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Optimal Layout Planning for School Consolidation ~Elementary Schools in Machida City, Tokyo~

Hiroto Yamazaki
Graduate School of Science and Engineering
Hosei University, Tokyo 184-8584, Japan
Hiroto.yamazaki.3n@stu.hosei.ac.jp

Hiroyuki Goto
Department of Industrial and Systems Engineering
Hosei University
Tokyo 184 8584, Japan

Abstract

Japan is facing a rapid and continuing decrease in the number of younger people as the outcomes of population decrease, low birthrate, and population aging. Even in the vicinities of Metropolitan districts, municipalities must begin or have started planning core school restructures including school consolidation and closure. Not only the decrease in the number of students but also the teacher shortage must be taken into account. To find an objective plan, mathematical methods known as facility layout optimization can be utilized. The maximum covering problem (MCLP) is adopted to seek optimal layout plans, amongst several candidates. Because the traditional model does not reflect the capacities of facilities, the traditional model is modified to incorporate the capacities of facilities. A case study is conducted for Machida City, located southernmost of Tokyo. The city plans to reduce the number of public elementary schools from 42 to 26 by the year 2040, considering the declining birthrate and aging school buildings. Comparisons are made on the traditional and capacitated models. The results highlight that even if the number of schools is reduced to 26 from the current number of 42, the population coverage decreases by only 1% with the help of the MCLP-based approach.

Keywords
Optimal layout, maximal covering location problem, capacitated facility, school consolidation, coverage rate.

1. Introduction

Many facility allocation models are available in urban planning. Given a set of users and candidate locations, a facility allocation problem aims to find the number of facilities and the corresponding optimal facility locations. Traditional facility location-allocation problems take into account the marginal distance between users and facilities. The maximum covering problem first formalized by Church and ReVelle (1974) is pioneering research; the problem maximizes the demand coverage within a standard distance. On the other hand, in actual urban planning, it is common to adapt to changes in social needs and conditions by establishing new facilities, consolidating, and/or abolishing existing facilities. Along with financial restrictions, an effective use of existing facilities and resources is pivotal. In Japan, the circumstances surrounding public facilities have become critical in recent years due to composite changes such as population decrease, low birthrate, population aging, and needs by diversification of residents. About the declining birthrate problem in Japan, the number of children in public elementary schools in Tokyo metropolitan is expected to decrease by 8% over the next five years, and by 18% over the next 10 years. Therefore, some municipalities in Tokyo have begun making restructuring plans along with consolidation and closures of schools. For example, Machida City located southward of Tokyo plans to reduce the number of public elementary schools from 42 to 26 by the year of 2040, accounting for both the declining birthrate and aging of school structures.

On the other hand, excessive consolidation might yield exceeding capacity, which leads to another type of problem. Actual examples of exceeding capacities highlight a major problem, for which it is necessary to abate both oversizing and inefficiency in order to retain a proper balance between the number of teachers and students. Sound classroom management can be achieved along with this balance. In Japan, natural disasters by earthquakes and floods often cause municipalities to issue evacuation orders when it is unsafe to stay at home. Municipalities plan to which site the residents should evacuate in such an emergency. However, there are two types of actual disasters: one that impacts only a part of an area, and the other that impacts a broad area. In the latter case, a large number of residents are forced to evacuate, and the number of evacuees may surpass the capacity of evacuation centers. In such a situation, some residents fail to evacuate to evacuation centers and are forced to stay at their homes even if they may not be safe. While establishing extra sites can be a solution to this, it requires of course additional cost with which the most efficient location-allocation plan must be suggested considering both premises and demographic constraints. Our research motivation came into existence from these recent social circumstances.

In light of these, in this study, elementary school consolidation planning is addressed in a maximum-coverage problem context. The pioneering research that considers capacity in the maximum covering problem context is formulated by Pirkul and Schilling (1989), Pirkul and Schilling (1991). On the other hand, this study constructs a model that takes into account the capacity for each candidate facility. A case study of Machida city, located in southernmost Tokyo, will be reported, implying potential applications to other similar infrastructural problems.

2. Technical Background

2.1 Facility layout problem

Various facilities exist in our daily lives, including convenience stores, restaurants, as well as public facilities such as schools and hospitals. The locations of such facilities impact significantly on users. Facility location problem is a common term for instances of determining locations of facilities that satisfy predetermined standards; a set of candidate locations of facilities and customers reflecting demand is given as an input. Typical problems include location set covering problem (Toregas et al. 1971), maximum covering problem, p-median problem, and p-center problem (Hakimi 1964). A formulation in which capacity is added to the p-center with respect to distance has been formulated by Espejo et al (2015). The p-median problem with capacity has also been formulated by Lorena and Senne (2004). Variants based on the above models have been proposed, some of which are commonly used for urban planning.

2.2 Traditional facility location-allocation problem

The traditional type of facility location-allocation problems is an instance without considering capacity constraints, called the uncapacitated facility location problem. The costs of demand flows between customer and facility, as well as costs of installing or building facilities, are given. In the problem, the optimal placement of facilities that minimizes the total cost is sought, in which flows between customers and facilities are considered with all demands satisfied.

2.3 Maximum covering location problem

The Maximal Covering Location Problem (MCLP) is a facility location problem that maximizes the demand coverage in which the number of facilities is given. By varying the number, the impact of the parameter in the context of coverage rate can be investigated. This problem is useful in cases where the full coverage of demand is unachievable or unrealistic due to budget constraint, for which a compromise with partial coverage is necessitated. The formulation is as follows.

MCLP:

Maximize

$$\sum_{i \in I} a_i y_i, \tag{1}$$

subject to:

$$\sum_{j \in J_i} x_j \geq y_i\,,\; i \in I, \tag{2}$$

$$\sum_{j \in J} x_j = p, \tag{3}$$

$$x_j, y_i \in \{0,1\}. \tag{4}$$

Objective (1) aims to maximize the amount of demand coverage. Inequality (2) restricts at least one neighboring facility to be chosen if demand i is covered. Constraint (3) indicates the number of facilities to be installed is fixed to p. Constraints (4) enforce the decision variables to be binary {0, 1}. The associated notations are as follows:

i: index of demand point,
I: set of all demand points,
j: index of potential facility point,
J: set of all potential facility points,
J_i: set of facility points capable of suppling demand i,
a_i: demand quantity at demand point i,
p: number of facilities to install,
s: service distance by a single facility (in meters).

Selection of facility j and coverage of demand i are formulated with the following binary decision variables.

$$x_j = \begin{cases} 1\text{: facility } j \text{ is installed,} \\ 0\text{: facility } j \text{ is not installed,} \end{cases}$$

$$y_i = \begin{cases} 1\text{: demand } i \text{ is covered by supply point(s),} \\ 0\text{: demand } i \text{ is not covered by any suppply points.} \end{cases}$$

The above formulation will be called the traditional model in this study.

2.4 Capacitated model

In elementary schools, the allocation of students to schools is determined by quantitative factors such as the number of teachers and classrooms. Easy school has its own capacity, which directly relates to the number of acceptable students. However, such capacity is not taken into account in the traditional model. This might result in exceeding capacity at several sites. We thus modify the traditional model to incorporate the capacity of facilities. The resulting formulation is as follows:

Maximize

$$\sum_{i \in I} b_i \overline{y_i}, \tag{5}$$

subject to:

$$\sum_{j \in J_i} x_j = \overline{y_i}\,,\; \forall i \in I, \tag{6}$$

$$\sum_{j \in J} x_j = p, \tag{7}$$

$$\left(\sum_{i \in I_j} b_i\right) x_j \leq c, \quad \forall j \in J, \tag{8}$$

$$x_j \in \{0,1\}, \tag{9}$$

$$\overline{y_i} \in \mathbb{Z}\; (\overline{y_i} \geq 0). \tag{10}$$

Objective (5) maximizes the amount of demand coverage, wherein a modified parameter b_i is accompanied. Both hand-sides of (6) indicate the number of facilities covering the demand i. Constraint (7) enforces the number of facilities to install to be p. Inequality (8) restricts the total usage at the facility j below or equal to the capacity c. Constraint (9) associates the selection of facility j with a binary variable {0, 1}. Constraint (10) signifies the number of selected facilities to cover demand i. The associated notations are as follows:

i: index of demand point,
I: set of all demand points,
I_j: set of demand points covered by j,
j: index of potential facility point,
J: set of all potential facility points,
J_i: set of facility points capable of serving demand i,
b_i: quantity demanded at demand point i,
c: capacity of facilities,
p: number of facilities to install,
s: service distance by a single facility (in meters).

Selection of facility j and coverage of demand i are reflected along with the following decision variables.

$$x_j = \begin{cases} 1\text{: facility } j \text{ is installed,} \\ 0\text{: facility } j \text{ is not installed,} \end{cases}$$

$\overline{y_i}$: number of facilities covering demand i.

If $\overline{y_i} = 0$, then it means that the demand i is not covered by any facility.

3. Case study

An assessment is conducted for Machida City, located southernmost of Tokyo, Japan. Figure 1 identifies the location of Tokyo Metropolitan and Machida City. The union of the yellow and green objects identifies Tokyo Metropolitan and Machida City, while the latter is delineated in light green.

Two types of datasets containing town boundaries and population are used. The boundary data is delineated in Figure 2. The northern and northwestern areas have rich natural resources and low populated, while the those between the central and south areas are highly populated.

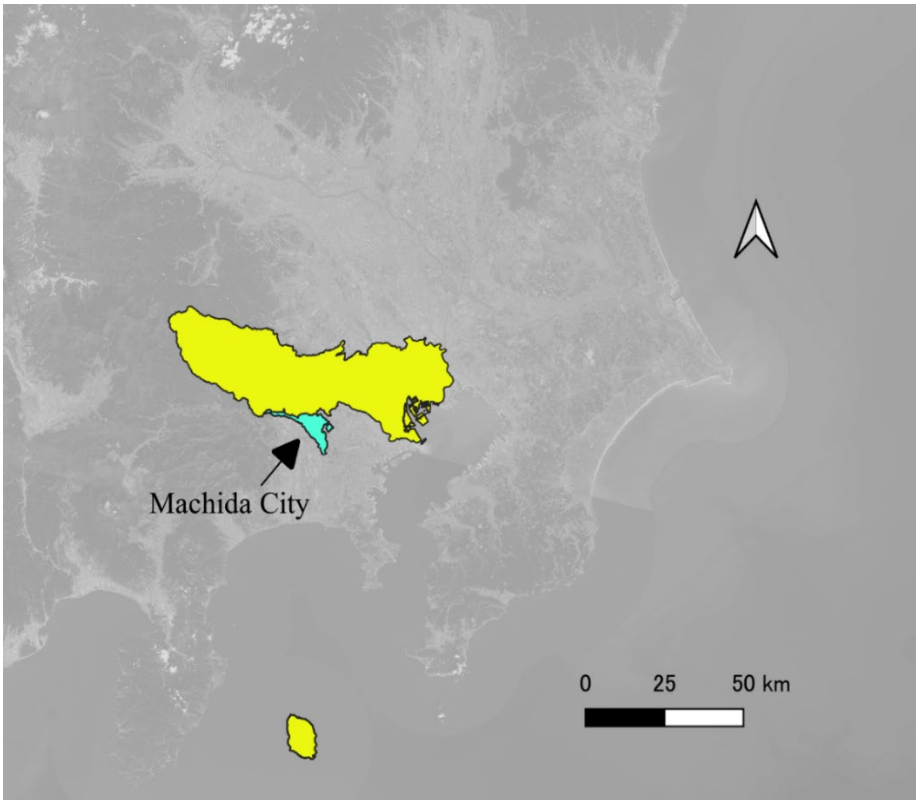

Figure 1. Location of Machida City, Tokyo

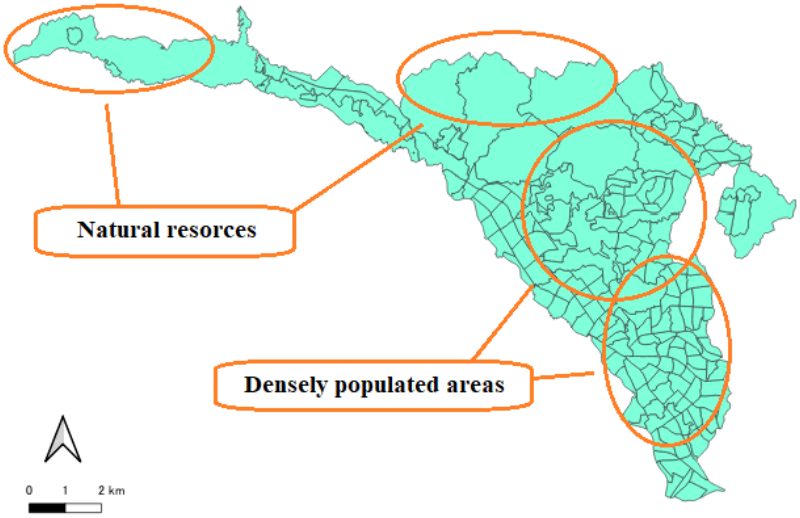

Figure 2. Town boundaries of Machida City

3.1 Demand and supply points

In facility location-allocation problems, a demand point is a group of persons receiving service, and a supply point is a facility capable of providing service. A potential facility corresponds to an existing elementary school. The demand points are delineated at the centroid of each town or street. If the population of a town is zero, then it is deemed a void area, on which no demand point is delineated. Supply points are delineated based on the coordinates of elementary schools, the locations of which can be retrieved through the website of Machida City.

The demand and supply points are delineated in Figure 3 and Figure 4, respectively. According to the standard in Japan, the distance to commute to a designated elementary school is supposed to be 4 km at the maximum. On the other hand, the actual average commuting distance in populated cities such as Tokyo is some 1 km, or even shorter. Considering this, we set the service distance of a single supply point to 1 km, which will be used in the succeeding experiments. With respect to the number of demand points and supply points, they are 195 and 42, respectively.

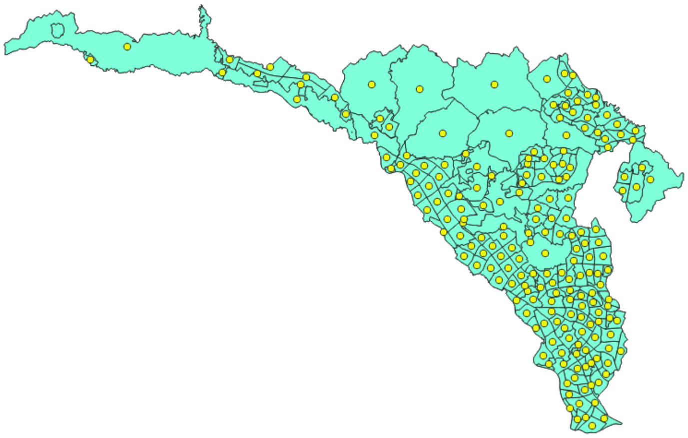

Figure 3. Demand Points

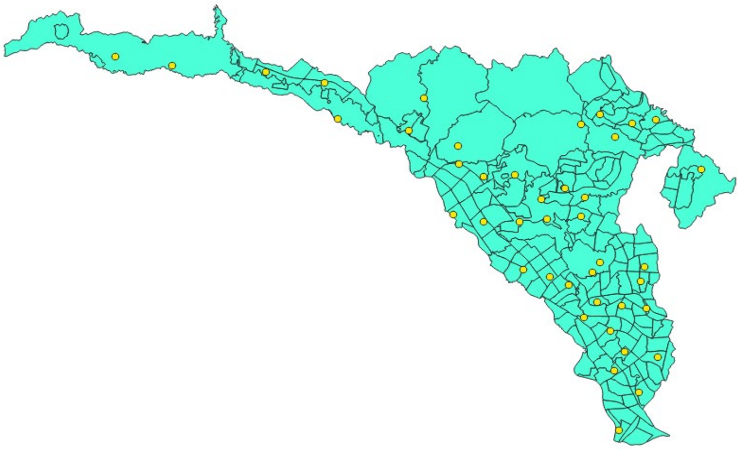

Figure 4. Supply Points

3.2 Population data manipulation

The population data between the ages of 0 and 14 are allocated to each demand point with a demand quantity. The data is obtained from the age population for each town and district. The dataset open to public is the Traditional Resident Registry (as of July 1, 2023), which is retrieved from the website of Machida City. The total population between the ages of 0 and 14 is 48,886. With respect to towns with multiple demand points, the population is divided and weighted evenly for each demand point. In this experiment, the population coverage, denoted by r, is defined by the ratio of the amount of demand covered to the total demand, as follows:

$$r = \begin{cases} \dfrac{\sum_{i \in I} a_i y_i}{\sum_{i \in I} a_i} : & \text{basic model,} \\ \dfrac{\sum_{i \in I} b_i \overline{y_i}}{\sum_{i \in I} a_i} : & \text{capacitated model.} \end{cases} \tag{11}$$

3.3 Population distribution

As shown in Figure 5, several demand points could be covered by multiple supply points. The population on a demand point is reduced by the number of facilities that are able to cover the demand point. If a demand point can be covered by multiple facilities, then a modified demand denoted by b_i is defined as the total population divided by the total number of facilities capable of covering it. The modified demand quantity b_i is defined as:

$$b_i = \begin{cases} \dfrac{a_i}{|J_i|} : & \text{if } |J_i| \geq 2, \\ a_i : & \text{otherwise.} \end{cases} \tag{12}$$

The average number of students in public elementary schools in Tokyo is 466, which is transferred to the capacity of facilities, c. The population data is regarding children aged 0-14. Since Japanese elementary school children are aged from 6 to 12, the capacitated model experiment is conducted as 2/5(= 6/15) of the demand. The objective function and constraint conditions are modified according to the newly set demand b_i and facility capacity c.

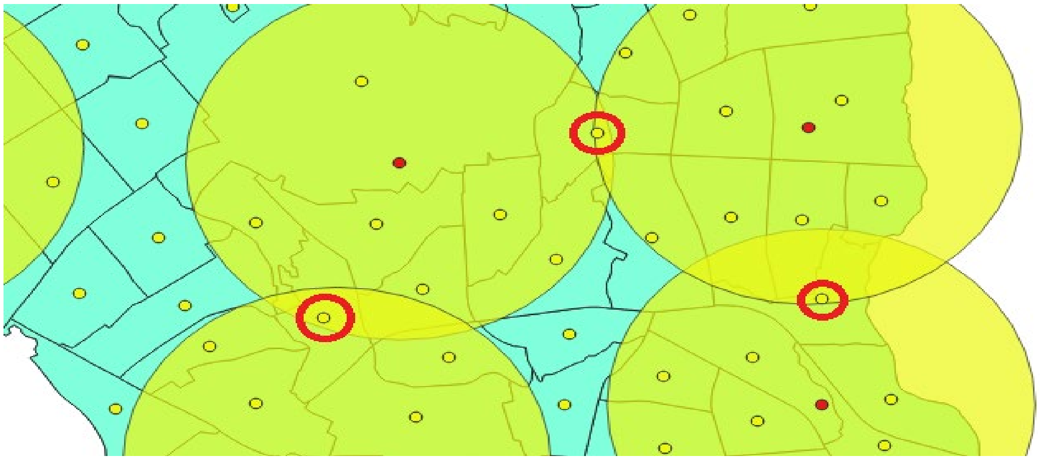

Figure 5. Demand points covered by multiple supply points

4. Assessment

4.1 Traditional model

The result based on the traditional model is depicted in Figure 6. Table 1 reports the optimal values and coverage rates with varying p=42, 27, 26, and 25. The first one is set equal to the current number of schools, whereas the latter three are based on the future plans of consolidation. Comparing the current situation and the optimal solution for p=26, the coverage rate does not decrease significantly. This is also similar to those for p=25 and p=27. The coverage rate does not change significantly even if the number of facilities is around the planned target number p=25. This fact may imply that a further decrease in the number of schools down to p=25 schools suggests that it might be possible to change the planned target p=26 to a smaller number of schools. This might be a realistic solution to cope with teacher shortage.

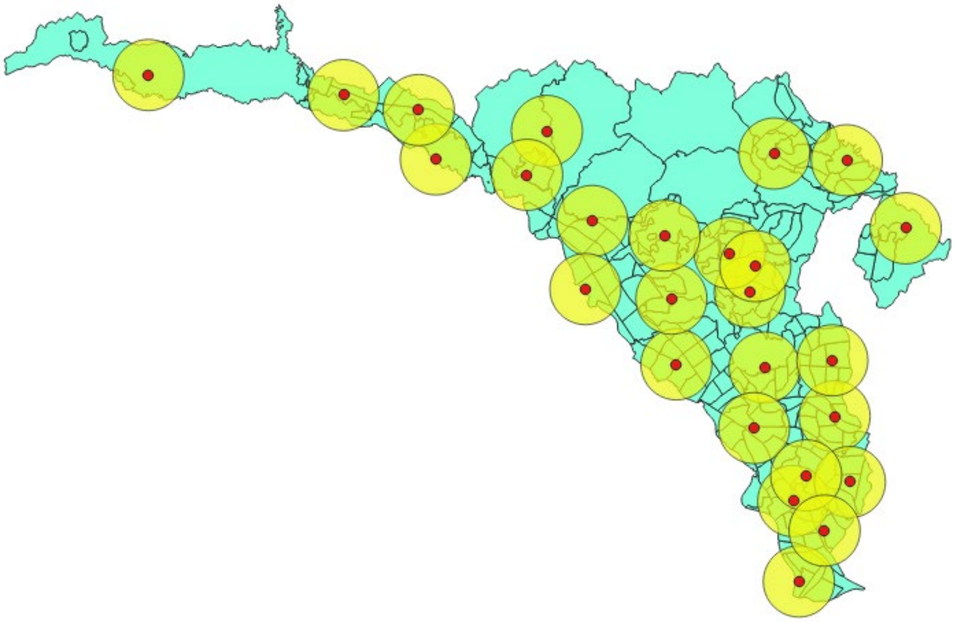

Figure 6. Optimal solution of the traditional model.

Table 1. Optimal values and coverage rates based on the traditional model

	p=42 (Current)	p=27	p=26	p=25
Optimal values	44,114	43,768	43,584	43,303
Coverage rate *r* (%)	90.2	89.5	89.2	88.6

4.2 Capacitated model

Figure 7 delineates the optimal locations obtained by the capacitated model. Table 2 summarizes the optimal values of the traditional basic model and proposed capacitated model, along with the current locations. The corresponding coverage rate for each case is also accompanied. The optimal placement of the capacitated model appears to be densely populated compared to the optimal placement of the traditional model. It also resulted in a population coverage rate of 49.7%. Table 3 shows the maximum number of people that can be covered and the capacity ratio for p=25, 26, and 27. Along with 26 facilities, the maximum number of people that can be covered is 12,116.

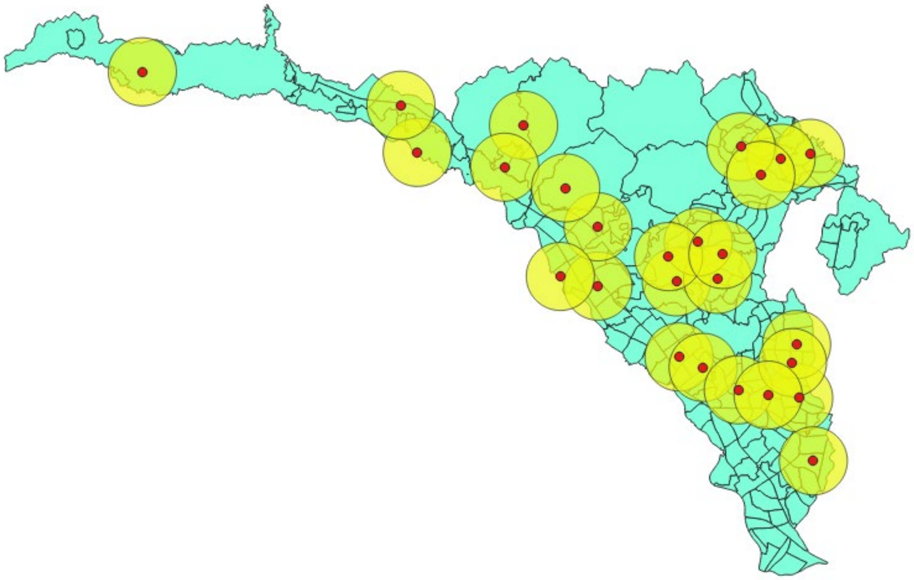

Figure 7. Optimal solution of the capacitated model.

For the planned target of 26 schools, the utilization rate is 80.3%, which means that there is still room of the remaining 19.7% to full utilization. Also, it can be seen that the utilization rate decreases as the number of facilities increases, while it increases as the number of facilities decreases.

Table 2. Optimal values and coverage rates for each case.

	p=42 (Current)	Basic model	Capacitated model
Opimal values	44,114	43,584	9,728
Coverage rates r(%)	90.2	89.2	49.7

Table 3. Maximum number of people to be covered and facility capacity utilization.

	p=25	p=26	p=27
Optimal values	9,442	9,728	9,985
Coverage rates r(%)	48.3	49.7	51.1
Maximum number of people that can be covered	11,650	12,116	12,582
Capacity utilization (%)	81.0	80.3	79.4

5. Conclusion

Two decades ago, Japan began facing a rapid and continuing decrease in the number of younger people as the multiple outcomes of population decrease, low birthrate, and population aging. Not a few municipalities must tackle this along with a school restructuring plan. Not only the continuing decrease in the number of students, but also the teacher shortage must be taken into account. School consolidation would be a central solution to achieve sustainability of elementary education, while residents must be patient with elongated commuting distances.

Considering these, a benchmark study was conducted for a city in Tokyo Metropolitan. To yield an objective consolidation, a traditional service coverage model called MCLP is adopted. Derived from this, a capacitated model that averts excessing capacity is constructed. A case study was conducted for Machida City, located southernmost of Tokyo, where there are currently 42 elementary schools. Experimental results with the traditional MCLP revealed that: even if the number of schools is reduced to 26 from the current number 42, the population coverage would not decrease by more than 1%. The results also show that the coverage rate does not change when comparing the cases for 25, 26, and 27 schools. A key finding is that there would not be a severe problem even if a further decrease is enforced: a solution better than the current plan might exist to cope with the aging and teacher shortage. The experimental results of the capacitated model resulted in a population coverage of 49.7%. The optimal placement of the capacitated model appears to be densely populated compared to the optimal outlet of the traditional model. For the planned target of 26 schools, the maximum number of students that could be covered was 12,116, while the actual number of students covered was 9,728, along with a capacity utilization of 80.3%.

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Biographies

Hiroto Yamazaki is a graduate student in Graduate School of Science and Engineering, Hosei University, Japan. He received his B.E. degree from Hosei University in 2024.

Hiroyuki Goto is a professor in the department of Industrial and Systems Engineering, Hosei University, Japan. He received his B.S. and M.S. degrees from The University of Tokyo in 1995 and 1997. He received his D.E. degree from Tokyo Metropolitan Institute of Technology in 2004. His research interests include operations research, geographic information science, and high-performance computing.