# A Time-Discretized Linear Integer Programming Model for Vacation Route Planning 

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#### Abstract

Budget-conscious tourists are travellers electing to plan their vacations by taking advantage of pricing variability to minimize the total cost of their desired multi-city vacation. This paper presents the formulation and analysis of a timediscretized linear integer programming (LIP) model to optimize total travel costs for travellers vacationing between major European cities by considering dynamic flight costs and accommodation expenses. The proposed timediscretized network flow model presented is an extension of classical Shortest-Route-Problems (SRP) and is solved using Excel's SIMPLEX LP algorithm. Flight data collected from the online travel agency Kiwi.com between May $1^{\text {st }}, 2022$, and May $30^{\text {th }}$ was found to show significant daily price fluctuation, whereas accommodation prices collected from Airbnb.com between May $1^{\text {st }}$, 2023, and May $13^{\text {th }}, 2023$, remained stable. This paper includes the model formulation, assumptions, and cost analysis of varying travel instances by rotating the origin city and altering the stay duration at each destination. Three solutions, the generous, greedy, and cost-minimized solutions, were calculated for each travel trip instance. By optimizing the route, the cost can be reduced by upwards of $20.3 \%$ and $6.7 \%$ relative to the generous and greedy solutions. The model evaluated the impact of varying staying durations on the optimal travel order and was found to be very robust in terms of stay durations such that the optimized cost remained constant. The results of this paper have implications for the global travel industry by lowering the barrier of entry for budget-friendly consumers.


## Keywords

Routing Optimization, Linear Integer Programming (LIP), Cost Minimization, Shortest Route Problem (SRP), Network Flow Optimization

## 1. Introduction

The aviation industry itself is one of the largest global markets accounting for upwards of 2.7 trillion in global GDP and 65 million jobs (Asquith 2020). It is a huge global employer and facilitates the ever-growing demand for transnational and trans-continental travel. Of the uses for air travel, tourism has been its lifeblood contributing direct and indirectly to over 9 trillion of global GDP (Constantin et al. 2020). During the COVID-19 pandemic, the airline industry was particularly affected with grounded fleets and travel restrictions halting consumer spending in the industry. Following the reversal of these restrictions where airlines could begin to fly under normal business
operations, labour market shortages, rising fuel costs, rising interest rates and high demand sent the industry into soaring inflation which priced many lower-budget consumers out. In November 2022 a Canadian poll with over 1000 people surveyed showed that $65 \%$ of Canadians cited that higher-than-normal accommodation and ticket fares were holding them back from travel plans (D'Andrea 2022). Many who previously could afford to travel were now unable to which further hampered the industry's economic recovery. These factors together emphasize the need for a consumer evaluation of cost-minimized travel itineraries which incentivize cost-effective travel supporting the aviation and tourism industries in normal business operations, crisis, and the subsequent recovery.

Tourists can opt to use travel agents for their travel itinerary, but these services incur additional costs and use generalpurpose travel plans which do not address unique tourists' preferences or take advantage of pricing dynamics. Ding et al. (2021) states that tourism service providers must be able to generate cost-effective customized travel itineraries to appeal to tourists with differing preferences to remain competitive. This further stresses the need for a systematic yet customizable framework for generating satisfactory travel itineraries for tourists while considering budget constraints.

The time-discretized linear integer programming (LIP) model developed here solves a variation of the Shortest Route Problem (SRP) directed towards budget-conscious tourists wanting to minimize the cost of a multi-city vacation which concludes with a return to the origin. Within this paper, the proposed LIP model will be evaluated to find the optimal travel itinerary for tourists choosing to travel between five major European cities (Frankfurt, Rome, Paris, London, and Barcelona) where the origin city rotates within this list for each instance evaluated (Figure 1). These cities were chosen based on their consistent inflow and outflow of flights among themselves. This work is original in that it addresses travel from the consumer's perspective rather than operationally and utilizes a distinct integer discretized time framework in which each travel day adds a layer of linked nodal connections building upon traditionally static SRP. The following illustration shows the bi-directional network flow diagram between nodes described in this paper with the origin $(\mathrm{O})$ and the 4 interconnecting destinations (A, B, C, D) for any day (T).


Figure 2. 5-node network flow diagram
The proposed model can be extended to a more generalized framework but is constrained within this paper to travellers planning on visiting four predetermined European cities from an origin within a 13-day travel period (May 2023) such that they specify how many consecutive days they will remain in a location. The optimal solution predicts the trip itinerary to minimize the total trip cost. This trip cost is composed of both the variable daily airfare as well as the variate accommodation costs of remaining overnight in a city. The travellers are constrained to start and return at the end of the trip to the origin city, visiting each destination city only once and cannot stay longer than planned. As air travel is the only transportation method considered, the differences in commute time of flights are not considered. This model assumption limits the cost-benefit consideration of using alternatives such as trains, automobiles, buses and ferries. This assumption becomes of increasing concern for destinations closer to each other such that these alternative transportation methods may be preferred by consumers. Resulting from this, the instances evaluated within
this paper are all destinations far enough away that the railway, bus and automobiles are not recommended modes of travel.

### 1.1 Objectives

The motivation behind this paper is to determine the optimized trip itinerary for tourists visiting predetermined destinations within a specified timeframe for a decided-upon number of days. The following plot highlights the clear motivation for this investigation as it shows the real dynamic flight costs between London (Lon), Barcelona (Bar), Frankfurt (Fra) and Rome (Rom) between May $1^{\text {st }}, 2023$ and May $30^{\text {th }}, 2023$ as seen on the online travel agency Kiwi.com as on February 19 ${ }^{\text {th }}$, 2023(Figure 2).


Figure 2. Inter-city airline fairs
From the above plot, it is clear to see where the optimization problem arises. Flight fares such as Frankfurt to Barcelona have minor fluctuations as they remain consistent day-to-day whereas other flights such as London to Barcelona experience much higher daily fluctuations from a low of $\$ 79$ to a high of $\$ 391$. The optimal solution will opt to travel on less costly days rather than more expensive ones.

Similarly, Airbnb (AirBnb.com) accommodation data is evaluated from May $1^{\text {st }}, 2023$ to May $13^{\text {th }}, 2023$ as of February $19^{\text {th }}, 2023$. The following plot shows the changes in daily accommodation costs for Barcelona, Paris, Rome, and London (Figure 3).


Figure 3. City accommodation prices
Although there are slight variations in the daily cost of accommodation, these not nearly as varied as those seem for flight data. Therefore, the optimal solution will place more emphasis on minimizing flight costs as this has the highest potential for savings. To ensure data collected for airfare and accommodation remained consistent, the cheapest direct flights and rentals with similar characteristics such as 2-bedroom, laundry, and within the city centre etc. were chosen.

## 2. Literature Review

The classical problem of the Travelling Salesman (TS) is a ubiquitous variation of the Shortest Route Problem (SRP). This is a NP-Hard network flow optimization problem attempts to minimize the path cost between a source node and auxiliary destination nodes (Davendra et al. 2010). The origins of such problems are difficult to determine precisely but based on its utility in minimizing distances between provisions such as food, water, medicine and family, human societies have dealt with TS problems for a long time. Before the advent of rapid computing technologies, researchers proposed heuristic approaches which provided sub-optimal routing (Schriver 2012). In the 1950s SRP became of particular interest for telephone operators routing long-distance calls in the USA. During this time three well-known algorithms arose, the 'Bellman-Ford', 'Dijkstra' and 'Floyds' algorithms but these relied on solving iterative distance function (Schriver 2012). In later years, Lagrangian Linear Integer Programming (LIP) methods were developed as an alternative to inaccurate heuristic methods. Simple LIP TS models can be solved by minimizing the cost function under a nodal balance between the source, destination, and all other transitory nodes. Overall, LIP formulations can be significantly slower than solutions found using 'Dijkstra' and 'Floyd Warshall' algorithms (Apoorva et al. 2013). This slowness in LIP becomes of greater concern for larger network flow problems. More efficient algorithms such as three state-of-the-art algorithms proposed by Santos et al., Zhu and Wilhelm, and Joksch address problems of higher dimensionality handling problems with thousands of nodes and tens of thousands of arcs (Lozano and Medaglia 2013). Within the low dimensionality context of the TS problem addressed in this paper, the SIMPLEX LP method yielded solutions at a reasonable speed.

With the rapid advance of computing power and data analysis tools, faster and more versatile optimization algorithms have arisen to solve variations of the Travelling Salesman Problem. More recent machine learning algorithms such as Genetic Algorithm, K-Shortest Path Algorithm, Multimodal Shortest Path Algorithm and K-Nearest Neighbor Query have been used in GPS (Global Positioning Systems) car navigation to minimize the total distance travelled for commuters (Bhaskoro et al. 2021). These new methods show promise in solving the entire plethora of optimization problems. Other researchers have investigated solving SRP under stochastic conditions in which an agent is directed between states such that the cost of transitions is governed by probability distributions (Guillot and Stauffer 2020). These model formulations are excellent at evolving dynamically and provide adaptive solutions under uncertainty and disturbances.

Within optimization and operations research, the Orienteering Problem (OP) is seen as a combination of the TSP and Knapsack Problem (KP) to create an Optimum Personalized Tourism Package (OPTP). The OP optimizes for the subset and order of nodes to be visited by maximizing a collected score (Gunawan et al. 2016). This OP framework is successful in creating personalized travel itineraries as it considers personalized preferences while staying within budget and time constraints. Aslam et al. (2015) solve the tourist TSP by formulating separate objective functions addressing firstly the budget-minimizing tourist, as evaluated in this paper, and secondly, the reward-maximizing tourist who maximizes enjoyment based on a destination scoring system. Additionally, Al Maimani et al. (2023) have extended this work for the tourist TSP by developing four multi-objective meta-heuristic algorithms, namely the NSGA-II, NSGA-III, grey wolf optimization, and imperialist competitive algorithms which optimize along multiple conflicting criteria.

Within the airline industry itself, classical optimization problems like TS arise in flight scheduling, fleet assignment, aircraft routing and crew scheduling (Bazargan 2004). Following the United States airline deregulation in 1978, the airline industry rapidly adopted the hub-and-spoke network design to provide broader airport coverage for consumers (Yang and Yu 1998). To avoid unprofitable flights between smaller markets, flights are routed through a central airport which can service smaller secondary flights to passengers heading to less in-demand airports. Within the context of this problem, major cities were chosen as nodes to take advantage of the hub-and-spoke network design. Generally, tourist cities facilitate the most reliable airports for routing travel between other urban centers.

## 3. Methods

The following network diagram is the nodal representation of the integer optimization problem discussed in the above sections. The traveller's first decision is the initial departure ( $\mathrm{OA}, \mathrm{OB}, \mathrm{OC}$ or OD ) followed by daily departure decisions between nodes ( $\mathrm{AB}, \mathrm{CB}, \mathrm{BA}, \mathrm{DD}$ etc.) (Figure 4). Regardless of the total length of the stay, n days, the travellers must leave the final destination, returning to the origin and concluding the trip. Starting with $\mathrm{T}=0$, time is
discretized daily such that every new day requires a new decision of departing to a new destination, for example going from A to B on day $5\left(X_{A, B, 5}=1\right)$ or remaining at A for a consecutive day $\left(X_{A, A, 5}=1\right)$.


Figure 4. Time discretized network flow diagram

## Model Formulation:

Excel's computation limitations hingers the problem size to a maximum of 200 decision variables. The number of binary decision variables for a problem can be found using:

$$
\begin{equation*}
\operatorname{num}\left(X_{d, T}\right)=T d^{2}+2 d \tag{1}
\end{equation*}
$$

Where $d$ is the number of destinations to visit, and $T$ is the length of the trip in days. Thus, having chosen 4 cities to visit for a trip length of 13 days, the number of decision variables is 200 .

## Objective Function:

The objective function finds the sequence of trips to minimize the overall cost of visiting the specified destinations in a closed loop. $C_{i, j, T}$ is the associated cost of going from $i$ to $j$ on day T and $X_{i, j, T}$ is the binary decision variable of going from $i$ to $j$ on day T . Time is discretized by each day relative to a starting date where n is the total length of the trip in integer days. When $i=j, C_{i, j, T}$ is the cost of accommodation in location $i$ whereas if $i \neq j, C_{i, j, T}$ is the flight cost from $i$ to $j$. The objective function to be minimized is as follows:

$$
\begin{equation*}
\min \left(\sum_{j=1}^{d} C_{0, j, 0} X_{0, j, 0}+\sum_{T=1}^{n-1} \sum_{i=j}^{d} \sum_{j=1}^{d} C_{i, j, T} X_{i, j, T}+\sum_{i=1}^{d} C_{i, 0, n} X_{i, 0, n}\right) \tag{2}
\end{equation*}
$$

## Constraints:

## Binary Constraints

All decision variables ( $X_{i, j, T}$ ) must be binary.

$$
X_{i, j, T}=\left\{\begin{array}{l}
1 ; \quad \text { if action is taken }  \tag{3}\\
0 ; \text { if action is not taken }
\end{array} \forall i, j, T\right.
$$

## Initial Departure Constraint

The initial departure constraint ensures that travellers start from the origin and immediately go to one of the destinations.

$$
\begin{equation*}
\sum_{j=1}^{j=d} X_{0, j, 0}=1 \tag{4}
\end{equation*}
$$

## Changeover Constraints

The following set of changeover constraints are nodal flow balances on each destination for each day. They ensure that if travellers visit a city, the next day they must either depart to another city $(i \neq j)$ or remain at the same city by remaining in it $(i=j)$.

## Origin Departure Constraint

The starting change over on day 1 is based on starting from the origin and travelling to the first destination.

$$
\begin{equation*}
\sum_{j=1}^{j=d} X_{i, j, 1}-X_{0, i, 0}=0 ; \forall i \in\{1, d\} \tag{5}
\end{equation*}
$$

## Change Over Constraint

For all the remaining days of the trip except the last one, the travellers must decide to remain at the same destination or depart to a new one. This can be represented by the following constraint.

$$
\begin{equation*}
\sum_{i \in[1, d], j=1}^{j=d} X_{i, j, T+1}-\sum_{i=1, j \in[1, d]}^{i=d} X_{i, j, T}=0 ; T \in\{1, n-2\} \tag{6}
\end{equation*}
$$

## Origin Return Constraint

Like the initial change-over constraint mentioned above, on the final day, the travellers must return to the origin and thus conclude their trip.

$$
\begin{equation*}
X_{j, 0, n}-\sum_{i=1}^{i=d} X_{i, j, n-1}=0 ; \forall j \in\{1, d\} \tag{7}
\end{equation*}
$$

## Return Constraint

On the final day, the travellers must return to the origin to conclude the trip.

$$
\begin{equation*}
\sum_{i=1}^{i=d} X_{i, 0, n}=1 \tag{8}
\end{equation*}
$$

## Visitation Contraint

To avoid looping where travellers return to the same city after leaving, they are constrained to only visit a city once starting from the origin.

$$
\begin{equation*}
X_{0, j, 0}+\sum_{T=1}^{T=n-1} \sum_{i=1, i \neq j}^{i=d} X_{i, j, T}=1 ; j \in\{1, d\} \tag{9}
\end{equation*}
$$

## Departure Constraint

Similarly, to avoid looping, the travellers are only able to leave the same city once for all days of the trip up to the return to the origin.

$$
\begin{equation*}
\sum_{T=1}^{T=n-1} \sum_{j=1, i \neq j}^{j=d} X_{i, j, T}+X_{i, 0, n}=1 ; i \in\{1, d\} \tag{10}
\end{equation*}
$$

## Timed Stay Constraint

The following constraint is specified by the travelers and accounts for how many consecutive days, $s_{l}$, the travellers plan on remaining at a specified destination.

$$
\begin{equation*}
\sum_{T=1}^{T=n-1} X_{i, i, T}=\sum_{T=1}^{T=n-1} X_{j, j, T}=s_{l} ; i, j \in\{1, d\} \tag{11}
\end{equation*}
$$

## Required Stay Constraint:

The following constraint enables travellers to specify whether they must or cannot be in a city on a certain date.

$$
X_{i, j, T}=\left\{\begin{array}{l}
1 \quad \text { must stay }  \tag{12}\\
0 \text { cannot stay }
\end{array}\right.
$$

## 5. Results and Discussion

### 5.1 Scheduling

The generalized model proposed is used in the following analysis where the origin and four destination nodes include London, Rome, Paris, Frankfurt and Barcelona as they are major hub-and-spoke tourism airports servicing daily flights. In the following instances, one of the destinations is set as the origin city whereas the remaining cities are defined as destinations. The greedy solution is evaluated by always selecting the cheapest flight option from one destination to another whereas the generous solution always selects the most expensive. It is foreseeable the generous solution will be more expensive than the greedy solution (Figure 5).


Figure 3. European map showing London, Rome, Paris, Frankfurt and Barcelona
The following three cases showcase the model in which Frankfurt, Paris and London are rotating origin cities. In all three cases, the travellers spend the same sequence of 2-3-2-2 days in each of the destinations before returning to the origin.

### 5.2 Origin in Frankfurt

Table 1 indicates the city node encoding for the first test case examined in which the origin is Frankfurt along with the stay durations for London, Rome, Paris and Barcelona Table 1.

Table 1. City encoding for test case 1 (Origin: Frankfurt)

| City | Encoding | Stay Duration |  |
| :---: | :---: | :---: | :---: |
| London | A | 2 |  |
| Rome | B | 3 |  |
| Paris | C | 2 |  |
| Barcelona | D | 2 |  |
|  |  |  |  |



Figure 4. Greedy, Generous and Optimized Travel Itinerary (Origin: Frankfurt)
Figure 6 presents the greedy, generous, and optimized travel itinerary for this instance. It is important to note that the lighter and darker coloured cells indicate days remaining in a city and those dedicated to travelling, respectively. This color coding of the travel itineraries is consistent with all subsequent instances. The greedy itinerary $(O A \rightarrow A D \rightarrow$ $D C \rightarrow C B \rightarrow B O$ ) costing $\$ 1960$ and generous itinerary $(O D \rightarrow D C \rightarrow C B \rightarrow B A \rightarrow A O)$ costing $\$ 2240$ are both higher than the optimized itinerary $(O C \rightarrow C D \rightarrow D A \rightarrow A B \rightarrow B O)$ costing $\$ 1858$. This is a savings of upwards to $\$ 382$ from the generous itinerary relative to the optimized one. The three routes have three distinct starting departure and return trips, yet both the greedy and optimized itineraries go from Barcelona (D) to Rome (C) as intermediary steps.

### 5.3 Origin in Paris

Table 2 indicates the city node encoding in which Paris is the origin city along with the stay duration of each city. This data is the basis for the model evaluation Table 2.

Table 2. City encoding for test case 2 (Origin: Paris)


Figure 5. Greedy, Generous and Optimized Travel Itinerary (Origin: Paris)
Figure 7 showcases the generous itinerary $(O B \rightarrow B C \rightarrow C D \rightarrow D A \rightarrow A O)$ costing $\$ 1970$ is $\$ 280$ more expensive than both the greedy $(O C \rightarrow C A \rightarrow A D \rightarrow D B \rightarrow B O$ ) and optimized ( $O C \rightarrow C A \rightarrow A D \rightarrow D B \rightarrow B O$ ) itineraries both costing $\$ 1690$. This instance is particular to note because both the greedy and optimized itineraries follow the same route. Although this is not always the case as shown above when Frankfurt was the origin city, the greedy solution can exist close to or exactly on the optimized one.

### 5.4 Origin in London

Table 3 displays the encoding of city nodes, with London as the origin city, and it also shows the duration of stay in each city (Table 3).

Table 3. City encoding for test case 3 (Origin: London)

| City | Encoding | Stay Duration |
| :---: | :---: | :---: |
| Paris | A | 2 |
| Rome | B | 3 |
| Frankfurt | C | 2 |
| Barcelona | D | 2 |



Figure 6. Greedy, Generous and Optimized Travel Itinerary (Origin: London)
In a similar manner shown before, Figure 8 includes the travel itineraries for the generous $(O A \rightarrow A B \rightarrow B C \rightarrow C D \rightarrow$ $D O$ ), greedy ( $O D \rightarrow D B \rightarrow B C \rightarrow C A \rightarrow A O$ ) and optimized ( $O D \rightarrow D B \rightarrow B A \rightarrow A C \rightarrow C O$ ) itineraries costing $\$ 1987, \$ 1780$ and $\$ 1671$, respectively. As expected, there is no agreement between the routing of the generous and optimized solutions. In contrast, both the greedy and optimized routes will first visit Barcelona (D) followed by a trip to Rome (B). The above three instances show that the generous solution will likely lie furthest away from the optimized solution even with similarities in routing.

### 5.5 Savings Analysis

To determine the success of the optimization model it is important to identify the savings provided by it. The following section details the potential savings through using the proposed model. Figure 9 shows how rotating through Frankfurt, Paris, Barcelona, Rome, and London as origin cities affects the minimized travel cost. Although the instances in which Barcelona and Rome as origin cities are not described above, the results in the following figure are generated from the same analysis.


Figure 7. Rotating origin-optimized travel cost
From Figure 9, it is clear how significant the origin city is on the travel cost. By starting in Frankfurt rather than London, travellers would spend an additional $\$ 187$ or $11.2 \%$ more for the trip.

Additionally, it is also important to evaluate the improvement between the optimized solution to all other alternatives. To illustrate this, Figure 10 includes the associated costs of all alternative route sequences in which the travellers originate from London.


Figure 8. Cost Comparison between all route sequences (Origin: London)
To note in Figure 10, DBAC is the minimized route whereas ABCD and DBCA are the generous and greedy itineraries mentioned in Section 5.4. By optimizing the route, the cost is reduced from $\$ 1971$ and $\$ 1780$ to $\$ 1671$ for the generous and greedy solution, respectively. That is an $18.9 \%$ and $6.5 \%$ savings rate between the sub-optimal to optimal route. It is important to note that there exist alternative routes (BDAC, CADB and DACB) which exist close to the optimal solution but there is no discernible relationship in where these lie.

Figure 11 summarizes the total saving percentage by selecting the optimal route over the greedy and generous routes for the 5 cities discussed (Frankfurt, Paris, Barcelona, Rome, and London) with a rotating origin city.


Figure 11. Optimization savings results

From the above plot, the optimized solution shows an improvement between $16 \%-21 \%$ relative to the generous solution and $0 \%-\% 6.5$ relative to the greedy solution. In the instances where Paris and Barcelona are the origin cities, the greedy and optimal routes are identical. In the other three instances, there is a clear improvement over $5 \%$.

### 5.6 Variable Stay Duration on Optimal Schedule

With an origin from Frankfurt, Table 4 shows the impact of changing the stay duration of each intermediary city on the total cost of the optimal solution.

Table 4. Frankfurt (O), London (A), Rome (B), Paris (C) and Barcelona (D)

| Stay Duration (Days) <br> A-B-C-D | Route | Total Cost <br> (\$CAD) |
| :---: | :--- | :---: |
| $\mathbf{1 - 2 - 3 - 3}$ | $\mathrm{OC} \rightarrow \mathrm{CB} \rightarrow \mathrm{BD} \rightarrow \mathrm{DA} \rightarrow \mathrm{AO}$ | 1994 |
| $\mathbf{1 - 1 - 3 - 4}$ | $\mathrm{OC} \rightarrow \mathrm{CD} \rightarrow \mathrm{DB} \rightarrow \mathrm{BA} \rightarrow \mathrm{AO}$ | 1964 |
| $\mathbf{3 - 1 - 3 - 2}$ | $\mathrm{OC} \rightarrow \mathrm{CB} \rightarrow \mathrm{BD} \rightarrow \mathrm{DA} \rightarrow \mathrm{AO}$ | 1948 |
| $\mathbf{4 - 1 - 1 - 3}$ | $\mathrm{OC} \rightarrow \mathrm{CA} \rightarrow \mathrm{AB} \rightarrow \mathrm{BD} \rightarrow \mathrm{DO}$ | 1940 |

It is apparent from Table 4 that varying the staying durations at each location will impact the route of travel but will have a limited impact on the total cost. This is one instance of the model's applicability, but further work should be done to generalize the relationship between stay duration and total cost as this depends on both the travel and accommodation prices.

### 5.6 Proposed Improvements

Within the proposed model, there are important assumptions and considerations taken which need to be addressed to improve and widen the breadth of the model's applicability. Firstly, the model takes advantage of the hub-and-spoke network design to ensure the daily availability of flights between all destinations and that travel between them is bidirectional. If the travellers wish to visit a destination with limited availability, the model must be adjusted with additional constraints to accommodate more infrequent travel. Additionally, the model does not consider travel time which is a parameter of great interest to travellers as they may wish to consider the trade-off between less expensive but more time-consuming and indirect travel. Lastly, the model can be expanded to respond dynamically to disturbances such as flight cancellations due to weather and maintenance.

## 6. Conclusion

In conclusion, this paper introduced a linear integer programming (IP) approach to optimize the travel itineraries of European tourists, minimizing total travel expenses encompassing flight and accommodation costs. The proposed discretized time network flow model, an extension of traditional Shortest-Route-Problems (SRP), was employed and solved using Microsoft Excel's integer optimization capabilities. The research unveiled considerable daily fluctuations in flight costs, reaching up to $\$ 312$ within a brief timeframe, while accommodation expenses from AirBnB.com remained more stable. This paper provided the mathematical model formulation, underlying assumptions, and explored diverse travel scenarios involving changes in the origin city and stay durations at each destination. Three strategies-generous, greedy, and cost-minimized-were examined for each travel route originating from different locations. Optimizing the itinerary yielded cost reductions of up to $20.3 \%$ and $6.7 \%$ when compared to the generous and greedy approaches, respectively. The most cost-effective trip itinerary was achieved when the journey started in London, totalling $\$ 1671$, whereas the most expensive route was initiated in Frankfurt, costing $\$ 1858$. The implications of these findings extend to the global tourism industry, where even small enhancements in travel planning can result in substantial savings for both businesses and budget-conscious consumers.

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## Biographies

Luca Romagnoli, a recent graduate of the University of Waterloo, holds a Bachelor of Applied Science in Chemical Engineering (with distinction). His academic excellence is evident through accolades such as the NSERC, Randy Duxbury Memorial Award, President's Scholarship the Deans List. Under the guidance of Dr. Ali Elkamel, Luca has made his debut in academic contributions with his first conference paper submission to the14th IEOM Society International Conference on Industrial Engineering and Operations Management.

Ali Elkamel is a Full Professor of Chemical Engineering. He is also cross appointed in Systems Design Engineering. He holds a BSc in Chemical Engineering and BSc in Mathematics from Colorado School of Mines, MSc in Chemical Engineering from the University of Colorado, and PhD in Chemical Engineering from Purdue University. His specific research interests are in computer-aided modeling, optimization, and simulation with applications to energy planning, sustainable operations, and product design. His activities include teaching graduate and undergraduate courses, supervising post doctorate and research associates, and participation in both university and professional societal activities. He is also engaged in initiating and leading academic and industrial teams, establishing international and regional research collaboration programs with industrial partners, national laboratories, and international research institutes. He supervised over 120 graduate students (of which 47 are PhDs ) and more than 45 post-doctoral fellows/research associates. He has been funded for several research projects from government and industry. Among his accomplishments are the Research Excellence Award, the Excellence in Graduate Supervision Award, the Outstanding Faculty Award, and IEOM Awards. He has more than 425 journal articles, 175 proceedings, 50 book chapters, and has been an invited speaker on numerous occasions at academic institutions throughout the world. He is also a co-author of six books.

