captures the range of *v* and *w*.

Remark The following inequalities are valid for *P*:

$$\sum_{l\in L}\sum_{q\in R\setminus\{r\}} v_{rq}^l \le |O_r| - \sum_{o\in O} y_{ro} \quad r \in R$$

where $O_r$ is the set of orders that have at least one item in common with $r$. This inequality follows from the fact that, in an optimal solution, each store $r$ is the consolidation point of $\sum_{o\in O} y_{ro}$ items and at most $|O_r| - y_{ro}$ are transhipped from $r$ to other stores.
Note that even for one single order and one given consolidation point, selecting the optimal set of stores to fulfill the order reduces to a set-cover problem, which is also NP-Hard (Akyuz et al. (2022)). Furthermore, numerical experiments have indicated that obtaining optimal solutions of ($P$) for large instances is computationally intractable. Therefore, we propose a Variable Neighborhood Search (VNS) algorithm that is faster to implement and produces reasonable solutions.

# 4. Solution Method

## 4.1 Nested Variable Neighborhood Search Heuristic (NVNS)

The general framework of the Nested Variable Neighborhood Search Heuristic is presented in Algorithm 1. The algorithm sequentially changes the consolidation points and uses $l_{max} = 3$ neighborhood structures to optimize the stores for a given set of consolidation points. Each neighborhood focuses on different aspects of the solution.
The algorithm starts with an initial feasible solution $s = (x, y, z)$ containing the consolidation points $CO$ for the set of orders $O$ and the set of stores $SO$ to serve these orders.

To find an initial solution (Line 1), we solve the linear programming relaxation (LP) of $P$. Let $(\acute{x}, \acute{y}, \acute{z})$ be the optimal solution to this (LP). For each order $o$, we choose the store $r$ that achieves max $\{\acute{y}_{ro}\}$ as the consolidation point. Let $CO$ be the set of consolidation points obtained. For each order $o$, let $I_o$ be the set of items in $o$,$I_{or}$ be the set of items in $o$, available at store $r$ and $I_o^{nd}$ be the set of items in $o$ for which no delivery store has been decided. Initially,

$I_o^{nd} = I_o$. For each order $o$, we assume that the corresponding consolidation point will deliver any available items that are contained in $I_o$. While $I_o^{nd} \neq \emptyset$, we proceed as follows. For each store $r \in R$, we calculate the costs of transporting $I_{o,r} \cap I_o^{nd}$ over the arc $(r, q)$, while taking into account the previous transport decisions on $(r, q)$. The store $r^*$ with minimum average cost on $(r, q)$ is added to the set of stores that deliver items in $o$. The items delivered by $r^*$ are excluded from $I_o^{nd}$, and the weight and number of items on arc $(r, q)$ are updated.

After the initial solution is constructed, at each iteration, the algorithm explores different sets of consolidation points until certain termination criteria are met (Lines 5-28 in Algorithm 1). For each new set of consolidation points, the set of delivery stores are improved by exploring several neighborhoods (Lines 12-20 in Algorithm 1). If no improvement is obtained in any of the neighborhoods, a Shake procedure is employed (Lines 21-26 in Algorithm 1). Next, we explain in more detail the components of this algorithm. Throughout the algorithm, $f(.)$ will denote the objective value.
New sets of consolidation points are constructed as follows. For each order $o$, let $CO_o$ be the consolidation point for $o$ in the current solution. Every store $r \in R \setminus \{CO_o\}$ is considered as a possible consolidation point for $o$. The stores are sorted in decreasing order of the number of available inventory and in case of ties, the number of overlapping items with $o$, is considered. The items in $o$ that were initially delivered to $CO_o$ will be delivered to $r$ in the new solution. The stores that deliver items to $r$ are chosen in a greedy manner: the closest stores to $r$ where items in $o$ are available will be chosen for delivery (Lines 9-10 in Algorithm 1).

For each new set of consolidation points, the algorithm tries to improve the delivery stores by exploring three neighborhoods: Store Swap, Add Store, and Item Exchange Neighborhood.

## 4.2 Store Swap Neighborhood

For a feasible solution $s$, the neighborhood $N_1(s)$ explores the solutions that can be generated from $s$ by exchanging the stores that deliver an item $i$ between two different orders. The consolidation points of the respective orders are not considered for exchanges. Clearly, if all the orders contain disjoint items, this neighborhood is empty. The algorithm for the construction of $N_1(s)$ is given in Algorithm 2.

Algorithm 1 NVNS (*s*, N))

```
Construct an initial solution s
CO: set of consolidation points in s
No improvement ← True
iter ← 1
while No Improvement OR iter < maxiter do
    for o ∈ O do
        CO_o : consolidation point of o
        Select r ∈ R \ CO_o
        Construct s_1 by assigning r as the new consolidation point of o
        s' ← Greedy store selection(s_1, r)
        l ← 1
        while 1 ≤ l ≤ l_max do
            s'' ← First Improvement(s', N_l)
            if f(s'') < f(s) then
                s ← s''
                l ← 2
            else
                l ← l + 1
                s ← Shake(s'')
            end if
        end while
        if f(s') < f(s) then
            s ← s'
            No Improvement← False
        else
            s ← Shake(s')
        end if
        iter ← iter + 1
    end for
end while
return s
```

### 4.3 Add Store Neighborhood

The neighborhood $N_2(s)$ of a feasible solution $s$ is found by adding a new store to serve items consolidated at a specific consolidation point. $N_2(s)$ contains solutions constructed as follows. For a consolidation point $c$, let $O_c$ be the set of orders consolidated at $c$, $R(O_c)$ the set of stores that have items contained in $O_c$ and $R_c$ the set of stores used by orders in $O_c$. A new solution is obtained by adding to $R_c$ a store $r \in R(O_c) \setminus R_c$. All the items in $O_c$ available at the new store will be delivered from the new store and items that cannot be delivered form new store, are sent from other stores close to the consolidation. The inventory at *r,* the added stores and the stores that no longer deliver items are updated accordingly.

When considering the first improvement in the neighborhood of add stores, for a consolidation point $c$, stores are considered in decreasing order of their distance to $c$.

Algorithm 2 Store swap neighborhood $N_1(s)$

Input:
    Incumbent solution $s$
    $r_{i,o}(s)$: store delivering item $i$ in order $o$ in solution $s$, for $i \in I, o \in O$
    $CO(o)$: consolidation point of order $o$ for $o \in O$
Output: $N_3(s)$
for $(o_1, o_2) \in O \times O$ do
    for $i \in o_1 \cap o_2$ do
        $s' \leftarrow s$
        if $r_{i,o1} \neq r_{i,o2}$ and $r_{i,o1} \neq CO(o1)$ and $r_{i,o2} \neq CO(o2)$ then
            $r_{i,o1}(s') \leftarrow r_{i,o2}(s)$
            $r_{i,o2}(s') \leftarrow r_{i,o1}(s)$
            $N_3(s) \leftarrow N_3(s) \cup \{s'\}$
        end if
    end for
end for

## 4.4 Item Exchange Neighborhood

The role of this neighborhood is to improve the delivery assignment among stores selected to deliver to the same consolidation centre. For a feasible solution $s$, the neighborhood $N_3(s)$ contains solutions $s'$ obtained from $s$ by changing the store from which an item is delivered to another store that delivers to the same consolidation centre. More precisely, let $c$ be a consolidation centre, $R_c$ be the set of stores that deliver items to $c$ in the present solution, and $I_c$ the set of items delivered at $c$, which are contained in more than two stores in $R_c$ . A new solution is obtained by changing the store that delivers an item $i \in I_c$ to another store in $R_c$ that contains that item.

## 4.5 Shake

The shake operator is utilized to introduce diversity in the search process and prevent being trapped in local optima. After exploring the neighborhood sequentially and not finding a better solution, we applied the shake operator. This transforms the NVNS into a multi-start search process. By reassigning a subset of orders from the given set to randomly selected stores, the shake operator generates a new solution $s$. We implemented two shake operators. One for the outer loop that randomly select new consolidation point for randomly selected orders. The shake for the inner loop select randomly new set of stores to serve randomly selected orders. Specifically, the set of stores currently serving subset $O_{so}$ is denoted as $R(O_{so})$, and the set of potential stores that can serve $O_{so}$ is denoted as $S(O_{so})$. A new solution is obtained by replacing the stores in $R(O_{so})$ with randomly selected stores from $S(O_{so})$.

# 5. Numerical experiments

In the numerical experiments, we consider a set of stores $R$ with $|R| \in \{15, 25, 40\}$. Each location may store items or only serve as a consolidation point. The stores contain a set of items $I$ with $|I| \in \{3,5\}$. The total number of orders considered $O$ varies in $|O| \in \{3,5\}$. The number of items in each order is randomly generated in $\{1, \ldots, |I|\}$. Each item is allocated to a random set of $\frac{|R|}{2}$ stores. The available inventory at each chosen store is generated as follows; let $d_i$ be the total demand for each item $i$ in the set $O$. For each $i \in I$, we install the inventory $C_i = kd_i$ on the network, where $k$ is a factor that controls the inventory-to-demand ratio. Then we choose $p = \min\{kd_i, |R_i|\}$ locations/stores with item $i$ at random from the stores that contain item $i$ and install $\{C_i^r = \frac{kd_i}{|R_i|}\}$ of item $i$ in each $r$ in these locations. The remaining amount of $i$ is assigned to the subset of the selected stores according to the initial installation sequence. The values of $k$ are set from $\{1, 1.5\}$. The weight $a_i$ of each item $i \in I$ is uniformly generated between [2,15]. The locations from where orders are generated are randomly located in a 2D square coordinate of $[0, 1000]^2$.

In total, we ran 24 combinations of parameters, and, for each combination, we generated 5 random instances. All experiments were run on an Intel(R) Xeon® 2.40GHz CPU using Gurobi solver version 11.0.0. The time limit for the MIP was set at 30 min.

## 6. Numerical Results

In the subsequent analysis, we examine the results obtained after solving models *P* and *NV NS* for different scenarios that cover varying numbers of stores, orders, and items configurations. Firstly, we examine the effect of the inventory to demand ratio $(k)$ on the fulfillment and consolidation costs as shown in Figure 1. The results shows that the average cost is higher when $k = 1$ as compared to $k = 1.5$. The relative percentage increase on average is 71% for all the stores, orders and items combinations. The high cost for $k = 1$ is due to the limited options to consolidate items and the high chances of fulfilling them from a farther store which will result in high transshipment cost.

Figure 2. presents the result of the relative percentage increase in the average costs obtained by the NVNS algorithm. This percentage increase is calculated using the expression $\frac{Obj_{NVNS}-Obj_P}{Obj_P}$, where $Obj_{NVNS}$ and $Obj_P$ represent the objective values obtained from the models *NVNS* and *P*, respectively. These results are presented for various levels of the capacity ratio $k$ as discussed in the preceding section. Furthermore, we present the ratio of the average computational times for *NVNS* relative to *P*.

The *NVNS* shows an overall increase in the average cost of 1.56% compared to *P* for $k = 1$. As the inventory to demand ratio increases ($k = 1.5$) the relative percentage increase in cost compared to *P* increases to 2.98%. Furthermore, the average running time of *NVNS*, relative to *P*, is still a bit slower for the case of $k = 1.5$ which takes about 5 times more than $P$. On the other hand, the NVNS is approximately 2 times faster than the IP for $k = 1$.

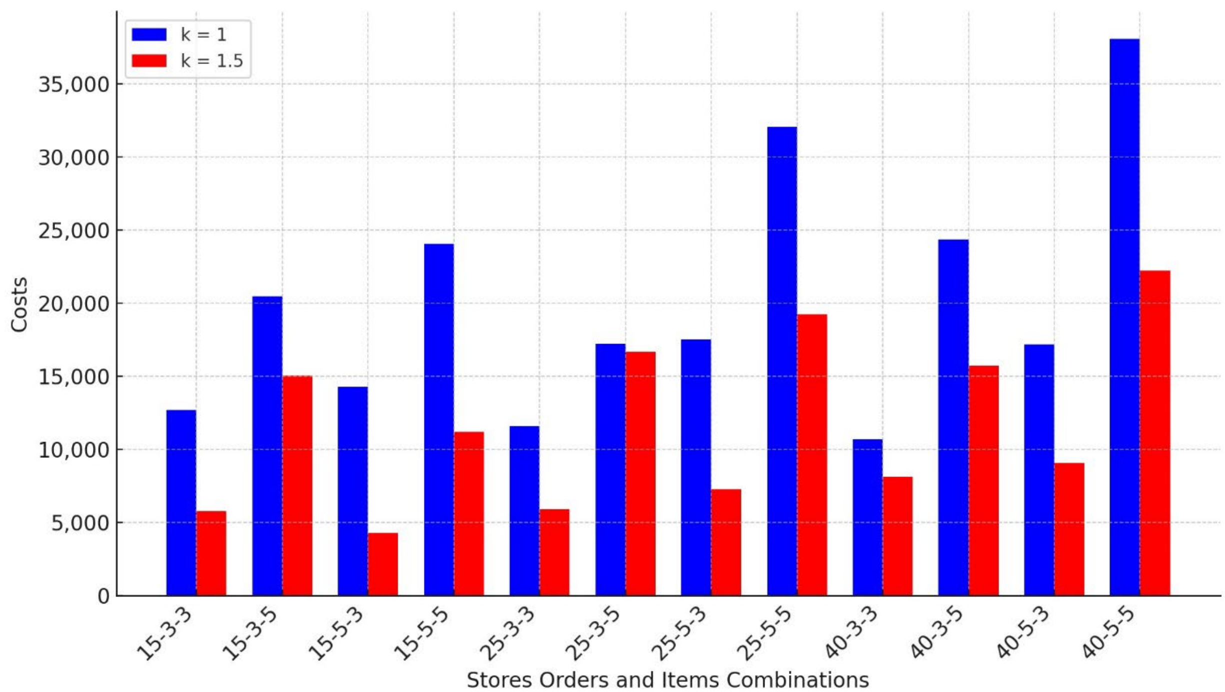


Figure 1. Fulfilment and Consolidation Costs for Various Combinations of Nr. of Stores, Nr. Orders and Nr. Items for inventory to demand ratio of $k = 1$ and $k = 1.5$.

The reasons for the decrease in *NVNS* performance as more inventory is introduced into the network may be due to several factors, one of which might be the complexity of the solution space. *NVNS*, being a heuristic, relies on local search and neighborhood structures to find improved solutions. In the scenario where the capacity constraint is tight ($k = 1$), the problem becomes the effective use of limited resources, which might lead to straightforward decisions in the *NVNS* search process. On the other hand, when there is more inventory ($k = 1.5$) it is more about optimizing among many feasible options, which is more challenging and requires more robust neighborhood structures.

The Nested Variable Neighborhood Search (NVNS) algorithm gives an overall average relative cost increase of 1.88% for 15 stores, 2.64% for 25 stores, and 2.30% for 40 stores. It shows a decline in performance with more orders: the average relative cost increase is 1.36% for 3 orders and 3.19% for 5 orders. Additionally, the performance varies with the number of items, exhibiting an average relative increase of 1.43% for 3 items and 3.39% for 5 items. The overall running time of the NVNS is on average 3 times slower indicating that the implementation still requires further improvement.

## 7. Conclusion

This study proposes a mat-heuristics approach to solve the fulfillment and consolidation problem of a retailer with an online platform and a network of physical stores. The retailer needs to decide for a given set of orders the consolidation points and the stores serve the orders. We formulated the problem as a capacitated multi-order mixed-integer linear programming (MILP) with piecewise linear costs. We propose a nested variable neighborhood search approach to solve the problem and compare the result with the solution of a MILP solver. The performance of the NVNS was examined for different values of the parameter that controls the inventory-to-demand ratio. Compared to the MILP solver, the NVNS gave a relative percentage increase in cost as 1.56% and 2.98% for the inventory to demand ratio $k$ = 1 and 1.5, respectively. Overall, the NVNS shows an average increase of 2.28% in cost compared to the IP. While The performance of the NVNS is within acceptable limit in terms of it relative percentage increase in cost to the optimal cost of the IP, its computational time is still a bit higher than the IP. Since this study is a work in progress, we hope to refine the implementation and make the algorithm robust for large scale implementation. In this study, we have considered the case of more items that overlap between stores in the network. We hope to consider scenarios where there is limited overlap among stores.

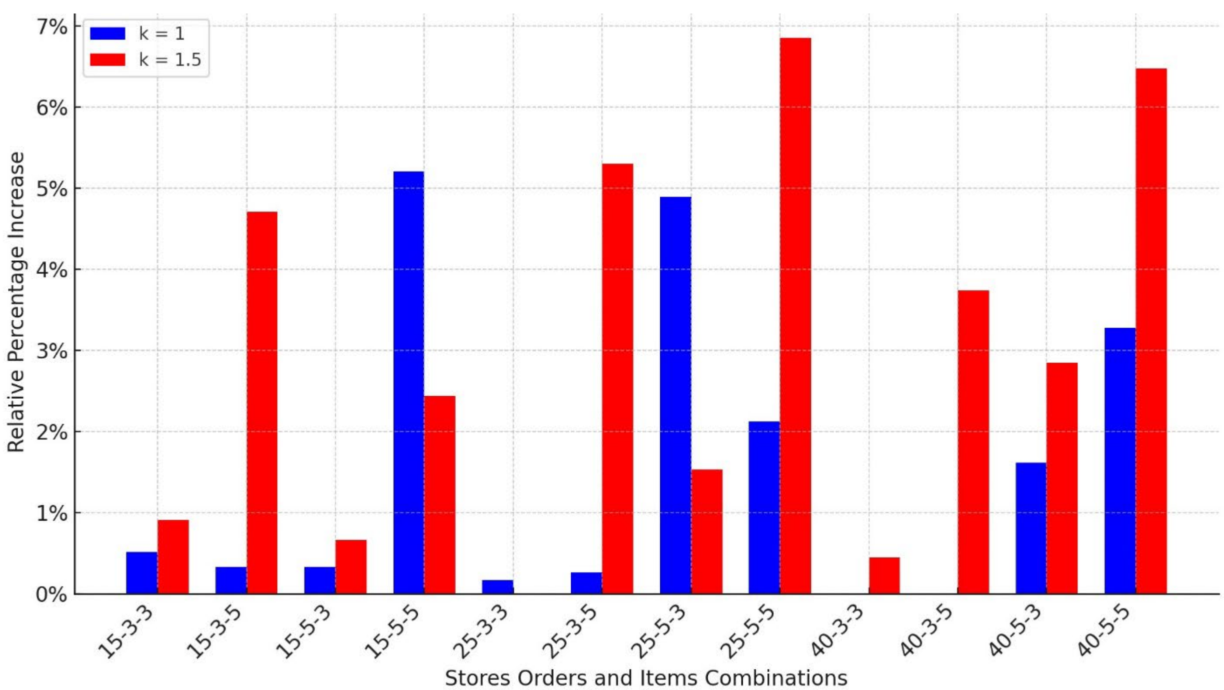


Figure 2. The Relative Percentage Difference of NVNS to P For Various Combinations of Nr. of Stores, Nr. Orders and Nr. Items for inventory to demand ratio of $k$ = 1 and $k = 1.5$.

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