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Synergistic Strategy for Group Decision-Making in Materials Selection Problem Using Fuzzy MCDM, A Case Study

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Abstract

Selecting the right material for a product is crucial, impacting performance and manufacturing. This research offers a valuable tool for engineers and designers facing complex material selection challenges within group decision making process. It introduces a novel framework for material selection that tackles uncertainties in the decision-making process. The proposed approach combines Fuzzy COPRAS with existing aggregation techniques. Fuzzy COPRAS allows for the inclusion of imprecise information, making it suitable for real-world scenarios with inherent ambiguities. A case study showcases the practical application of this model by selecting the optimal material for high-pressure turbine blades. The model efficiently evaluates different materials like Nickel-based super alloys, Titanium alpha beta super alloys, and Stainless steels. A comparative analysis is conducted to show the validity of the framework.

Keywords
COPRAS, MCGDM, Materials Selection, Fuzzy Sets, Crisp Numbers

1.Introduction
Despite its critical role in optimizing product performance and cost (Thakker et al. 2008), material selection is susceptible to errors during the initial design phase (Edwards 2005; Deng and Edwards 2007). A poor decision at this stage can lead to premature component failure and unnecessary expenses. The constant emergence of new materials presents exciting possibilities for maximizing performance and minimizing costs across various industries. However, this rapid development creates a vast pool of potential materials for a single product or component, overwhelming engineers and designers with the challenge of selecting the best option while considering a multitude of performance-critical criteria (Jajimoggala and Karri 2013). Prioritizing these criteria, and determining which properties are most crucial, is equally important (Zhao et al. 2016). A key hurdle in material selection is the lack of information. Implementing appropriate decision-making procedures alongside suitable material samples can mitigate this challenge. These procedures involve selecting the most fitting material for the intended product, prioritizing available options, and ranking them accordingly. Selecting the right decision-making tool is fundamental for effective material selection and prioritization (Baghel et al. 2014).

Choosing the most suitable material for complex design problems remains a persistent challenge for materials engineers, designers, and decision-makers. Fortunately, Multi-Criteria Decision Making (MCDM) methods can assist decision-makers in establishing efficient and effective designs by strengthening the decision-making process in material selection for engineering applications. Over the past few decades, MCDM methods have demonstrated significant potential in addressing intricate design problems within the realm of engineering materials selection. These methods enhance existing quantitative approaches, such as selection charts, by enabling the simultaneous consideration of design criteria, component configurations, and material types (Jahan et al. 2016).

2. Literature Review

Several MCDM approaches are available for material selection, including Ashby (Prashant Reddy et al. 2010; Rashedi et al. 2012), TOPSIS (Shanian and Savadogo 2006; Dağdeviren et al. 2009; Momena & Nidal 2021; Momena 2024), VIKOR (Jahan et al. 2011; Jeya Girubha and Vinodh 2012), ELECTRE (Shania and Savadogo 2006), PROMETHEE (Chatterjee and Chakraborty 2012), COPRAS (Chatterjee et al. 2011; Maity et al. 2012), and COPRAS-G (Maity et al. 2012). These methods aim to identify optimal material solutions for engineering components and products, thereby enhancing manufacturing performance and efficiency.

Jahan et al. (2010) identified TOPSIS, ELECTRE, and AHP as the most popular MCDM approaches for material selection in the preceding decade. However, they highlighted drawbacks in implementing ELECTRE and AHP, making TOPSIS the preferred choice. ELECTRE's limitations include the absence of clear numerical values to differentiate material alternatives and the rapid increase in computational complexity with a growing number of options. Additionally, ELECTRE analyzes alternatives under each criterion individually, which is impractical for material selection where all criteria must be assessed for each alternative. AHP also has limitations, including a restricted number of material alternatives and criteria (not exceeding 15) unsuitable for the vast set typically encountered in material selection problems (Mousavi-Nasab and Sotoudeh-Anvari 2017). Furthermore, both ELECTRE and AHP cannot handle benefit and cost criteria simultaneously, rendering them inapplicable to material selection.

Complex proportional assessment (COPRAS), introduced by Zavadskas et al. (1994), is another recommended MCDM approach for material selection. This method identifies the most favorable solution by comparing the direct and proportional ratio of the best solution to the ideal-worst solution. Recent studies extensively reviewing MCDM applicability in material selection suggest that COPRAS, alongside TOPSIS, is among the most suitable techniques (Mousavi-Nasab and Sotoudeh-Anvari 2017). They proposed a comprehensive MCDM approach using TOPSIS, COPRAS, and data envelopment analysis (DEA) for general material selection problems. Additionally, Mousavi-Nasab and Sotoudeh-Anvari (2018) investigated rank reversal phenomena in applying MCDM to material selection, concluding that decision-makers should employ TOPSIS, COPRAS, and simple additive weight (SAW) in a comparative approach to verify ranking results. Chatterjee et al. (2011) evaluated the performance of various MCDM approaches, indicating COPRAS as a top material selection technique due to its straightforward procedure and minimal calculation time.

Researchers have explored COPRAS implementation in diverse material selection applications. Zavadskas et al. (2014) reviewed various MCDM methods, highlighting COPRAS as a rapidly developed method well-suited for real-world problems. Chatterjee and Chakraborty (2012) investigated implementing COPRAS and additive ratio assessment (ARAS) techniques in a comparative analysis for gear material selection. Their study demonstrated the effectiveness of both methods for various industrial material selection processes, including problems with numerous criteria and alternatives. Aghdaie et al. (2013) employed a hybrid model integrating step-wise assessment ratio analysis (SWARA) and COPRAS-G to evaluate and select the most suitable machine tool. The results of implementing the hybrid model in a machine tool selection problem validated COPRAS-G's effectiveness in precisely ranking alternatives from best to worst. Petković et al. (2015) incorporated COPRAS with weighted aggregated sum product assessment (WASPAS) in a novel MCDM approach for non-conventional machining of ceramics. Despite the challenge of handling a large number of criteria and alternatives in most NCMP selection problems, this method proved efficient in selecting the optimal NCMP for ceramics machining, suggesting its potential application to various manufacturing decision-making problems. Yazdani (2015) tested the impact of implementing various normalization methods in COPRAS on material and design selection decisions. He justified selecting COPRAS for this study due to its recognition as a promising approach in the material selection field, designed to handle complex real-world problems with conflicting attribute properties.

2.1 Applications of Fuzzy COPRAS to MCDM Problems in Material Selections
Due to its simplicity, the COPRAS method has gained recognition as an effective MCDM technique for real-world decision-making problems (Zavadskas et al. 2014). Its successful applications have spanned various fields (Podvezko 2011; Antucheviciene et al. 2012; Mulliner et al. 2013; Gadakh 2014; Mousavi-Nasab and Sotoudeh-Anvari 2017; Chatterjee et al. 2011). Researchers have explored integrating fuzzy logic with COPRAS (Fuzzy COPRAS) to address material selection problems. Dursun and Arslan (2018) proposed a fuzzy multi-criteria group decision-making (MCGDM) framework incorporating customer and expert input. This framework utilizes fuzzy COPRAS to rank material alternatives. In the initial assessment phase, they leverage the principles of quality function deployment (QFD), 2-tuple fuzzy linguistic representation, and linguistic hierarchies. Fuzzy COPRAS is then applied to rank and select the most suitable material.

Another study by Nguyen et al. (2014) developed a hybrid approach combining fuzzy ANP (Analytic Network Process) and COPRAS-G (COmplex Proportional ASsessment of alternatives with Grey relations) for machine tool selection. Their three-phase model involves setting criteria and alternatives, determining criteria interaction and weights using fuzzy ANP, and selecting the best option through COPRAS-G ranking. The authors highlight the effectiveness of COPRAS-G in incorporating uncertainty through interval values, aiding in accurate ranking and selection. Further research by Nguyen et al. (2015) presented an MCDM framework integrating fuzzy AHP and fuzzy COPRAS for machine tool evaluation. This framework allows for the processing of uncertain information during data collection. Fuzzy linguistic preference relations are integrated into AHP to determine the MCDM matrix elements based on decision-maker thoughts and opinions. Fuzzy COPRAS is then employed in the ranking phase.

The study emphasizes the value of this integrated approach for machine tool selection, particularly its ability to handle uncertainty and model decision-maker judgments for the ranking process. However, a thorough examination of the existing literature reveals a scarcity of research specifically focused on implementing the fuzzy COPRAS method for material selection problems. This research gap necessitates the development of a more robust model. To address this limitation, we propose a Hybrid Pythagorean fuzzy MCGDM COPRAS model tailored for real-world material selection applications that are related to group decision-making.

2.2 Research Significance
To the best of our knowledge, the contribution in the research area regarding the implementation of the fuzzy COPRAS method in material selection problems is still insufficient. For this reason, this research will introduce an effective Hybrid Pythagorean fuzzy MCGDM COPRAS model for the application of real life materials selection problems. The objectives of this research can be summarized as follows:
1. To build a hybrid MCGDM framework through integrating an aggregation approach into a ranking method that deal with PFNs such as COPRAS.
2. To implement the hybrid Pythagorean Fuzzy MCGDM method into real world materials selection problem.
3. To evaluate the accuracy and sensitivity of the proposed method decision making outcomes.

3. Methodology
3.1 Pythagorean Fuzzy Sets (PFS)
DEFINITION 1. Let a set X be a universe of discourse. A PFS P is an object having the form

$$P = \{ < x, P\left(\mu_P(x), v_P(x)\right) > | x \in X\} \tag{1}$$

where the function μ_P: X → [0, 1] describes the degree of membership and v_P : X → [0, 1] describes the degree of nonmembership of the element x ∈ X to P, respectively, and for every x ∈ X, it holds that

$$(\mu_P(x))^2 + (v_p(x))^2 \leq 1 \tag{2}$$

For any PFS P and x ∈ X, $\pi_P(x) = \sqrt{1 - \mu_P^2(x) - v_P^2(x)}$ is called the degree of indeterminacy of x to P. Moreover, in order to simplify it, $P(\mu_P(x), v_P(x))$ is called a Pythagorean fuzzy number (PFN) denoted by $\beta = P(\mu_\beta, v_\beta)$, where $\mu_\beta, v_\beta \in$ [0, 1], $\pi_\beta = 1 - (\mu_\beta)^2 - (v_\beta)^2$, and $(\mu_\beta)^2 - (v_\beta)^2 \leq 1$.

Yager (2013, 2014), and Yager & Abbasov (2013) stated the major operations on three PFNs, which are $\beta_1 = P\left(u_{\beta_1}, v_{\beta_1}\right), \beta_2 = P\left(u_{\beta_2}, v_{\beta_2}\right)$ and $\beta = P(\mu_\beta, v_\beta)$ as follows:

1. $\beta_1 \cup \beta_2 = P(\max\{\mu_{\beta_1}, \mu_{\beta_2}\}, \min\{v_{\beta_1}, v_{\beta_2}\})$

2. $\beta_1 \cap \beta_2 = P(\max\{\mu_{\beta_1}, \mu_{\beta_2}\}, \min\{v_{\beta_1}, v_{\beta_2}\})$
3. $\beta^C = P(v_\beta, u_\beta)$

Zhang and Xu (2014) developed some operations on the basis of a relationship between PFNs and IFNs, which can be expressed as follows:

4. $\beta_1 \oplus \beta_2 = P\left(\sqrt{\mu_{\beta 1}^2 + \mu_{\beta 2}^2 - \mu_{\beta 1}^2\mu_{\beta 2}^2},\ v_{\beta_1}v_{\beta_2}\right)$
5. $\beta_1 \otimes \beta_2 = P\left(u_{\beta_1}u_{\beta_2}, \sqrt{v_{\beta 1}^2 + v_{\beta 2}^2 - v_{\beta 1}^2 v_{\beta 2}^2}\right)$
6. $\lambda\beta = P\left(\sqrt{1-(1-\mu_\beta^2)^\lambda},\ (v_\beta)^\lambda\right), \lambda > 0$
7. $\beta^\lambda = P\left((\mu_\beta)^\lambda,\ \sqrt{1-(1-v_\beta^2)^\lambda},\right), \lambda > 0$

THEOREM 1.

For three PFNs $\beta_1 = P(u_{\beta_1}, v_{\beta_1}), \beta_2 = P(u_{\beta_2}, v_{\beta_2})$, and $\beta = P(\mu_\beta, v_\beta)$, the following ones are valid:

1. $\beta_1 \oplus \beta_2 = \beta_2 \oplus \beta_1$
2. $\beta_1 \otimes \beta_2 = \beta_2 \otimes \beta_1$
3. $\lambda(\beta_1 + \beta_2) = \lambda\beta_1 \oplus \lambda\beta_2,\ \lambda > 0$ λ (β1 + β2) = λβ1 ⊕ λβ2, λ > 0.
4. $\lambda_1\beta \oplus \lambda_2\beta = (\lambda_1 + \lambda_2)\beta, \lambda_1, \lambda_2 > 0$
5. $(\beta_1 \otimes \beta_2)^\lambda = {\beta_1}^\lambda \otimes {\beta_2}^\lambda,\ \lambda > 0$
6. $\beta^{\lambda_1} \otimes \beta^{\lambda_2} = \beta^{(\lambda_1+\lambda_2)}, \lambda_1, \lambda_2 > 0$

DEFINITION 2.

Let $\beta_j = P\left(u_{\beta_j}, v_{\beta_j}\right)$ $(j = 1,2)$ be two PFNs; then the nature quasi-ordering on the PFNs can be defined as follows:
$\beta_1 \geq \beta_2$ if and only if $u_{\beta_1} \geq u_{\beta_2}$ and $v_{\beta_1} \leq v_{\beta_2}$.
Also, a score function is developed and implemented in order to compare the magnitudes of two PFNs as defined in what follows.

DEFINITION 3.

Let $\beta = P(u_\beta, v_\beta)$ be a PFN; then the score function of β can be defined as follows:

$$s(\beta) = (u_\beta)^2 - (v_\beta)^2 \tag{3}$$

It is observed that the score function $s(\beta)$ contains some desirable properties as below.
Rule 1. For any PFN $\beta = P(u_\beta, v_\beta)$, the proposed score function $s(\beta) \in [-1, 1]$.
Rule 2. For two PFNs $\beta_j = P\left(u_{\beta_j}, v_{\beta_j}\right)$ $(j = 1,2)$, if $\beta_1 \leq \beta_2$, then $s(\beta_1) \leq s(\beta_2)$.
The subsequent laws are introduced on the basis of the score function of PFNs in order to make a comparison between two PFNs.

DEFINITION 4.

Let $\beta_j = P\left(u_{\beta_j}, v_{\beta_j}\right)$ $(j = 1,2)$ be two PFNs, $s(\beta_1)$ and $s(\beta_2)$ be the scores of β_1 and β_2, respectively; then

1. If $s(\beta_1) < s(\beta_2)$, then $\beta_1 \prec \beta_2$;
2. If $s(\beta_1) > s(\beta_2)$, then $\beta_1 \succ \beta_2$;
3. If $s(\beta_1) = s(\beta_2)$, then $\beta_1 \sim \beta_2$;

Example 4.1.

Let $\beta_1 = P(\frac{\sqrt{3}}{2},\frac{1}{2})$ and $\beta_2 = P(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2})$, according to Definitions 4.4-4.5, we have $s(\beta_1) = (\frac{\sqrt{3}}{2})^2 - \left(\frac{1}{2}\right)^2 = \frac{1}{2}, s(\beta_2) = \left(\frac{\sqrt{2}}{2}\right)^2 - \left(\frac{\sqrt{2}}{2}\right)^2 = 0$

Evidently, $s(\beta_1) > s(\beta_2)$, thus $\beta_1 \succ \beta_2$.

3.2 The Fuzzy COPRAS Approach

This approach has been proposed by Peng & Selvachandran (2017) to effectively address MCDM problems with PFNs (Peng & Selvachandran 2017). Also, the illustrated method is developed on the concept that the optimal alternative should have the highest utility level.

Initially, we need to set the Pythagorean fuzzy decision matrix $R = P(\mu_{ij}, v_{ij})_{mxn}$ according to the following equation:

$$R' = P(\mu'_{ij}, v'_{ij})_{mxn} = \begin{cases} P(\mu_{ij}, v_{ij}) & is\ benefecial\ element \\ \\ P(v_{ij}, \mu_{ij}) & is\ non-benefecial\ element \end{cases} \quad (4)$$

Then, we need to measure the weighted Pythagorean fuzzy decision matrix by multiplying criteria C_j weights to the collective decision making matrix in Model (4.4). The new weighted Pythagorean fuzzy decision matrix $S = (s_{ij})_{m\times n}$ can be calculated using the following equation:

$$s_{ij} = w_j R' = \left(\sum_{i=1}^{m} w_j \mu'_{ij}, \sum_{i=1}^{m} w_j v'_{ij}\right) \quad (5)$$

After that, the summation of beneficial criteria values will be calculated for each alternative. Let BA refers to beneficial criteria, the higher values of which are better. As well, the summation of beneficial criteria values will be calculated for each alternative. Let CA refer to non-beneficial criteria, the lower values of which are better. The equations to calculate the sum of beneficial criteria and non-beneficial criteria for each alternative can be written, respectively, as follows:

$$P_{+i} = \underset{j\in BA}{\oplus} \boldsymbol{s_{ij}} \quad (6)$$

$$P_{-i} = \underset{j\in CA}{\oplus} \boldsymbol{s_{ij}} \quad (7)$$

where $P_{+i}\ and\ P_{-i}$ represent the beneficial criteria values and non-beneficial criteria values, respectively. In fact, $CA \cup BA$ is all the criteria.

The next step is to measure the score of $P_{+i},\ P_{-i}\ and\ P_{-min}$ using the score function in equation (3). It is important to note that P_{-min} refers to alternative i with the lowest value that belongs to a non-beneficial criterion. The calculated score functions can be referred to as $s(P_{+i}),\ s(P_{-i})\ and\ s(P_{-min})$. Also, $s(P_{-min})$ can be written as $\min_i s(P_{-i})$.

Now, we can calculate the relative weight of each alternative. Thus, the relative priority $Q(x_i)$ for each alternative can be determined by the following formula:

$$Q(x_i) = s(P_{+i}) + \frac{s(P_{-min}) \sum_{i=1}^{m} e^{s(P_{-i})}}{e^{s(P_{-i})} \sum_{i=1}^{m} \frac{s(P_{-min})}{e^{s(P_{-i})}}} = s(P_{+i}) + \frac{\sum_{i=1}^{m} e^{s(P_{-i})}}{e^{s(P_{-i})} \sum_{i=1}^{m} \frac{1}{e^{s(P_{-i})}}}$$

$$,\ i = 1,2,\dots,m;\ j = 1,2,\dots,n \quad (8)$$

After that, we obtain the maximum relative priority $Q_{max}(x_i)$ to calculate the level of utility $U(x_i)$ for each alternative i as follows:

$$U(x_i) = \frac{Q(x_i)}{Q_{max}(x_i)} \times 100 \quad (9)$$

Finally, the alternatives can be ranked based on the level of utility $U(x_i)$ for each one: the larger the value of $U(x_i)$, the more preference of the alternative i. Accordingly, the best alternative is going to have the highest utility degree $U_{max}(x_i)$.

3.3 The Algorithm of the Proposed Fuzzy Pythagorean MCGDM TOPSIS Method

In this section, a practical algorithm of the Pythagorean fuzzy TOPSIS approach is demonstrated based on the previous methodology explanation. The algorithm can be applied by following the next steps:

Step 1. Establish group decision matrix $X_i = (r^i_{kj})_{t \times n}$ for each ith alternative in which each decision maker $D = \{d_k : k = 1,2, \ldots, t\}$ evaluates the importance of each given criterion $C = \{C_j : j = 1,2, \ldots, n\}$ with respect to each alternative $x = \{x_i : i = 1,2, \ldots, m\}$.
Step 2. Assign Criteria weights.
Step 3 Assign DMs weights according to their importance.
Step 4. Aggregate DMs crisp values into fuzzy numbers using the aggregation approach (Momena, A.F, 2023).
Step 5. Set collective evaluation Pythagorean fuzzy decision matrix $R = P(\mu_{ij}, v_{ij})_{mxn}$ into $R' = (\mu'_{ij}, v'_{ij})_{mxn}$.
Step 6. Determine the weighted Pythagorean fuzzy decision matrix $S = (s_{ij})_{m \times n}$ using equation (5).

Step 7. Calculate the sum of beneficial criteria P_{+i} and non-beneficial criteria P_{-i} for each alternative by implementing equations (6) and (7), receptively.

Step 8. Use the score function in equation (3) to measure the score of P_{+i}, P_{-i} and P_{-min}. The calculated score functions will be represented as $s(P_{+i})$, $s(P_{-i})$ and $s(P_{-min})$, respectively.

Step 9. Measure the relative priority $Q(x_i)$ for each alternative as well as determine $Q_{max}(x_i)$ using equation (8).

Step 10. Calculate the level of utility $U(x_i)$ for each alternative i by applying equation (9).

Step 11. Define the optimal ranking order of the alternatives and find the optimal alternative. Based on the utility degree $U(x_i)$ achieved from Step 10, the alternatives will be ranked into order with respect to the declining values of $U(x_i)$ $(i = 1,2, \ldots, m)$ and the alternative with the highest utility degree $U(x_i)$ is the optimal one, namely,

$$x^* \coloneqq \{x_i : \left(i : U(x_i) = \max_{1 \le i \le m} U(x_i) \right)\}$$

Application of the Proposed Method in a Real World Material Selection Problem
The following section describes a case study that is based on practical issues in the aerospace industry (inspired by real problems). The case study application aims to examine the applicability of the proposed method for group decision making in the context of materials selection problem.

3.3 Case Study

Aircraft engines consist of three main sections: compressor, combustion chamber, and gas turbine. High-temperature gases from the combustion chamber can significantly decrease the long-term efficiency of turbine blades in turbojet engines. Corrosion and fatigue cracking are additional threats to turbine blade performance. A real-world example of this challenge occurred in 2017 with Rolls-Royce's Trent 1000 high-pressure turbine blades used in Boeing 787 turbofan engines. These blades experienced significant corrosion issues (Johnson & Raina 2018). These blade problems resulted in numerous Boeing 787 groundings and substantial financial burdens for Rolls-Royce. The company incurred an estimated \$35 million for blade inspections and replacements, with the total repair cost projected to reach \$504 million in 2018 (Kollewe 2018). To prevent similar issues, material selection, and design must be optimized for the harsh environment of high-pressure turbine blades. Modern jet engines operate their high-pressure turbines at temperatures exceeding 1350-1450°C (Boyer et al. 2015). Furthermore, the aerospace industry prioritizes lightweight materials; hence, the ideal material should balance high-temperature resistance with minimal weight. Accordingly, in order to select an efficient material for the turbine blade, the material selection process requirements and criteria will be shown in (Table 1) as follows:

Table 1. Material criteria descriptions and their level of importance

Criteria	Material requirements	Material indices	Importance
C1	It should be high-resistant to high temperature environment	Max service temperature	Very important (VI)
C2	It should be low in weight	Low density	Very Important (VI)
C3	It should be high-resistant to tensile stresses	High yield strength	Important (I)
C4	It should be high-resistant to bending stresses	High young's modulus	Important (I)
C5	It should be economic in its price	Low cost	Unimportant (U)

Based on the requirements provided in the previous table, the candidate materials have been narrowed down to titanium and nickel-based super alloys, along with stainless steels. These alternatives were chosen due to their exceptional ability to endure the extreme pressures and temperatures encountered in high-pressure turbine blades. The material candidates can be displayed as follows:

Table 2. Material candidates and their commercial names

Symbol	Material alternative	Commercial name
X1	Nickel-based Super alloy	Inconel 625
X2	Nickel-based Super alloy	Pyromet 680
X3	Nickel-based Super alloy	Haynes R-41
X4	Nickel-Chromium Super alloy	Inconel 706
X5	Nickel-Co-Cr Super alloy	Udimet 700
X6	Titanium alpha beta Super alloy	Ti-6Al-2Sn-2Zr-2Mo (Ti-6-6-6-6)
X7	Austinitic Stainless steel	AISI 202, Wrought
X8	Nickel-Cr-Co-Mo Super alloy	Rene 41
X9	Austinitic Stainless steel	AISI 302, Wrought
X10	Nickel based Super alloy	Rene 80

The weights for the criteria are assigned to ensure the efficiency in the design focus in the decision-making process. Thus, the weights of the selected criteria are service temperature ($\boldsymbol{w_{C_1} = 0.2686}$), density ($\boldsymbol{w_{C_2} = 0.2686}$), yield strength ($\boldsymbol{w_{C_3} = 0.1940}$), young's modulus ($\boldsymbol{w_{C_4} = 0.1940}$) and cost ($\boldsymbol{w_{C_5} = 0.07476}$). A committee of decision makers is formed for the evaluation process as follow: d1: Operation director, d2: Senior consultant, d3: Production manager, d4: Manufacturing engineer, d5: Materials engineer. The committee members' initial subset weights are assigned as $\boldsymbol{d_1} = 0.8$, $\boldsymbol{d_2} = 0.4$, $\boldsymbol{d_3} = 0.4$, $\boldsymbol{d_4} = 0.2$, and $\boldsymbol{d_5} = 0.2$. The primary weight of the DMs are assigned as $\boldsymbol{\mu}(\{\boldsymbol{d_1}\}) = 0.45762$, $\boldsymbol{\mu}(\{\boldsymbol{d_2}\}) = 0.18517$, $\boldsymbol{\mu}(\{\boldsymbol{d_3}\}) = 0.18517$, $\boldsymbol{\mu}(\{\boldsymbol{d_4}\}) = 0.08601$, and $\boldsymbol{\mu}(\{\boldsymbol{d_5}\}) = 0.08601$.

Each DM evaluates every material alternative with respect to each criterion by crisp numbers using a scale of 0 (being worst) to 100 (being best). Decision makers' assessments are provided in Table 3.

Table 3. Decision makers' assessments

Material	Decision maker	C1	C2	C3	C4	C5
Inconel 625	d1	80	10	40	60	95
	d2	80	12	35	60	90
	d3	79	15	46	66	92
	d4	82	5	32	70	80
	d5	80	10	25	68	85
Pyromet 680	d1	15	18	20	62	75
	d2	10	12	25	60	70
	d3	5	14	10	70	72
	d4	12	10	20	70	70
	d5	8	5	18	68	68
Haynes R-41	d1	81	17	75	65	90
	d2	80	12	70	67	82
	d3	78	15	75	70	90
	d4	80	10	68	75	80
	d5	79	5	70	70	85
Inconel 706	d1	65	20	95	100	85
	d2	55	15	95	95	75
	d3	70	20	92	92	88
	d4	65	18	90	90	73
	d5	70	20	92	95	85
Udimet 700	d1	55	25	100	85	60
	d2	50	20	100	80	70
	d3	60	18	95	84	65
	d4	40	20	97	79	35
	d5	61	10	95	90	40
Ti-6-6-6-6	d1	70	100	95	10	40
	d2	70	100	96	15	50
	d3	75	90	100	20	45
	d4	60	95	90	22	35
	d5	81	100	95	8	25
AISI 202	d1	70	40	45	65	60
	d2	73	30	35	72	70
	d3	75	25	55	68	65
	d4	65	20	33	75	30
	d5	80	15	57	69	35
Rene 41	d1	93	20	83	100	10
	d2	95	16	80	95	5
	d3	90	12	82	92	13
	d4	95	10	83	90	15
	d5	90	10	75	95	18
AISI 302	d1	50	35	40	62	30
	d2	45	25	30	60	15
	d3	60	25	55	71	15
	d4	45	18	30	70	18
	d5	48	15	55	65	16

	d1	100	40	90	95	10
	d2	100	28	95	95	5
Rene 80	d3	95	25	90	88	15
	d4	95	22	95	92	15
	d5	95	15	85	90	12

The aggregation method is performed and crisp numbers are converted successfully into final aggregated PFNs a collective evaluation Pythagorean fuzzy multiple criteria group decision making matrix is formed as shown in Table 4.

Table 4. The collective PF-MCGDM matrix

Material	**C1**	**C2**	**C3**	**C4**	**C5**
X1	(0.77442,0.00000)	(0.00000,0.88469)	(0.00000,0.48565)	(0.50317,0.00000)	(0.90959,0.00000)
X2	(0.00000,0.87906)	(0.00000,0.84448)	(0.00000,0.78864)	(0.53507,0.00000)	(0.67061,0.00000)
X3	(0.76980,0.00000)	(0.00000,0.84770)	(0.67885,0.00000)	(0.59306,0.00000)	(0.85850,0.00000)
X4	(0.53859,0.00000)	(0.00000,0.78864)	(0.93548,0.00000)	(0.96231,0.00000)	(0.80359,0.00000)
X5	(0.31042,0.12875)	(0.00000,0.76082)	(0.98372,0.00000)	(0.81892,0.00000)	(0.44961,0.00000)
X6	(0.64692,0.00000)	(0.97691,0.00000)	(0.94413,0.00000)	(0.00000,0.85278)	(0.00000,0.42291)
X7	(0.66198,0.00000)	(0.00000,0.60827)	(0.16002,0.34645)	(0.59949,0.00000)	(0.44243,0.23086)
X8	(0.92443,0.00000)	(0.00000,0.82392)	(0.79462,0.00000)	(0.96231,0.00000)	(0.00000,0.88475)
X9	(0.18676,0.16965)	(0.00000,0.66161)	(0.15995,0.43438)	(0.53371,0.00000)	(0.00000,0.74438)
X10	(0.98197,0.00000)	(0.00000,0.61153)	(0.89442,0.00000)	(0.92966,0.00000)	(0.00000,0.88475)

Then fuzzy COPRAS will be applied by finding the weighted Pythagorean fuzzy decision matrix following steps 2 and step 3. After that, to proceed with the ranking process, we will calculate the summations of the beneficial criteria P_{+i} and the non-beneficial criteria P_{-i}, and calculate their score functions $s(P_{+i})$, $s(P_{-i})$ and $s(P_{-min})$ as shown in step 4 and 5. Then, in order to rank the material alternatives, we should calculate the relative priority $Q(X_i)$ and level of utility $U(X_i)$ as explained in steps 6, 7 and 8. All the calculations are displayed in the next table:

Table 5. Results of the Pythagorean fuzzy COPRAS method

Material alternative (X_i)	P_{+i}	P_{-i}	$s(P_{+i})$	$s(P_{-i})$	$s(P_{-min})$	$Q(x_i)$	$U(x_i)$	*Ranking*
Inconel 625 (X_1)	(0.30567,0.09424)	(0.23765,0.06791)	0.08455	0.05186	-0.06787	1.11109	78.97	8
Pyromet 680 (X_2)	(0.10382,0.38917)	(0.22685,0.05006)	-0.14067	0.04895		0.88886	63.18	10
Haynes R-41(X_3)	(0.45359,0.00000)	(0.22772,0.06410)	0.20574	0.04774		1.23652	87.89	5
Inconel 706 (X_4)	(0.51292,0.00000)	(0.21185,0.05999)	0.26309	0.04128		1.30056	92.44	3
Udimet 700 (X_5)	(0.43317,0.03458)	(0.21185,0.03356)	0.18644	0.04691		1.21808	86.58	6
Ti-6-6-6-6 (X_6)	(0.35698,0.16547)	(0.03157,0.26243)	0.10005	-0.06787		1.25718	89.36	4
AISI 202 (X_7)	(0.32520,0.06722)	(0.18063,0.03303)	0.10124	0.03153		1.14886	81.66	7
Rene 41 (X_8)	(0.58924,0.00000)	(0.28738,0.00000)	0.34720	0.08259		1.34269	95.43	2
AISI 302 (X_9)	(0.18476,0.12986)	(0.23330,0.00000)	0.01727	0.05443		1.04119	74.00	9
Rene 80 (X_{10})	(0.61773,0.00000)	(0.00000,0.23033)	0.38159	0.05305		1.40692	100	1

For more illustration of the proposed method and aforementioned steps, processes can be clearly simplified in (Figure 1) as follow:

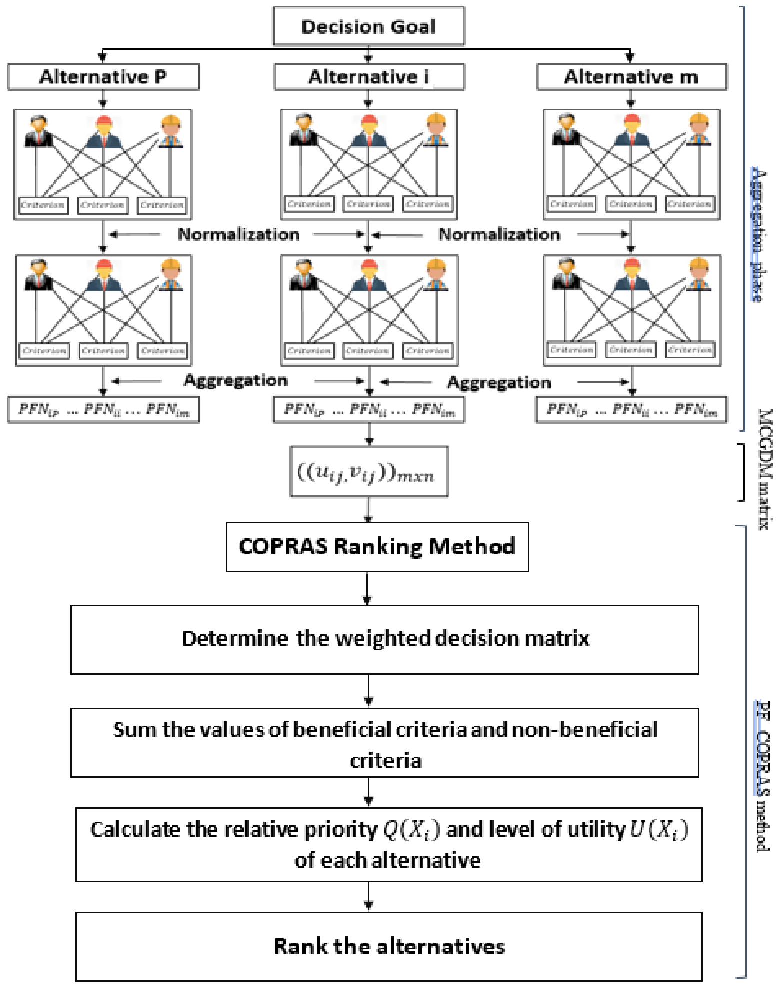

Figure 1. Illustration of the Pythagorean Fuzzy MCGDM COPRAS method

4. Solution and Discussion

The PFMCGDM COPRAS method determined the final ranking of candidate materials based on their utility scores. These scores are calculated by summing the "maximization" and "minimization" indices for each material, representing its relative strengths and weaknesses. The final ranking is $X_{10} \succ X_8 \succ X_4 \succ X_6 \succ X_3 \succ X_5 \succ X_7 \succ X_1 \succ X_9 \succ X_2$. The analysis showed Nickel-based Superalloy Rene 80 as the optimal choice (100% utility) for high-pressure turbine blades, with Nickel-based Superalloy Pyromet 680 (lowest utility) being the least suitable option. Interestingly, the difference in utility between the top two materials (Rene 80 and the second-best option) is relatively small (5%), suggesting comparable performance. Furthermore, Figure 2 indicates minimal variations in utility scores between other material pairs. This implies that several materials exhibit close performance levels, offering some flexibility in the selection process.

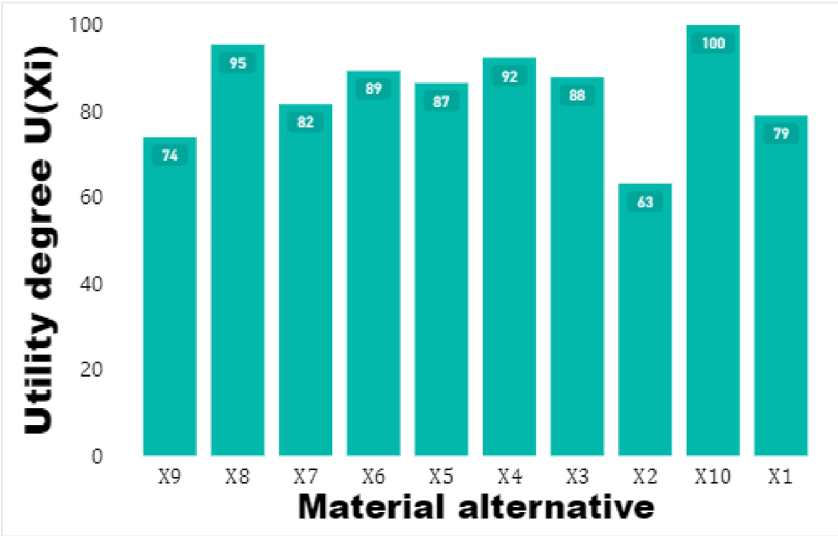

Figure 2. Utility degree $U(X_i)$ for all material alternatives

4.1 Comparative Analysis

A comparative analysis has been suggested to be performed between the fuzzy TOPSIS and the proposed method to validate for results validation. It can be clearly noticed both methods identified Nickel-based Superalloy Rene 80 (X_10) as the best material for high-pressure turbine blades in jet engines as displayed in Table 6.

Interestingly, the top three materials following the optimal choice (X_8, X_4, and X_6) were consistent between the methods, although their ranking order differed slightly. PFMCGDM TOPSIS ranks them as X_10 > X_8 > X_4 > X_6, while PFMCGDM COPRAS has the order X_10 > X_6 > X_8 > X_4 as shown in Figure 3. This difference highlights how the mathematical approaches of each method influence the optimal selection. For example, material X_6 (Titanium alpha beta Super alloy, Ti-6-6-6-6) is ranked second in PFMCGDM TOPSIS due to its proximity to the ideal solution in key criteria (density and tensile strength). However, PFMCGDM COPRAS prioritizes materials with the lowest sum of negative criteria and the highest sum of positive criteria. As X_6 falls short in maximizing positive criteria, its utility score decreases, placing it fourth in the COPRAS ranking.

Finally, both methods agree on the ranking of the bottom four materials (X_7 > X_1 > X_9 > X_2), with Nickel-based Superalloy Pyromet 680 (X_2) being the least favorable option.

Table 6. Comparative ranking analysis using Intuitionistic fuzzy number

Material (Alternative)	Proposed method Ranking	PF TOPSIS Ranking
Inconel 625 (X_1)	8	8
Pyromet 680 (X_2)	10	10
Haynes R-41(X_3)	5	6
Inconel 706 (X_4)	3	4
Udimet 700 (X_5)	6	5
Ti-6-6-6-6 (X_6)	4	2
AISI 202 (X_7)	7	7
Rene 41 (X_8)	2	3
AISI 302 (X_9)	9	9
Rene 80 (X_{10})	1	1

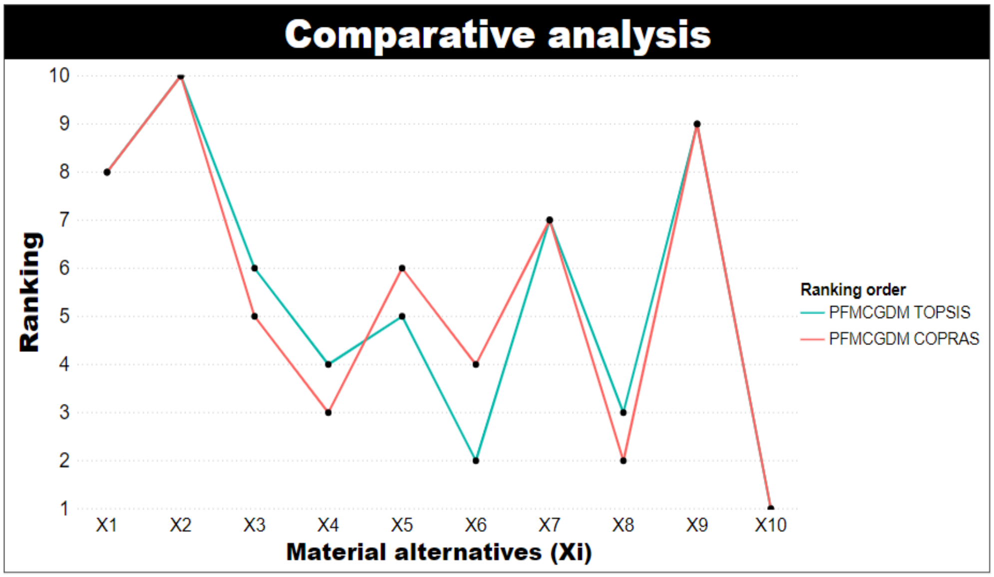

Figure 3. Comparison of final ranking outcomes between Hybrid PFMCGDM TOPSIS and hybrid PFMCGDM COPRAS model

5. Conclusion, limitations and future research scope

Multi-Criteria Group Decision-Making (MCGDM) has proven successful in various fields. This study develops a hybrid MCGDM approach for engineers and designers to tackle group decision-making problems in engineering and manufacturing.The contribution of the paper lies in an MCGDM framework that integrates decision-makers' final judgments into a Pythagorean Fuzzy Number (PFN)-based group decision. To calculate hesitation levels in decision-maker information, the framework incorporates an aggregation approach based on the Takagi-Sugeno method and fuzzy inference rules. Building upon this integrated approach, a new hybrid MCGDM methodology is proposed by integrating the aggregation strategy with Fuzzy COPRAS for ranking final choices.The effectiveness of the proposed hybrid MCGDM PF-COPRAS method is demonstrated through a realistic material selection case study, showcasing its applicability in manufacturing. A comparison with Fuzzy TOPSIS further validates its efficiency in information gathering for real-world group decision-making. The potential applications of the proposed methodologies extend beyond manufacturing to logistics, project management, healthcare, finance, and facility placements. Future improvements include refining the aggregation strategy to handle both subjective and objective data and validating findings against other aggregation approaches.

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