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A Comparative Study on Heuristic Methods for Transportation Problem

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Abstract

Transportation cost accounts for one to two-thirds of the logistics cost for many organizations. So, minimizing the transportation cost in a Transportation Problem (TP) would help the organization to maximize profit. Though there are various methods, such as linear programming approach, network approach, transportation method, etc., available for addressing TP, the transportation method – particularly any simple and efficient heuristic method (which is expected to give very near optimal solution to TP) – is widely used as this type of method could easily be coupled with any another logistics/supply chain decision-making methods/tools to have an integrated decision-making process. Due to this, many researchers are continuously proposing simple heuristic method(s) for solving TP to get near-optimal solutions. However, from the analysis of the literature, it is observed that there is no good comparative study with good computational experiments considering all heuristic methods, which are (a) claimed as relatively 'better heuristic method' and published in reputed journals during the year 2013-2022, (b) evaluated using a large number of tiny test data taken from the literature, and (c) evaluated with large scale test data - generated from popular experimental design available in the literature. To address this research gap, this study considers 10 'better heuristic methods' reported in the literature and proposed 4 heuristic methods. Further, from the detailed performance analyses carried out using 640 randomly generated problem instances, this study identified the 'best' heuristic method(s) and the same is reported with insights.

Keywords
Transportation Problem, Transportation Cost, Total Opportunity Cost Matrix, Heuristics, Computational Analysis

1. Introduction
Transportation Problem (TP), also known as the distribution problem, is one of the widely-addressed optimization problems with an objective to minimize the total cost of shipping goods from origins to destinations or delivery time for any business organization while satisfying all the supply constraints and the demand requirements. Many applications which are not physically moving/sending/distributing goods from origins to destinations can also be formulated as a TP (Mathirajan et al. (2010); Mathirajan et al. (2021); Tayyeh and Hussien (2018).

Though there are many solution methodologies for a TP like (a) developing network model and solving using the network method, (b) linear programming (LP) approach and (c) transportation method, the transportation method is widely used. The transportation method consists of a two-step procedure to obtain the optimal transportation cost for any TP. The first step is finding the initial solution using any existing heuristic method available in the literature – the North West Corner Method (NWCM) (Charnes et al. (1953)), Least Cost Method (LCM) (Hitchcock (1941)), Vogel's Approximation Method (VAM) (Reinfeld and Vogel (1958)) etc. The second step involves implementing the optimality test on the initial solution found using the afore-mentioned methods using the Stepping Stone Method (Charnes and Cooper (1954)), Modified Distribution Method (MODI) (Dantzig (1963)) etc.

The computational time required to reach the optimal solution using the second step of the transportation method largely depends on the quality of the heuristic method. This implies that the initial solution obtained using the heuristic method should be efficient so that it is somewhat near to the optimal solution. Hence, researchers are continuously trying to introduce new heuristic methods to prove their efficiency over existing heuristic methods in solving large-scale transportation problems and obtaining near optimal solutions. Despite continuously developing heuristic methods, almost every published research paper does not have a serious computational evaluation process to prove that the heuristic method(s) proposed by them consistently gives better results (Mathirajan and Rani (2021)).

Accordingly, this study uses (a) 4 newly proposed variants of the existing heuristic methods, and (b) 10 existing heuristic methods, reported as better heuristic methods, in the literature for getting near optimal solutions. A series of computational experiments carried out in this study indicate that the newly proposed heuristic methods and its variants outperform in comparison with the existing heuristic methods considered in this study.

The paper is organized as follows: Section 2 presents a related review of the existing literature on the recently published as well as existing heuristic methods pertaining to a transportation problem. The research gaps and the proposed methodology followed in this study is discussed in Section 3. Section 4 briefly presents the required details for the performance analysis of the proposed methodologies. The analysis of the results with discussions is presented in Section 5. Finally, we conclude the paper in the last section.

2. Literature Review

Except a few studies Singh et al. (2012); Mathirajan et al. (2022), most of the studies do not have a serious computational evaluation process to prove that their proposed heuristic method is giving consistently better results in comparison with the benchmark procedure(s) considered in their studies. As this study is related to considering recently published efficient heuristic methods from the period 2013 – 2022, the review is restricted to the recent studies on heuristic methods with well-defined computational experiments.

The Vogel's Approximation Method (VAM) is a widely-discussed heuristic method in all textbooks on Operations Research, as it provides very efficient near optimal solutions. The concept of Total Opportunity Cost Matrix (TOCM) was introduced by Kirca and Satir (1990). This concept was incorporated with VAM and presented as a new variant of the existing heuristic method, called as VAM – TOCM, by Mathirajan and Meenakshi (2004). The authors considered 640 randomly generated problem instances based on the experimental design framework of Kirca and Satir (1990) and found VAM – TOCM to yield better solutions in comparison with VAM. Similar to VAM, the next popular heuristic method is the Russell's Approximation Method (RAM) discussed by Hillier and Lieberman (2010). The method was an extension of the Dantzig's algorithm to finding a near-optimal basis for the transportation problem by Russell (1969).

A simpler heuristic method than VAM was conceived by Kasana and Kumar (2013) known as the Extremum Difference Method (EDM). The algorithm worked on the principle that if an allocation was not made to the corresponding row or column with the highest extreme difference, then the cost penalty per unit cost would be higher for other rows or column with other extreme differences. A new heuristic method known as the Global Minimum Method (GMM) was developed by Harrath and Kaabi (2018). The method: GMM was compared with three other heuristic methods (including the popular VAM). The results indicated that GMM was outperforming all the other methods for large-scale test instances. However, only balanced TPs were considered in the study.

Mathirajan et al. (2022) recently proposed many new variants of heuristic methods and considered various existing methods that are empirically proved to be better heuristics in the literature for performance analysis following an extensive computational experiment. The authors considered 34 heuristic methods – 23 newly proposed heuristics and 11 existing heuristic methods in the literature. Out of these 34 heuristics, the two variants of the Row-Column Weighted Minimum-Cost-Allocation Method (RCWMCAM) i.e., one coupled with the original cost matrix and one coupled with the TOCM, were empirically and statistically proven to give superior results. The method computed a composite index with respect to row and column for deciding an allocation quantity. Amaliah et al. (2022) proposed a new heuristic method called the Bilqis Chastine Erma (BCE) method. Their experimental results indicated that BCE obtained even lower total costs than VAM as well as 3 other heuristic methods consider in their study. The same set of authors further developed a new heuristic – the Supply Selection Method (SSM) which yielded lower total costs than BCE as well as 4 other existing heuristic methods.

3. Research Methodology

From the analysis of the literature, it is observed that there is no good analysis with extensive computational experiments considering all heuristic methods for TP towards deciding the efficiency of the heuristic methods. To address this research gap, this study considers 10 heuristic methods reported in the literature - which are (a) claimed as relatively 'better heuristic method' and published in reputed journals during the year 2018-2022, (b) evaluated using a large number of tiny test data taken from the literature, and (c) evaluated with large scale test data - generated from popular experimental design available in the literature. In addition, there are 4 heuristic methods are proposed in this study. The brief details of the existing heuristic methods considered and the proposed heuristic methods are presented in the following section.

3.1 Existing Heuristic Methods

The name of the each of the 10 existing heuristic methods, published in reputed journals during the year 2018-2022 (including VAM), with its reference are as follows:

1. **VAM – TCM**: Vogel's Approximation Method applied on the Total Cost Matrix (TCM) (Reinfeld and Vogel (1958))
2. **VAM – TOCM**: VAM applied on TOCM (Mathirajan and Meenakshi (2004))
3. **RAM – TCM**: Russell's Approximation Method applied on TCM (Hillier and Lieberman (2010))
4. **RAM – TOCM**: RAM applied on TOCM (Storozhyshina et al. (2011))
5. **EDM – TCM**: Extremum Difference Method applied on TCM (Kasana and Kumar (2013))
6. **GMM – TCM**: Global Minimum Method applied on TCM (Harrath and Kaabi (2018))
7. **RCWMCAM – TCM**: Row-Column Weighted Minimum-Cost-Allocation Method applied on TCM (Mathirajan et al. (2022))
8. **RCWMCAM – TOCM**: RCWMCAM applied on TOCM (Mathirajan et al. (2022))
9. **BCE – TCM**: Bilqis Chastine Erma method applied on TCM (Amaliah et al. (2022))
10. **SSM – TCM**: Supply Selection Method applied on TCM (Amaliah et al. (2022))

3.2 Proposed Heuristic Methods

Each of the 4 proposed heuristic methods is the variant of the existing heuristic methods. Further, basically the step-by-step details of each of the 4 proposed heuristic methods are exactly similar to the corresponding existing version of the heuristic methods except that the basic input to proposed heuristic methods and its existing version of the heuristic methods for allocation decision varies. That is, the basic input to each of the 4 proposed heuristic methods for allocation decision is "total opportunity cost matrix (TOCM)", whereas the basic input to each of the corresponding existing heuristics methods is "transportation cost matrix (TCM)". Due to this, the step-by-step details of each of the 4 proposed heuristic methods is not presented. Accordingly, the names of each of the 4 proposed heuristic methods are as follows:

1. **EDM – TOCM**: EDM applied on TOCM
2. **GMM – TOCM**: GMM applied on TOCM
3. **BCE – TOCM**: BCE method applied on TOCM
4. **SSM – TOCM**: SSM applied on TOCM

4. Performance Evaluation of the Heuristic Methods

For carrying out performance analysis, there are 3 components: problem instances, bench mark procedure(s), and performance measure(s) need to be defined. The details of these 3 components involved in this study are given as follows:

Problem Instances: In the absence of real-life data, the actual research practice is to propose a suitable experimental design for generating pseudo-random problem instances. Accordingly, in this study, the experimental design defined in Mathirajan et al. (2022) is considered. The summary of this experimental design is given in Table 1. A Python code is written for randomly generating problem instances using the given experimental design. Accordingly, for each combination of the problem factors: problem size (m x n) where m: number of rows and n: number of columns, degree of imbalance (K) and cost structure – range (R), 10 problem instances are randomly generated for performance evaluation of the heuristic methods pertaining to a traditional TP considered in this study. A total of 640 TP instances are randomly generated following the experimental design and each of the 640 TP instances generated randomly are unbalanced TPs.

Table 1. A summary of the experimental design [**Source**: Mathirajan et al. (2022)]

Problem Factor	**No. of Levels**	**Values**
Problem Size (***m x n***)	4	{10 × 20; 10 × 40; 10 × 60; 10 × 100}
Cost Structure – Range (***R***)	4	{20, 100, 500, 1000}
Degree of Imbalance (***K***)	4	{1, 2, 5, 10}
Number of Problem Configurations		(4 x 4 x 4) = 64
Number of Instances per Configuration		10
Number of Problem Instances		640
Cost Structure (C_{ij}): Uniform Distribution: U (C_{ij}: Mean Cost − R/2, Mean Cost + R/2) Where Mean Cost = 500 Supply (S_i): Uniform Distribution: U (S_i: 0.75 × Mean Supply, 1.25 × Mean Supply), Where Mean Supply = [(K × n × Mean Demand)/m] and Mean Demand = 100 Demand (D_j): Uniform Distribution: U (D_j: 75, 125)		

Performance Measure: The performance analysis of the 14 heuristic algorithms considered in this study is carried out using the performance measure: average relative percentage deviation (ARPD), which is computed using the following equations:

$$\mathrm{ARPD_j} = \sum_{i=1}^{N} \mathrm{RPD_{ij}}/N \qquad (1)$$

$$RPD_{ij} = \left(D_{ij}/OTC_i\right) * 100 \qquad (2)$$

$$D_{ij} = \left(TC_{ij} - OTC_i\right) \qquad (3)$$

Where *i:* *Problem instances* $i \in [1, 640]$ *instances*

j: *Variants of heuristic methods and* $j \in [1, 14]$

TC_{ij}: Total Cost yielded by the j^{th} heuristic method for i^{th} problem instance

OTC_i: Optimal TC for 'i^{th}' problem instance, yielded by the ILP Model

D_{ij}: Difference between the 'Total Cost' obtained for the i^{th} instance from the j^{th} variant of the heuristic method and the ILP for the i^{th} instance.

RPD_{ij}: Relative percentage deviation of 'j^{th} variant of the heuristic method for 'i^{th}' problem instances

$ARPD_j$: Average relative percentage deviation of 'j^{th}' variant of the heuristic method

N : = 160 when ARPD is computed considering the number of problem instances w.r.t. each of the problem configuration

N : = 640 when ARPD is computed considering entire problem instances

Bench Mark Procedure: For absolute performance evaluation of the 14 heuristic algorithms considered for the TP problem, the LP model proposed in Raghavendra and Mathirajan (1987) is considered as bench mark procedure.

5. Results and Discussion

Each of the 640 problem instances is solved using each of the 14 heuristic algorithms and the obtained 'total cost' yielded for i^{th} problem instance by the j^{th} heuristic method is stored in TC(i,j). Each of the 640 problem instances is solved using the ILP model and the 'optimal total cost' obtained for each problem instance 'i' is stored in OTC(i). Using the results TC(i,j) and OTC(i), the relative percentage deviation of the solution obtained for each of the problem instances from each of the heuristic algorithms w.r.t. the optimal solution is computed using equation (2). The ARPD score over 10 instances for each of the 64 configurations, as mentioned in Table 1, is computed using equation (1) and the same is presented in Table 2. Further, for every cost structure range (R), the ARPD score over 160 instances is computed using equation (1) and the same is presented in Figure 1(a) to 1(d). Finally, the ARPD score over 640 instances (that is irrespective of the cost structure) is computer using equation (1) and the same is presented in Figure 1(e). From the results presented in Table 2 and Figure 1, the following observations can be made;

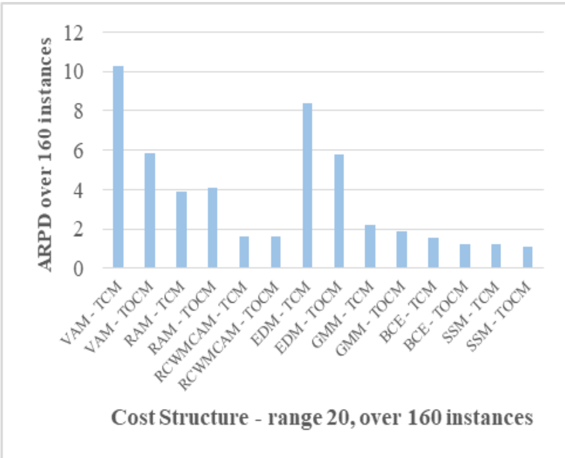
- ❖ The recent study by Mathirajan et al. (2022) empirically and statistically proved that (a) the 2 variants of the proposed heuristic methods: RCWMCAM (out of 34 heuristic methods considered for performance analysis in their study) are outperforming, and (b) existing heuristic methods: RAM are relatively performing well in comparison with other existing heuristic methods in the literature. But these findings/observations are NOT matching with the results presented in this study. Table 3 shows the computed ARPD Scores for each of the existing and proposed heuristic methods w.r.t. optimal solution.

Table 2. The computed ARPD Scores for each of the existing and **proposed heuristic methods** w.r.t. optimal solution

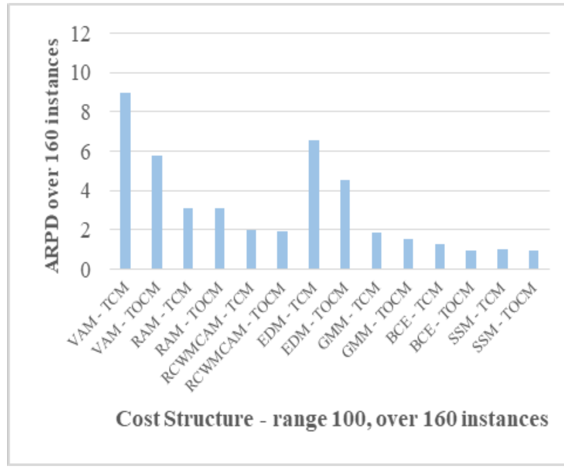
No.	Problem Configuration			Each of the 64 Problem configuration wise the ARPD Scores obtained for TP by the Heuristic Methods													
				Existing Heuristic Methods										**Proposed Heuristic Methods**			
	m x n	R	K	VAM – TCM	VAM – TOCM	RAM – TCM	RAM – TOCM	EDM – TCM	GMM – TCM	RCWMCAM-TCM	RCWMCAM-TOCM	BCE-TCM	**SSM-TCM**	EDM - TOCM	GMM-TOCM	BCE - TOCM	**SSM - TOCM**
1	10 x 20	20	1	9.35	5.01	3.32	3.52	7.08	1.90	1.97	1.78	1.33	**1.04**	4.78	1.64	1.03	**1.02**
2	10 x 20	20	2	8.43	5.49	3.42	3.32	6.89	2.06	1.89	1.72	1.37	**1.07**	4.73	1.67	1.05	**1.03**
3	10 x 20	20	5	8.78	5.62	3.54	3.67	6.63	1.92	1.86	1.80	1.46	**1.05**	4.69	1.63	1.07	**1.04**
4	10 x 20	20	10	8.65	5.35	3.21	3.24	6.98	1.98	1.78	1.68	1.56	**1.01**	4.68	1.60	1.01	**1.01**
5	10 x 20	100	1	9.01	5.22	3.68	3.59	7.15	1.87	1.91	1.75	1.30	**1.06**	4.82	1.65	1.04	**1.02**
6	10 x 20	100	2	9.12	5.14	3.37	3.40	6.81	2.01	1.82	1.70	1.41	**1.02**	4.75	1.59	1.06	**1.04**
7	10 x 20	100	5	8.98	5.48	3.45	3.22	6.69	1.94	1.75	1.79	1.52	**1.03**	4.81	1.66	1.08	**1.05**
8	10 x 20	100	10	9.21	5.27	3.58	3.48	6.93	1.99	1.80	1.67	1.47	**1.04**	4.74	1.68	1.02	**1.03**
9	10 x 20	500	1	8.72	5.59	3.63	3.14	6.72	2.04	1.96	1.76	1.35	**1.07**	4.71	1.61	1.01	**1.01**
10	10 x 20	500	2	8.81	5.41	3.15	3.42	6.56	1.97	1.79	1.74	1.48	**1.01**	4.70	1.62	1.03	**1.05**
11	10 x 20	500	5	8.84	5.12	3.50	3.60	6.78	1.86	1.85	1.82	1.55	**1.02**	4.84	1.70	1.06	**1.04**
12	10 x 20	500	10	8.88	5.73	3.29	3.29	7.01	2.02	1.74	1.71	1.32	**1.06**	4.77	1.56	1.05	**1.02**
13	10 x 20	1000	1	8.76	5.16	3.72	3.36	6.85	1.89	1.94	1.69	1.49	**1.05**	4.72	1.63	1.04	**1.06**
14	10 x 20	1000	2	9.04	5.38	3.12	3.12	6.91	2.05	1.87	1.77	1.43	**1.03**	4.66	1.69	1.02	**1.03**
15	10 x 20	1000	5	9.11	5.17	3.48	3.55	6.76	1.93	1.83	1.73	1.38	**1.09**	4.80	1.57	1.07	**1.02**
16	10 x 20	1000	10	8.89	5.54	3.17	3.46	7.09	1.95	1.77	1.65	1.29	**1.01**	4.76	1.64	1.05	**1.05**
17	10 x 40	20	1	8.79	5.63	3.56	3.23	6.67	2.03	1.92	1.84	1.50	**1.07**	4.79	1.68	1.03	**1.04**
18	10 x 40	20	2	9.05	5.31	3.61	3.34	6.82	1.91	1.81	1.81	1.54	**1.05**	4.67	1.58	1.01	**1.01**
19	10 x 40	20	5	8.92	5.08	3.25	3.44	6.95	1.88	1.98	1.66	1.35	**1.02**	4.83	1.70	1.09	**1.03**
20	10 x 40	20	10	8.87	5.67	3.46	3.39	6.70	1.96	1.88	1.85	1.48	**1.06**	4.65	1.57	1.06	**1.06**
21	10 x 40	100	1	8.90	5.23	3.18	3.15	6.88	2.00	1.84	1.63	1.39	**1.03**	4.76	1.63	1.04	**1.02**
22	10 x 40	100	2	8.99	5.42	3.40	3.57	6.62	1.89	1.72	1.83	1.44	**1.08**	4.80	1.66	1.02	**1.05**
23	10 x 40	100	5	8.96	5.19	3.69	3.51	6.74	2.06	1.90	1.68	1.52	**1.04**	4.70	1.68	1.08	**1.03**
24	10 x 40	100	10	9.09	5.76	3.13	3.61	6.97	1.93	1.76	1.77	1.32	**1.09**	4.68	1.60	1.05	**1.01**
25	10 x 40	500	1	8.94	5.04	3.33	3.19	6.84	1.94	1.95	1.70	1.41	**1.02**	4.72	1.59	1.03	**1.04**
26	10 x 40	500	2	8.82	5.29	3.51	3.37	6.79	1.99	1.93	1.79	1.49	**1.01**	4.74	1.71	1.07	**1.06**
27	10 x 40	500	5	8.73	5.68	3.23	3.30	6.90	1.97	1.73	1.75	1.43	**1.08**	4.67	1.57	1.09	**1.02**
28	10 x 40	500	10	8.91	5.07	3.41	3.68	6.66	1.91	1.89	1.72	1.54	**1.03**	4.69	1.62	1.02	**1.05**
29	10 x 40	1000	1	8.97	5.52	3.66	3.27	7.00	2.02	1.99	1.76	1.37	**1.09**	4.73	1.69	1.08	**1.03**
30	10 x 40	1000	2	8.95	5.06	3.47	3.45	6.73	1.86	1.82	1.82	1.28	**1.05**	4.81	1.63	1.05	**1.04**
31	10 x 40	1000	5	8.86	5.74	3.64	3.18	6.96	2.04	1.85	1.67	1.52	**1.04**	4.75	1.64	1.06	**1.02**
32	10 x 40	1000	10	8.93	5.16	3.20	3.41	6.71	1.85	1.97	1.71	1.30	**1.02**	4.82	1.66	1.04	**1.06**

Table 3. The computed ARPD Scores for each of the existing and **proposed heuristic methods** w.r.t. optimal solution (Contd.)

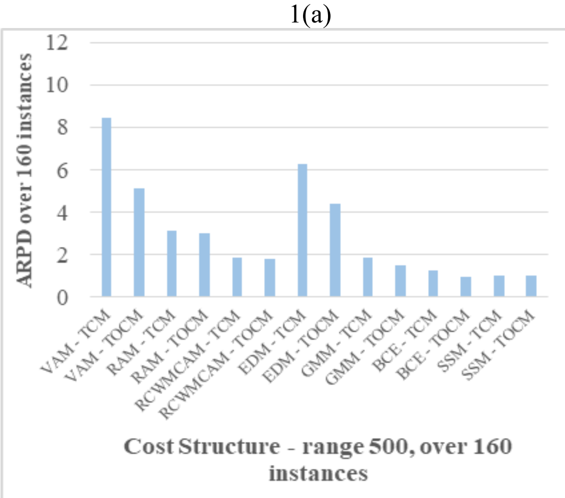
S. No	Problem Configuration			Each of the 64 Problem configuration wise the ARPD Scores obtained for TP by the Heuristic Methods													
				Existing Heuristic Methods										**Proposed Heuristic Methods**			
	m x n	R	K	VAM – TCM	VAM – TOCM	RAM – TCM	RAM – TOCM	EDM – TCM	GMM – TCM	RCWMCAM-TCM	RCWMCAM-TOCM	BCE-TCM	**SSM-TCM**	EDM - TOCM	GMM-TOCM	BCE - TOCM	**SSM - TOCM**
33	10 x 60	20	1	8.85	5.33	3.34	3.38	6.65	1.98	1.79	1.74	1.43	**1.07**	4.78	1.68	1.02	**1.01**
34	10 x 60	20	2	9.14	5.45	3.60	3.50	6.86	1.90	1.88	1.73	1.37	**1.08**	4.69	1.55	1.07	**1.05**
35	10 x 60	20	5	9.02	5.28	3.22	3.47	6.99	1.95	1.91	1.79	1.47	**1.06**	4.73	1.70	1.03	**1.03**
36	10 x 60	20	10	8.68	5.53	3.53	3.26	6.83	1.92	1.84	1.68	1.55	**1.01**	4.80	1.62	1.09	**1.04**
37	10 x 60	100	1	8.83	5.61	3.35	3.73	6.75	1.93	1.76	1.65	1.38	**1.04**	4.74	1.59	1.05	**1.02**
38	10 x 60	100	2	8.74	5.11	3.70	3.58	6.77	1.96	1.94	1.81	1.29	**1.09**	4.65	1.61	1.01	**1.06**
39	10 x 60	100	5	8.71	5.58	3.59	3.43	6.92	2.01	1.72	1.69	1.40	**1.05**	4.83	1.64	1.06	**1.01**
40	10 x 60	100	10	9.08	5.64	3.27	3.13	6.80	1.88	1.78	1.80	1.50	**1.03**	4.71	1.67	1.04	**1.03**
41	10 x 60	500	1	8.80	5.09	3.71	3.64	6.94	1.94	1.96	1.83	1.45	**1.02**	4.77	1.69	1.08	**1.04**
42	10 x 60	500	2	9.03	5.66	3.28	3.66	6.87	1.99	1.80	1.74	1.34	**1.07**	4.85	1.56	1.03	**1.05**
43	10 x 60	500	5	8.77	5.26	3.26	3.20	6.68	1.97	1.73	1.76	1.39	**1.08**	4.68	1.58	1.09	**1.02**
44	10 x 60	500	10	9.13	5.36	3.55	3.65	6.61	1.92	1.95	1.70	1.48	**1.06**	4.79	1.70	1.05	**1.06**
45	10 x 60	1000	1	8.75	5.18	3.44	3.71	7.04	2.00	1.75	1.78	1.51	**1.01**	4.72	1.61	1.07	**1.03**
46	10 x 60	1000	2	9.07	5.46	3.19	3.35	6.64	1.91	1.98	1.64	1.44	**1.05**	4.76	1.65	1.01	**1.01**
47	10 x 60	1000	5	8.70	5.47	3.24	3.31	6.55	1.95	1.86	1.82	1.36	**1.03**	4.81	1.57	1.04	**1.05**
48	10 x 60	1000	10	9.15	5.13	3.38	3.16	7.11	1.96	1.92	1.79	1.53	**1.09**	4.76	1.67	1.08	**1.04**
49	10 x 100	20	1	8.69	5.25	3.31	3.63	6.58	2.05	1.83	1.75	1.31	**1.04**	4.83	1.60	1.02	**1.02**
50	10 x 100	20	2	9.06	5.43	3.65	3.21	7.05	1.87	1.87	1.67	1.42	**1.08**	4.66	1.68	1.06	**1.03**
51	10 x 100	20	5	9.10	5.71	3.30	3.54	7.14	1.93	1.70	1.84	1.47	**1.02**	4.79	1.61	1.03	**1.06**
52	10 x 100	20	10	8.67	5.24	3.39	3.25	6.59	1.94	1.81	1.71	1.31	**1.06**	4.75	1.65	1.09	**1.01**
53	10 x 100	100	1	8.66	5.56	3.57	3.62	6.60	1.98	1.93	1.66	1.40	**1.07**	4.70	1.57	1.05	**1.04**
54	10 x 100	100	2	8.64	5.34	3.43	3.53	6.57	1.89	1.74	1.87	1.53	**1.05**	4.74	1.67	1.01	**1.05**
55	10 x 100	100	5	8.63	5.69	3.36	3.56	7.07	2.06	1.89	1.63	1.34	**1.03**	4.68	1.60	1.07	**1.02**
56	10 x 100	100	10	8.62	5.39	3.16	3.17	7.02	2.03	1.82	1.72	1.51	**1.09**	4.71	1.68	1.04	**1.03**
57	10 x 100	500	1	8.61	5.44	3.49	3.72	7.12	1.91	1.97	1.77	1.45	**1.01**	4.85	1.55	1.08	**1.06**
58	10 x 100	500	2	8.60	5.55	3.67	3.33	6.54	2.04	1.88	1.85	1.38	**1.04**	4.82	1.71	1.02	**1.04**
59	10 x 100	500	5	8.59	5.72	3.52	3.28	6.53	1.86	1.80	1.68	1.29	**1.08**	4.77	1.63	1.06	**1.02**
60	10 x 100	500	10	8.58	5.57	3.62	3.69	7.13	1.92	1.75	1.76	1.36	**1.02**	4.73	1.66	1.03	**1.01**
61	10 x 100	1000	1	8.57	5.51	3.14	3.49	6.87	2.02	1.84	1.74	1.42	**1.06**	4.79	1.60	1.09	**1.05**
62	10 x 100	1000	2	8.56	5.65	3.73	3.70	6.76	1.95	1.90	1.81	1.50	**1.03**	4.75	1.69	1.05	**1.03**
63	10 x 100	1000	5	8.55	5.37	3.67	3.30	6.72	1.97	1.78	1.69	1.33	**1.07**	4.70	1.59	1.07	**1.04**
64	10 x 100	1000	10	8.54	5.15	3.55	3.74	7.03	2.00	1.91	1.78	1.46	**1.05**	4.66	1.64	1.01	**1.02**



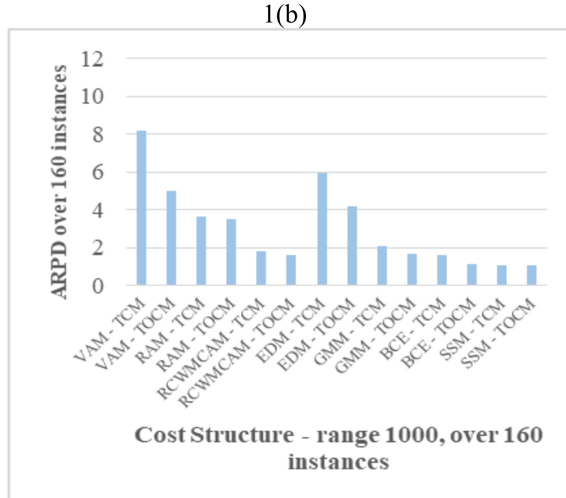
1(a)



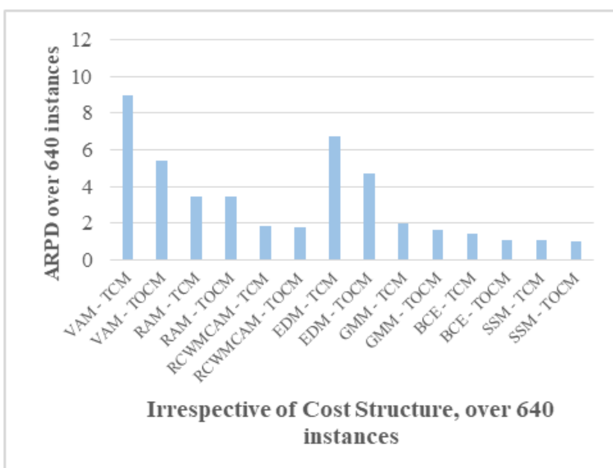
1(b)



1(c)



1(d)



1(e)

Figure 1. The average performance of the heuristics w.r.t. Optimal Solution

- ❖ On average, it appears that both variants of the Supply Selection Method (SSM) – SSM applied with TCM matrix (SSM – TCM) and SSM coupled with TOCM (SSM – TOCM) are performing relatively well. Further, out of 14 heuristic algorithms considered in this study, both variants: BCE-TCM and BCE-TOCM yielded next better results, followed by SSM. This is consistent with the findings of Amaliah et al. (2022) where SSM applied with TCM and BCM applied with TCM are proved to outperform the other heuristics considered in their study.

- This study endorsed the earlier observation that any heuristic method coupled with TOCM is expected to yield results that are either equal to or superior to those achieved by the same heuristic method coupled with TCM.
    - The 14 heuristic algorithms considered in our study can be grouped into 7 categories – heuristics applied on TCM and heuristics applied on TOCM. It is inferred that when the cost structure – range (***R***) varies, all the 7 heuristic algorithms coupled with TOCM give better results than the same heuristic algorithm coupled with TCM.
    - The above observation is consistent even when the problem size (***m x n***) varies, or the degree of imbalance (***K***) varies.

One practical implication of the present study is that the best-performing set of heuristics can now be computerized in a decision support system (DSS) environment, as the computational time required to solve real-life sized problem takes very meager computational time.

6. Conclusions

This study presents a comparative analysis of the recently published heuristic methods for a traditional transportation problem (TP). A total of 10 existing heuristic methods along with 4 newly proposed variants of the existing heuristic methods are considered in this study. For this, a suitable performance analysis of all the 14 heuristic methods considered for the TP is carried out considering 640 randomly and systematically generated problem instances, which are generated from a suitable experimental design. The two variants of the recently published heuristic method SSM: SSM – TCM and SSM – TOCM are topping the first two places out of all the 14 heuristic methods considered in the study. The analysis of the results indicated that the newly proposed heuristic method in the literature – the SSM coupled with the Total Opportunity Cost Matrix (TOCM) overall outperforms for the TP problem.

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