

Use of Continuous Improvement Approaches for Quality Risk Analysis

Ramzi Abidi

Computer Engineering, Production and Maintenance Laboratory (LGIPM)
Lorraine University, Metz, France
Mechanical Laboratory of Sousse (LMS)
National School of Engineers of Sousse, Sousse, Tunisia
ramzi.abidi@univ-lorraine.fr

Mohamed Ali Kammoun

Computer Engineering, Production and Maintenance Laboratory (LGIPM)
Lorraine University, Metz, France
[mohamed-ali.kammoun @univ-lorraine.fr](mailto:mohamed-ali.kammoun@univ-lorraine.fr)

Sofiène Dellagi

Computer Engineering, Production and Maintenance Laboratory (LGIPM)
Lorraine University, Metz, France
[sofiene.dellagi @univ-lorraine.fr](mailto:sofiene.dellagi@univ-lorraine.fr)

Anis Chelbi

RIFTSI Laboratory LR20ES02
National Higher Engineering School of Tunis, ENSIT
University of Tunis
Mediterranean School of Business
South Mediterranean University, Tunis, 1053, Tunisia
anis.chelbi@planet.tn

Sami Bennour

Mechanical Laboratory of Sousse (LMS), National School of Engineers of Sousse
University of Sousse, Sousse, Tunisia
National School of Engineers of Monastir, University of Monastir, Monastir, Tunisia
Computer Engineering, Production and Maintenance Laboratory (LGIPM)
Lorraine University, Metz, France
sami.bennour@enim.u-monastir.tn

Taysir Rezgui

Applied Mechanics, and Systems Research Laboratory (LASMAP)
Tunisia Polytechnic School, University of Carthage
Carthage, Tunisia
taysir.rezgui@ept.ucar.tn

Abstract

This paper proposes a novel framework that integrates production planning and quality control within a context of continuous improvement. Process degradation is jointly monitored using X-bar control chart, which ensures process stability monitoring, along with an anomaly index designed to assess the overall health condition of the process and, consequently, the conformity risk of outgoing products. Unlike standard control charts that only signal out-of-control conditions, the proposed anomaly index enables early degradation assessment and supports proactive decision-making before process failure occurs. An in-depth analytical investigation is conducted to identify the root causes of degradation, distinguishing between machine-related and human-related sources. Considering a finite planning horizon, an analytical economic optimization model is developed. First, it determines the production plan with the minimum average cost. Then, it maximizes the total expected profit over the planning horizon by jointly finding the optimal values of the quality-related decision variables and the appropriate improvement actions, including both imperfect and perfect actions applied according to the identified causes of degradation. A numerical example is presented to illustrate the proposed approach. The results demonstrate the framework's effectiveness in monitoring the process, addressing the root causes of degradation, and maximizing profitability.

Keywords

Process health, Production, Quality, Process degradation, Continuous improvement

1. Introduction

In an increasingly competitive global market, manufacturers are under strong pressure to deliver high-quality products while controlling costs, which drives the need for continuous improvement of production systems. Achieving this objective requires the integration of production planning with quality and maintenance strategies under realistic operational constraints (Ait El Cadi et al., 2024; Azzabi, 2010). Although existing research has explored the joint modeling of these dimensions, comprehensive and practically applicable models remain limited. Many approaches rely on unrealistic assumptions, such as modeling process degradation solely as a function of machine operating time, neglecting systemic factors like human performance, methods, and material quality (Christopher, 2022). Additionally, corrective maintenance is often assumed to be immediate after quality degradation is detected, ignoring real delays in resource mobilization. Furthermore, traditional continuous improvement tools such as Statistical Process Control (SPC) provide lagging indicators that detect deviations without identifying their root causes (Kazaz and Sloan, 2013). While qualitative tools like the Ishikawa diagram support root-cause analysis, their subjective nature limits their use in quantitative and data-driven decision-making (Ishikawa, 1985). This lack of integration between monitoring and causal analysis tools represents a major limitation of current integrated production–quality–maintenance frameworks. To address these limitations, this paper proposes an integrated framework of production planning and quality control, within a continuous improvement framework. Process degradation is modeled using an MH-based anomaly index, which includes both Machine and Human factors, and quantifies process health and conformity risk in real time. Combined with the X-bar control chart, this index enables early detection of degradation and supports targeted intervention strategies aimed at maximizing total expected profit under realistic operational constraints.

2. Literature Review

The evolution toward Quality 4.0, driven by digitalization and the use of real-time data, has significantly transformed quality management practices and reinforced the need for integrated operational decision-making in modern manufacturing systems (Sony et al., 2020; Zonnenshain and Kenett, 2020). In this context, the integration of production, quality control, and improvement or maintenance policies has become a central research topic. Numerous studies have therefore proposed integrated production, maintenance, and quality models aimed at simultaneously optimizing operational decisions in order to reduce disruptions, improve resource utilization, and enhance overall system performance. However, despite their functional coordination, most of these models rely on simplifying assumptions such as perfect maintenance actions, instantaneous interventions, or simplified representations of quality degradation that limit their ability to detect the underlying causes of process deterioration and reduce their applicability in real industrial environments.

A major stream of research focuses on integrating production and maintenance. Several models have been developed to optimize perfect preventive maintenance PPM (Hadian et al., 2021; Nourelfath et al., 2016). Other studies introduced dynamic failure rates dependent on production load and system age. Imperfect preventive maintenance (IPM) models have also been developed to reflect partial system rejuvenation (Castro, 2009; Chen et al., 2015; Fakher

et al., 2018). Although these models link production decisions to machine reliability, they often oversimplify quality as a binary outcome, ignoring the root causes of degradation.

Other authors sought to strengthen the maintenance–quality link by using Statistical Process Control (SPC) as a trigger for maintenance, creating feedback loops between quality outputs and maintenance actions (Bahria et al., 2019; Salmasnia et al., 2024). Nevertheless, SPC-based approaches are reactive, providing lagging indicators without specifying whether deviations originate from machine, operator, or material. They also often assume instantaneous maintenance, disregarding real operational lead times.

The most advanced models aim to jointly optimize production, maintenance, and quality, balancing trade-offs among inventory, downtime, inspection, and rework costs (Kammoun et al., 2024; Ait-El-Cadi et al., 2021; Pandey et al., 2011). However, these approaches remain largely machine-centric, failing to model degradation as a systemic phenomenon. Ishikawa's framework demonstrates that quality failures arise from complex interactions among Machine, Method, Human, Material, and Environment. Existing models rarely quantify these causal factors, limiting their ability to prescribe targeted corrective actions, despite recent efforts using multi-cause or Bayesian Network approaches (Tasias, 2022).

Conventional quality metrics, such as Process Capability Indices (C_p , C_{pk} , P_p), provide simple numerical measures of output performance but assume Normal data distributions and focus on results rather than process health (Kane, 1986; Porter and Oakland, 1991). They fail to capture the dynamic effects of the five factors of Ishikawa's framework on process performance.

3. Targeted contributions

This study provides an integrated framework that jointly addresses production planning and quality control. Unlike most studies in the literature, which rely on machine-centric or output-based indicators without explicitly considering human-machine interaction causality, we propose a real-time causal assessment of process health through an "Anomaly Index." This index focuses on machine and human factors and is derived from cause-and-effect relationships. This integrated approach allows for early detection of degradation, root-cause diagnosis, and targeted intervention within a unified economic optimization framework.

4. Problem description

We consider a manufacturing process that involves interconnected machine and human resources that work together to meet random demands and required service levels. First and second-grade products are produced. Process anomalies arising from the machine and the operators, progressively degrade system reliability and product quality, increasing non-conforming production and failure rates. A portion of these defective products can be reworked, while the remainder is rejected due to excessive quality degradation. However, the degradation process is complex, and its root causes are often difficult to identify in real time. This makes the implementation of effective and timely improvement actions particularly challenging. The progressive deterioration increases production losses, operational costs, and negatively affects overall system profitability. Consequently, an effective monitoring and decision-support mechanism is required to detect process degradation at an early stage and rapidly guide appropriate improvement actions in order to maintain process stability, product quality, and economic performance. The development of such a strategy motivates the integrated optimization approach presented in the following section.

5. Integrated optimization framework

This section presents the mathematical formulation of an integrated strategy for production, quality, and improvement management. The objective is to determine the optimal operational policy that maximizes the expected total profit over a finite planning horizon while maintaining acceptable process stability and quality levels. The problem is formulated as a joint optimization model involving several decision variables. These include the production plan, the monitoring parameters of the dual-control framework, such as the sample size, sampling interval, and X-bar control limits; and the improvement policy, which is defined by the number of perfect and imperfect improvement actions to be performed and the delay period before initiating each action. To support this model, a new causal framework for quality analysis has been introduced that includes an anomaly index based on factors related to both machine and human. The proposed approach enables evaluation of the impact of these factors on process performance. Cost structures for production, improvement, and quality losses, as well as revenues from first- and second-grade products, are considered. Then, the total expected profit function is defined with the aim of maximizing it.

5.1. Notations

In what follows, we will develop the analytical expressions of the total sales revenue (C_{VT}), the total average costs related to production (C_{PT}), enhancement and maintenance (C_{ET}), and quality (C_{QT}) (Table 1).

Table1. Input data and decision variables

Production variables		Quality variables		Continuous improvement variables	
$u(k)$	Production rate during period k ($k = 1, \dots, T$)	$A_{IMC,k}(t)$	Anomaly index at instant t in each production period k .	$N_{CIE,q}$	Total number of improvement actions (decision variable) ($q = 1.., N$)
C_{PT}	Total expected production cost	ANS_q	Average number of samples required to detect the quality indicator shift outside the control limits of the X-bar control chart.	N	Number of production cycles. A cycle is the duration between consecutive perfect enhancement actions over the planning horizon H .
Γ_p	Production cost	C_{QT}	Total quality cost	τ_{PE}	Improvement action average duration
$S(k)$	Stock level at the end of period k ($k = 1, \dots, T$)	n, m	Sample size (decision variable), number of inspections over the planning horizon H .	$Fr_k(t)$	Process failure rate function
$d(k)$	Customer demand in period k ($k = 1, \dots, T$)	γ_q	Rate of reworked products. ($q = 1.., N$)	$\psi(k)$	Global improvement level
Δt	Length of each production period k ($k = 1.., T$)	Pc	Proportion of compliant products	A_{th}	Anomaly Index predefined threshold
c_p	Unit production cost	$Q_{NC,q}$	Quantity of non-compliant products ($q = 1.., N$)	$\lambda_k(t) = \lambda_{k-1}(\Delta t) + \frac{u(t)}{u_{max}} \lambda_n(t)$	Failure rate function during period k .
c_{sho}, c_{sto}	Shortage, Inventory holding cost per unit	I_s, k_p	sampling interval (decision variable) control chart factor that determines its width (decision variable)	$N_{IM,q}$	Number of imperfect maintenance actions to be performed during cycle q (decision variable), ($q = 1.., N$)
H	Finite planning horizon $H = T \cdot \Delta t$	Dq	Time interval between the detection of a process deviation and the execution of the corresponding perfect enhancement action during a cycle q . (decision variable) ($q = 1.., N$)	$N_{IH,q}$	Number of imperfect human-related improvement actions during cycle q (decision variable), ($q = 1.., N$)
T	The total number of production periods over the planning horizon H	v	Probability of non-detection of the process shift at a given sampling instant.	cp_{IE}	cost of an imperfect preventive improvement related to the human factor
u_{max}, u_{min}	Maximum and minimum production rates respectively	c_{in}, c_{rw}, c_r	Inspection cost Cost to rework, reject, a product unit, respectively	cp_{IM}, cp_{PM}	costs of imperfect and perfect maintenance actions performed on the machine, respectively.
$U(t)$	The vector of production rate $U = \{u(k), k = 1, \dots, T\}$	$Q_{NC,q}$	Quantity of non-confirming products related to a cycle q ($q = 1.., N$)	N_{PH}, N_{PM}	Number of perfect human-related improvement and maintenance actions, respectively
		Pnc	Proportion of non-conforming products	cc	Corrective improvement action cost
		δ	Number of samplings over Δt		

5.1. The production policy

To determine the project plan $u(k)$ ($k = 1, 2, \dots, T$) that satisfies a random customer demand $d(k)$ ($k = 1, 2, \dots, T$) over a predefined finite planning horizon H made of k equal periods, the expected total production-related cost is minimized. This cost includes the manufacturing cost (Γ_p), the inventory holding cost Γ_s^+ and the shortage cost (Γ_s^-) that are defined, respectively, by equations (1 to 3).

$$\Gamma_p = c_p \cdot \sum_{k=1}^T u(k) \cdot \Delta t \quad (1)$$

$$\Gamma_s^+ = c_{sto} \cdot s^+(k) \quad (2)$$

$$\Gamma_{s-} = c_{sho} \cdot s^-(k) \quad (3)$$

The decision variable $u(k)$ represents the production rate adopted during period k ($k = 1 \dots, T$). The actual produced quantity during this period is therefore given by $u(k) \cdot \Delta t$ ($k = 1 \dots, T$), where Δt denotes the duration of each production period.

The total expected production cost C_{PT} is then obtained as the result of the following minimization problem:

$$C_{PT} = \underset{U=\{u(k); k=1 \dots T\}}{\text{Min}} (\Gamma_p + \Gamma_{s+} + \Gamma_{s-}) \quad (4)$$

where $U = \{u(k), k = 1, \dots, T\}$ is the vector of production rates over the planning horizon

5.2. The quality control policy

To address process degradation effectively, it is essential to analyze not only output deviations but also their underlying causes. This motivates a dual-monitoring quality control framework combining statistical and causal analyses. First, conventional statistical process control tools are used to detect abnormal process variations. Second, causal analysis evaluates the intrinsic health of the process to anticipate conformity risks. Together, these dimensions enable proactive quality management and targeted corrective actions.

• Stability control chart

The manufacturing process is subject to progressive degradation due to the continuous variation of machine and human (MH) factors, leading to instability and an increase in non-compliant products. For this purpose, the process stability is controlled by periodic sampling of size n every $1s$ time units. Note that the quality indicator sample means follow a Normal distribution $N_s \sim (\bar{X}_s, \sigma_0/\sqrt{n})$, with $\bar{X}_s$ being the average of the sample means $\bar{X}_i$ ($i=1, 2, \dots, m$), and σ_0 representing the process standard deviation.

The upper and lower control limits (UCL and LCL) of the X-bar chart are given by:

$$UCL = \bar{X}_s + \sigma_0 \times \frac{K_p}{\sqrt{n}} \quad (5)$$

$$LCL = \bar{X}_s - \sigma_0 \times \frac{K_p}{\sqrt{n}} \quad (6)$$

With K_p is the number of standard deviations between the centerline and the control limits.

We will denote by ANS_q within cycle q ($q = 1 \dots, N$) the average number of samples required to detect the quality indicator shift outside the control ($LCL > \bar{X}_i$ or $UCL < \bar{X}_i$). When such a shift is detected, the process is considered out-of-control. In this situation, an improvement action lasting τ_{PE} will be launched to bring the process back under control.

ANS_q ($q = 1 \dots, N$) is given by:

$$ANS_q = \frac{1}{1-v} \quad (7)$$

Where v denotes the probability of non-detection of the process shift at a given sampling instant.

• Anomaly index-based causal conformity risk assessment

A dynamic and causal assessment of process conformity is proposed by explicitly modeling the impact of machine and human factors variations across successive production periods. Unlike most existing studies that rely on static and aggregated performance indicators, the proposed approach analyzes how temporal changes in causal factors affect the intrinsic ability of the process to meet tolerance requirements.

Within this context, an Anomaly Index, A_{IMC} , interpreted as a Process Health Index, is introduced as a real-time indicator for assessing nonconformity risk based on cause-effect relationships. Unlike conventional statistical monitoring, the proposed indicator directly evaluates conformity risk and identifies whether performance degradation originates from machine deterioration or human-related variability, enabling targeted and efficient interventions. To quantify process deterioration, a parametric relationship between the anomaly index and the machine and human factors variations within each production period k ($k = 1 \dots, T$), is formulated using an exponential multiple regression model given by Eq. (8).

$$AIMC_k(t) = AIMC_{k-1}(\Delta t) \cdot (1 - e^{-(\beta_0 + \beta_{Hum} \cdot M_{Hum,k}(t) + \beta_{Mach} \cdot M_{Mach,k}(t) + \varepsilon)}) \cdot (1 - \gamma(k)) + AIMC_0 \cdot \gamma(k) \cdot \psi(k) \quad (8)$$

$\forall t \in [0, \Delta t]$

where Δt denotes the duration of one production period.

The term $AIMC_{k-1}(\Delta t)$ represents the value of the Anomaly Index at the end of the previous production period ($k-1$), which serves as the initial condition for the evolution of the index during the current period k ($k = 1 \dots, T$).

The global improvement level $\psi(k)$ at period k ($k = 1 \dots, T$) represents the overall effectiveness of the enhancement actions applied to the MH factors. It is defined as the ratio between the achieved improvement and the total deviation of the process from its ideal state. $\psi(k) = 1$ corresponds to a perfect enhancement action, fully restoring the degraded factors, whereas $0 < \psi(k) < 1$ indicates an imperfect enhancement with only partial recovery. In Eq. (9), ψ_{jk} is a binary decision variable indicating whether an improvement action is applied to factor j ($j = \text{Hum, Mach}$) during period k ($k = 1 \dots, T$), and $\alpha_{jk} \in [0, 1]$ denotes the corresponding improvement degree.

$$\psi(k) = \frac{\sum_{j=1}^2 \alpha_{jk} \psi_{jk}}{\sum_{j=1}^2 (1 - M_{jk})} \quad (9)$$

The production process is considered as a new system at $t = 0$, with maximum anomaly index equal to $AIMC_0$, referred to as anomalies-free state, in which the machine and human factors do not yet influence the process. $M_{jk}(t)$ ($j = \text{Hum, Mach}$) represents the evolution over time of the MH factors; β_j are the regression coefficients representing the impact of each factor; β_0 is the regression intercept, and ε is the residual error.

$$\gamma(k) = \begin{cases} 1 & \text{if an improvement action is performed during period } k \\ 0 & \text{else} \end{cases}$$

The non-performance of the process due to machine-related degradation, $M_{mach,k}(t)$, is defined as a cumulative function depending on the failure rate of the previous periods (Kammoun et al., 2024), the usage time, and the production rate, as modelled by Eq. (10).

$$M_{mach,k}(t) = e^{-\left(\lambda_{k-1}(\Delta t) + \frac{u(t)}{u_{max}} \lambda_n(t)\right)} \quad (k = 1 \dots, T), \quad (10)$$

The performance of the machine and human factors directly determines the health status of the process for which any degradation in one or more factors leads to a decrease of the anomaly index $AIMC_k(t)$ and consequently an increase of the risk of non-conformity (Figure 1)). Notably, the threshold A_{th} is instead, a strategic, risk-based parameter defined by the company. This choice aligns with modern risk management frameworks, where acceptable risk levels depend on organizational objectives, economic constraints, and product criticality (Siwiec and Pacana, 2022). The process is considered capable of producing compliant products as long as $AIMC_k(t) > A_{th}$. In this case, the system remains in a healthy operating state and only imperfect improvement actions may be applied, leading to a partial recovery quantified by $\psi(k) < 1$. When $AIMC_k(t) \leq A_{th}$, the process is considered unhealthy, producing non-compliant products. In this situation a perfect improvement action is initiated during period k ($k = 1 \dots, T$), and the recovery level is set to $\psi(k) = 1$.

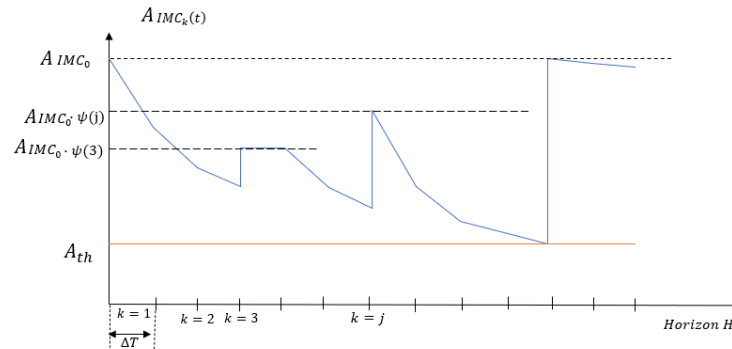


Figure 1. Anomaly index evolution monitoring chart

5.3. Continuous enhancement strategy

The continuous degradation of the manufacturing process adversely affects product quality, resulting in an increase in non-conforming items. To mitigate these risks, the process is continuously monitored using two control charts: anomaly control and $\bar{X}$ – bar charts. Based on their joint analysis, two possible scenarios are identified. The process is either in a healthy state (in control) or in an unhealthy state (out of control). Each scenario is associated with a specific improvement policy aimed at enhancing process performance and, consequently, ensuring the production of conforming products.

5.3.1. Process states and associated control strategies

Scenario 1 (process in control)

This scenario occurs if the conformity risk is low as indicated by the anomaly index being above the predefined threshold ($AIMC_k(t) \geq A_{th}$), and all the sample means $\bar{X}_i$ ($i = 1, \dots, m$) are within the control limits ($LCL \leq \bar{X}_i \leq UCL$) (see Figure (2)). A number N_{CIE} of imperfect enhancement actions are performed over the planning horizon H every period θ ($\theta = \frac{T \cdot \Delta t}{N_{CIE}}$) to ensure the continuity of high quality while the duration of these actions is assumed negligible. These actions are decomposed into imperfect maintenance actions, denoted by N_{IM} , and imperfect human-related improvement actions, denoted by N_{IH} . The decision of which type of action to perform depends on the source of degradation identified through the proposed causal analysis framework. More precisely, when the anomaly is primarily associated with machine-related factors, an imperfect maintenance action is applied; when it is primarily associated with human-related factors, an imperfect human-oriented enhancement action is performed. If both machine and human factors simultaneously contribute to process degradation, the model allows the combined implementation of both types of imperfect actions within the same period.

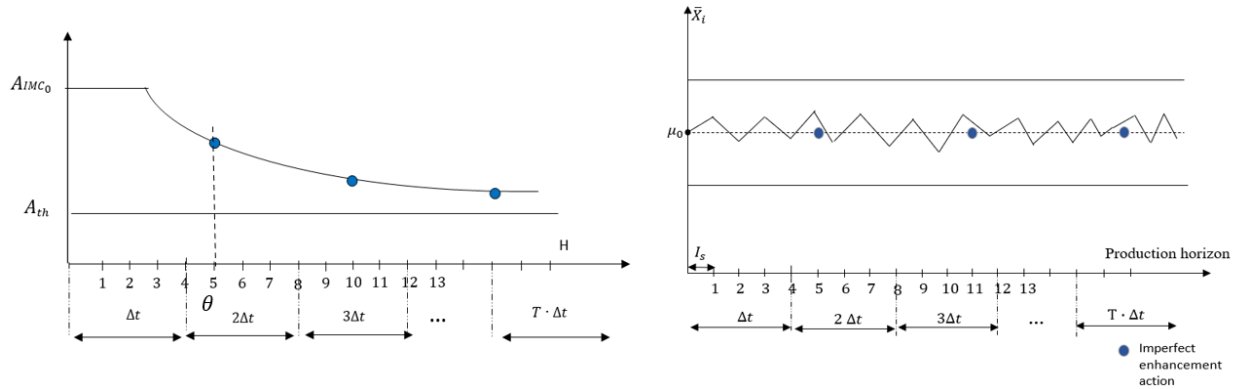


Figure 2. Process in control state

- **Scenario 2 (process out of control)**

This scenario corresponds to an out-of-control process state leading to non-conforming production, detected either by the $\bar{X}$ -bar chart or by the Anomaly Index whichever occurs first (Figure 3). While statistical instability identified by the $\bar{X}$ -bar chart directly indicates a high-risk condition, statistical stability does not necessarily mean a healthy process, as latent degradation of the underlying machine and human factors may occur without violating control limits. Such hidden deterioration cannot be captured by conventional Statistical Process Control tools, which motivates continuous monitoring of the Anomaly Index. The process is deemed unhealthy when either the quality indicator exceeds the control limits or the Anomaly Index drops below its threshold, triggering termination of the high-quality production cycle and the start of an HM-based diagnostic analysis. Depending on the root cause (machine or human related), a perfect maintenance of the machine or a human related improvement action of duration τ_{PE} is applied Dq time units later, restoring the process to an as-good-as-new state and the Anomaly Index returns to its maximum value $AIMC_0$. In this study, the durations of perfect machine and human-related actions are assumed to be identical for simplification purposes. Subsequently, periodic imperfect enhancement actions with negligible duration are performed to sustain an intermediate operating condition.

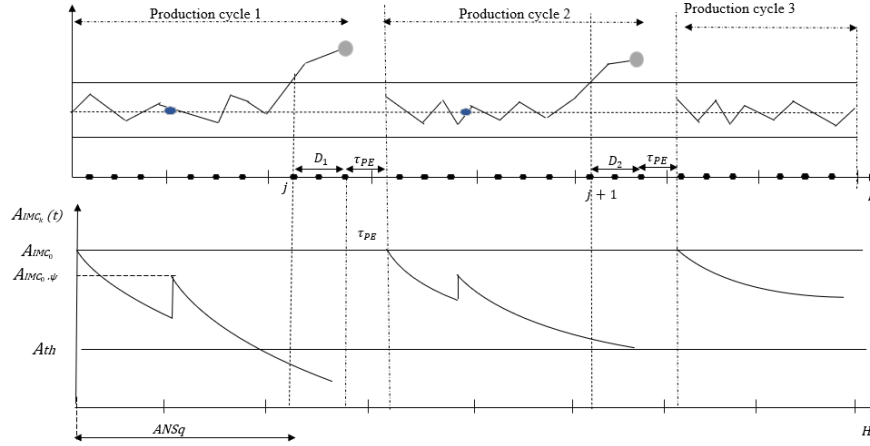


Figure 3. Process in out-of-control state

In practice, the anomaly index is not obtained through direct physical measurement but estimated from observable indicators related to machine and human factors. At each month period operational data such as machine condition indicators (e.g., vibration level, temperature drift, maintenance history) and human-related indicators (e.g., operator error rate, task repetition, non-compliance in operating procedures, or skill-related assessment scores) are collected and normalized. These variables are then introduced into the regression model to estimate the anomaly index, which provides a synthetic indicator of process health and is continuously compared with the threshold A_{th} .

5.3.2. Total continuous improvement and maintenance costs

The proposed continuous enhancement strategy aims to ensure, during the production horizon H , high process reliability and superior product quality, while minimizing the total continuous enhancement cost CE_T that is the sum of costs incurred in Scenario 1 (CE_{S1}) and Scenario 2 (CE_{S2}), which is formulated by Eq. (11).

$$CE_T = \varepsilon_1 \cdot CE_{S1} + \varepsilon_2 \cdot CE_{S2} \quad (11)$$

Where $\varepsilon_1 = \frac{m-N}{m}$ and $\varepsilon_2 = 1 - \varepsilon_1$ are the occurrence probabilities of Scenario 1 and Scenario 2, respectively.

• Total cost of scenario 1

To ensure continuity of process performance, N_{CIE} imperfect continuous preventive enhancement actions are applied (IPE), including N_{IH} human related imperfect preventive enhancement actions and N_{IM} imperfect preventive maintenance actions on the machine; $N_{CIE} = N_{IH} + N_{IM}$. The number of imperfect improvement actions is determined for each production cycle based on the degradation level observed during that cycle.

The numbers N_{IH} and N_{IM} are determined through the optimization program. They depend on the degradation evolution of the Machine and Human factors, and by the resulting Anomaly Index.

$$CE_{S1} = \underbrace{cc \cdot \varphi(u, N_{CIE})}_{\text{corrective enhancement}} + \underbrace{cp_{IE} \cdot N_{IH}}_{\text{IPE on Hum}} + \underbrace{cp_{IM} \cdot N_{IM}}_{\text{IPE on machine}} \quad (12)$$

$\varphi(u, N_{CIE})$ being the average number of failures that can occur during the planning horizon H , considering the production rate variation for each production period Δt which is expressed as follows (Zied et al., 2011).

$$\varphi(u, N_{CIE,q}) = \sum_{k=1}^T \int_0^{\Delta t} Fr_k(t) dt$$

In this study, a process failure refers to the production of non-conforming products, resulting from excessive degradation of the production process. Such degradation may originate from machine deterioration, human performance degradation, or their combined effects. After each failure, a corrective action (minimal repair of the machine or improvement of the human aspect) is applied to restore the process to an acceptable state without fully resetting its degradation level.

In previous studies, process degradation is mainly modeled based on machine deterioration as a function of the operating conditions (Kaplan and Mikes, 2016). However, in this study, the process failure rate $Fr_k(t)$ is formulated as a function of the combined states of machine and human factors, allowing a more realistic representation of the overall process health (Tashi et al., 2016.)

$$Fr_k(t) = b \cdot Fr_{k-1}(\Delta t) + e^{-\left(\frac{u(k)}{U_{max}} + M_{Hum,k(t)}\right)} \cdot \omega_j \cdot Fr_n(t) \quad \text{with } (j = \text{Hum, Mach}) \quad (13)$$

With:

$Fr_n(t)$: the nominal process failure rate when the manufacturing process operates with its maximal production cadence

b : A binary variable used to define the current state of the process, as defined below:

$$b = \begin{cases} 1 & \text{if } AIMC_k(t) > A_{th} \\ 0 & \text{otherwise} \end{cases}$$

$\omega_j \in [0,1]$ ($j = \text{Hum, Mach}$) : defines the improvement degree in the failure rate through continuous improvement actions on one of the machine or human related factors, which is defined as follows:

$$\omega_j = \begin{cases} 1 & \text{if } b = 1 \\ \in [0,1[& \text{otherwise} \end{cases}$$

The variables b and ω_j ($j = \text{Hum, Mach}$) act as multipliers increasing or decreasing the process failure rate $Fr_k(t)$ according to the operational conditions; the failure rate takes into account the accumulated failure rate as long as the process is healthy ($b = 1$) and the failure rate is decreased by ω_j ($j = \text{Hum, Mach}$) resulting from continuous enhancement if the process is unhealthy ($b = 0$).

• Total cost of Scenario 2

This scenario is triggered when process instability or high risk is detected, leading to a perfect enhancement action before the start of a new production cycle. In this framework, N denotes the total number of production cycles over the planning horizon H , where each cycle consists of a production phase followed by a perfect enhancement action; therefore, the number of cycles N is equal to the total number of perfect enhancement actions performed during the planning horizon. During a given cycle q , the production duration is defined as $\left[\frac{ANS_q}{\delta} - \left(\frac{ANS_{q-1}}{\delta} + \tau_{PE} + D_{q-1}\right) y_q\right]$, with $y_1 = 0$ for the first cycle and $y_q = 1$ for $q \in [2, N]$ (Kammoun et al., 2024). The proposed strategy optimizes improvement actions on the Human factor and maintenance decisions by minimizing the total enhancement cost expressed in (Eq.14), including corrective actions performed according to a minimal repair policy, as well as imperfect and perfect enhancement actions.

$$CE_{S2} = \sum_{q=1}^N \underbrace{cc \cdot \varphi(u, N_{CIE,q})}_{\text{corrective enhancement}} + \sum_{q=1}^N \underbrace{\left(cp_{IE} \cdot N_{IH,q}\right)}_{\text{IPE on Hum}} + \sum_{q=1}^N \underbrace{cp_{IM} \cdot N_{IM,q}}_{\text{IPM on machine}} + \sum_{q=1}^N \underbrace{\left(cp_{PE} \cdot N_{PH,q}\right)}_{\text{PPE on Hum}} + \sum_{q=1}^N \underbrace{cp_{PM} \cdot N_{PM,q}}_{\text{PPE on machine}} \quad (14)$$

With: PPE standing for Perfect Preventive Enhancement and IPE denoting Imperfect Preventive Enhancement

The total number N of production cycles over the planning horizon is determined incrementally as the anomaly index reaches the threshold A_{th} or one of the control limits of the X-bar chart is exceeded.

5.4. Quality-Related Costs

The total cost of quality represents all expenses associated with the decrease in quality of the product. It includes costs related to inspection, rework, and products rejected due to non-compliance. The total cost of quality is therefore defined as:

$$C_{QT} = C_{INS} + C_{RW} + C_{RP} \quad (15)$$

• Inspection cost (C_{INS}):

The inspection cost is defined by the average sampling cost which is related to the sample size (n) and the total number of inspections (m): it is given by:

$$C_{INS} = n \cdot m \cdot c_{in} \quad (16)$$

• Rework cost (C_{RW}):

As part of the data analysis, it is essential to determine the quantity of non-compliant products, attributable to process failure. This measure is calculated by multiplying the proportion of non-compliant products Pnc by the total quantity produced Q_T .

$$Q_{NC,q} = Pnc \cdot Q_T \quad (17)$$

In order to figure out the overall Proportion of Conforming products (Pc) within a certain time frame, we resort to the law of total probability, taking into account the condition of the system based on the anomaly index. Pc is given by

$$Pc = Pc_{Nominal} \cdot (Prob(AIMC_k(t) > A_{th})) + Pc_{deviated} \cdot (Prob(AIMC_k(t) < A_{th})) \quad (18)$$

The Proportion of Non-Conforming products is then:

$$Pnc = 1 - Pc$$

Where:

$Prob(AIMC_k(t) > A_{th})$: probability that the process is in the nominal state at time t , corresponding to the anomaly index exceeding the threshold A_{th}

$Pc_{Nominal}$: is the high conformity rate related to the nominal state: for example, $Pc_{Nominal} = 0.999$ (i.e., a basic defect rate of 0.1%).

$Pc_{deviated}$: is the proportion of compliant products when the system is in a deviated state.

Hence:

$$Q_{NC,q} = (1 - Pc_{Nominal} \cdot (Prob(AIMC_k(t) > A_{th})) + Pc_{deviated} \cdot (Prob(AIMC_k(t) < A_{th}))) \cdot Q_T$$

The total non-conforming quantity over the planning horizon is then obtained by summing over all cycles:

$$Q_{NC} = \sum_{q=1}^N Q_{NC,q}$$

A fraction γ_q of the total quantity inspected as non-compliant is subject to reworking and quality improvement in order to sell it at a lower price C_v' and reduce the company's expenses. The total cost of the reworked items is given by the equation:

$$C_{RW} = C_{rw} \cdot \gamma_q \cdot Q_{NC} \quad (19)$$

- **Rejected products cost (C_{RP}):**

The cost of rejected products represents the financial losses associated with non-compliant goods which, due to their insufficient quality, cannot be corrected or reused and must therefore be disposed of. The total cost lost due to rejection is defined as follows:

$$C_{RP} = C_r \cdot (1 - \gamma_q) \cdot Q_{NC,q} \quad (20)$$

- **Total profit**

The overall objective is to maximize the total profit (PT), which is the total sales revenue (C_{VT}) minus the total costs related to production (C_{PT}), enhancement and maintenance (C_{ET}), and quality (C_{QT}).

$$PT = C_{VT} - C_{QT} - C_{ET} - C_{PT} \quad (21)$$

Where total sales cost C_{VT} equal to the sum of sales of first- and second-order products.

6. Numerical application

A numerical example is presented to illustrate the application and evaluate the performance of the proposed integrated production–quality–enhancement framework. The analysis focuses on a single-machine manufacturing system over a finite 12-month planning horizon ($H = 12$ months with 184 working hours per month), using a monthly discretization ($\Delta t = 1$ month). The main input parameters are specified as follows in Table 2:

Table 2. Numerical parameters used in the case study

Category	Parameter	Value
Maximum and minimum production rates (product units /month)	u_{max}, u_{min}	1000, 200
Initial stock level	S0	100 units.

Costs(in monetary units)	$C_{pu}, C_{sto}, C_{sho}, c_{in}, c_{PM}, c_C, c_r, C_{rw}, C_v, c_v'$	2, 1.5, 10, 1, 300, 500, 2, 3, 10, 6
Reliability: Nominal failure rate of the machine in the nominal state (Weibull law): $\lambda_n(t) = \frac{\alpha}{\beta} \cdot \left(\frac{t}{\beta}\right)^{\alpha-1}$	α : shape parameter β : scale parameter λ_0 : initial failure rate	2 0.75 month 0.5 failure/month Mean time between failures approximately equal to 0.665 months (about 20 days).
The regression coefficients	$\beta_0, \beta_{mach}, \beta_{hum}$	0.1, 0.8, 0.65
Anomaly index threshold	A_{th}	0.5
Perfect maintenance or improvement action average duration	τ_{PE}	2 h
Proportion of Conforming and non-Conforming	$P_{C_{Nominal}}, P_{C_{deviated}}$	0.98, 0.02
Improvement degree	ω_j (j = Hum, Mach)	0.3
Probability of non-detection of the process shift at a given sampling instant.	v	0.4
The process nominal failure rate	$Fr_n(t)$	0.5 failure per month

The nominal failure rate $Fr_n(t)$ is assumed constant and equal to $Fr_n = 0.5$, representing the baseline failure intensity when the manufacturing system operates under nominal conditions and maximal production cadence. This assumption provides a reference failure level independent of degradation effects, while the actual failure behavior of the system evolves according to degradation related to the machine and human factors (Table 3).

The human-related factor is modeled as a Beta-distributed random variable defined on the interval [0,1], with mean 0.5 and standard deviation approximately equal to 0.17.

In contrast, the machine-related factor evolves deterministically according to cumulative degradation as described in Eq. (8). The baseline regression coefficients are set to $\beta_0 = 0.1$, $\beta_{mach} = 0.8$, and $\beta_{hum} = 0.65$, reflecting the dominant influence of machine and human factors on process quality. Initially, both the machine factor and the human factor are set to 1, representing an As-Good-As-New condition, while the residual error term is fixed at $\varepsilon = 0.1$. In this example, the threshold anomaly index is set to $A_{th} = 0.5$.

In the first step, a simple numerical optimization procedure is used to determine the economic production plan over the 12-month planning horizon. The objective is to minimize the total expected cost being equal to the sum of the costs related to production, inventory holding and shortage, expressed by the objective function in Eq. (4).

Table 3. Resulting stock levels and optimal production plan

Period (month) (k)	Forecasted Demand	Optimal production plan ($u(k)^*$) during one month	Stock $s(k)^*$
1	900	993	100
2	1000	974	274
3	850	846	375
4	750	752	189
5	800	613	222
6	750	876	208
7	720	589	197
8	600	765	-38
9	800	794	86
10	890	880	16
11	555	600	-239
12	900	817	-54

Once the optimal production plan $u(k)$ is determined, a numerical procedure is implemented to determine the optimal values of the decision variables that maximize the total expected profit. The decision variables include the sample size n , the sampling interval I_s , the control chart factor that determines its width k_p , the number of imperfect improvements N_{CIE} , and the delay D before launching a perfect improvement action. Due to the high dimensionality of the decision space, a grid search algorithm has been developed in Python to systematically explore feasible combinations of these variables (Ataei and Osanloo, 2004). The results obtained correspond to the best profit found within the explored grid. They are summarized in Table 4.

Table 4. Optimal values of the decision variables and resulting system performance.

n^* (decision variable)	21	Optimal sample size
I_s^* (decision variable)	18 hours	Sampling interval (hours)
m^* (decision variable)	128	Optimal number of inspections to do over the planning horizon.
k_p^* (decision variable)	4	X-bar chart factor that determines its width
$\varepsilon_1, \varepsilon_2$	98.4%, 1.6%	Probabilities of in control and out of control states
N_{CIE}^* (decision variable)	5	Number of imperfect enhancement actions over the planning horizon
D^* (decision variable)	3	Optimal delay before performing a perfect improvement action
C_{QT}^*	6048	Total quality cost (mu.)
PT^*	392,006	Total profit (mu.)
N	2	Number of production cycles

These results demonstrate that the proposed strategy enables early detection of process degradation and supports timely corrective interventions, leading to reduced quality-related costs and improved overall profitability, as illustrated in Figure 4. The detection of process deviation is ensured by the dual-monitoring framework. In this context, the anomaly index progressively decreases and reaches the predefined threshold during the 7th month (approximately 1248 hours), indicating a transition to a degraded state. This enables the implementation of appropriate corrective actions, confirming the effectiveness of the integrated monitoring and improvement policy in maintaining a balance between quality risk and economic performance.

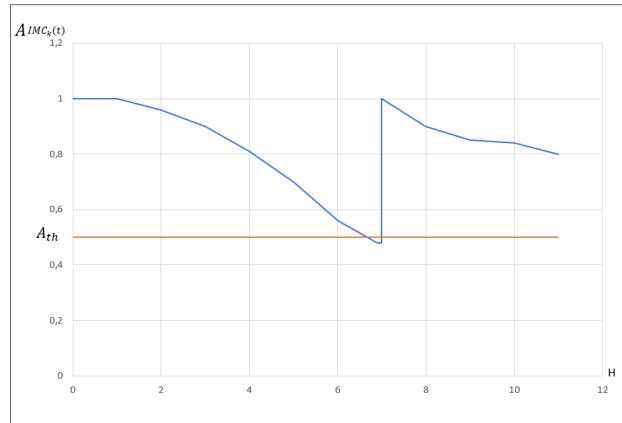


Figure 4. Temporal anomaly index evolution

Although 5 highlights the severe degradation and subsequent recovery in month 7, it does not reveal the underlying Figure causes. A root-cause analysis is therefore performed by decomposing the global Anomaly Index into its individual HM components, as illustrated by Figure 5. A normalized threshold equal to 0.5 is considered as a neutral reference separating acceptable and degraded performance, consistent with normalized health indicators commonly adopted in industrial monitoring frameworks (Zhu et al., 2018)

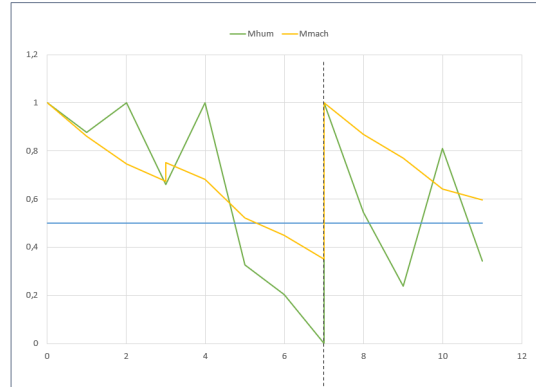


Figure 5. The evolution of human and machine factors

The sudden loss of process stability is caused by simultaneous declines in Machine performance, due to cumulative wear, and in the human performance, likely from fatigue or reduced vigilance. A targeted intervention at month 7, combining perfect preventive maintenance on the machine and corrective actions for the operators, restore both factors to nominal levels. As a result, system health and conforming production are immediately recovered.

Interpretation

The numerical results provide key insights into the benefits of a diagnostically aware, integrated framework. A major finding is the strong complementarity between the X-bar chart and the proposed Anomaly Index. While the X-bar chart detects statistical instability, the Anomaly Index identifies process degradation at an earlier stage, confirming its role as a leading indicator of systemic failure, whereas SPC acts as a lagging indicator of output deviation. This early warning capability creates a valuable window for proactive intervention, significantly reducing the risk of large-scale non-conforming production.

The degradation point observed at month 7 corresponds to the time at which the cumulative degradation level causes the Anomaly Index to exceed the predefined threshold A_{th} . This crossing indicates the onset of significant process deterioration and triggers corrective actions within the proposed control framework. Unlike SPC-only approaches, which merely signal an out-of-control state and require time-consuming root-cause analysis, the proposed framework embeds diagnosis directly into the monitoring process, allowing a seamless transition from detection to targeted corrective action. The successful recovery of process health validates this diagnostic capability.

The results also highlight the strategic role of the health threshold A_{th} , which governs the trade-off between intervention costs and quality losses. Its predefined value reflects the firm's tolerance level to process degradation and determines when corrective actions should be triggered according to the monitoring policy.

Finally, this study has limitations that motivate future work. The stochastic modeling of the human and machine factors and the predefined impact coefficients β_j serve as a methodological foundation, but empirical calibration using industrial data and data-driven modeling of the factors are essential steps toward deploying the framework as a practical, factory-specific decision-support system.

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7. Conclusion

This paper overcomes the limitations of traditional machine-centric approaches to integrated production and quality management by introducing a diagnostic framework based on an Anomaly Index analysis. Unlike conventional SPC or capability indices, the proposed indicator provides a real-time, causal assessment of process health, enabling explicit

identification of degradation sources. The integrated optimization model demonstrates its effectiveness in jointly optimizing production, quality control, and continuous enhancement decisions.

More precisely, an analytical model expressing total profit integrating sales revenue, and costs related to production, enhancement, maintenance and quality, has been developed. Using a general solving procedure designed for the model, the optimal values of the decision variables have been determined considering an illustrative numerical example.

Future research will focus on empirical validation using industrial data, enhanced modeling of the human factor through fatigue and cognitive-load dynamics, and extension to multi-machine and multi-product systems to further support intelligent and resilient manufacturing operations.

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