

# **A Hybrid Adaptive $\epsilon$ -Constraint Framework for Pareto Front Approximation in Multi-Objective Mixed-Integer Optimization**

**Özen Ergezer**

Department of Industrial Engineering  
Gazi University, Ankara, Türkiye  
[ozen.ergezer@gazi.edu.tr](mailto:ozen.ergezer@gazi.edu.tr)

**Gonca Yıldırım**

Assistant Professor, Department of Industrial Engineering  
Gazi University, Ankara, Türkiye  
[goncayildirim@gazi.edu.tr](mailto:goncayildirim@gazi.edu.tr)

**Fulya Altıparmak**

Professor, Department of Industrial Engineering  
Gazi University, Ankara, Türkiye  
[fulyaal@gazi.edu.tr](mailto:fulyaal@gazi.edu.tr)

## **Abstract**

This study focuses on improving Pareto front approximation methods for solving three-objective mixed-integer optimization problems under realistic computational constraints. In particular, a sustainable facility location problem with a mixed transportation fleet is considered, incorporating economic, environmental, and social objectives. To efficiently generate high-quality Pareto solutions, a hybrid solution framework is proposed that integrates a nested  $\epsilon$ -constraint method with several enhancements, including an AUGMECON-based scalarization, adaptive grid generation, and a refinement mechanism. The adaptive strategy dynamically updates  $\epsilon$ -levels based on intermediate solutions to better explore the feasible region, while the refinement stage identifies sparse regions of the Pareto front using both distance- and hypervolume-based criteria and intensifies the search locally. A comprehensive computational study is conducted on benchmark instances, where multiple solution variants are evaluated under a fixed runtime budget. In addition, a one-at-a-time sensitivity analysis is performed to assess the impact of key algorithmic parameters, including grid density,  $\epsilon$  penalty coefficient, and refinement depth. The results show that the proposed hybrid framework improves Pareto front quality in terms of hypervolume and solution distribution compared to the standard  $\epsilon$ -constraint method. In particular, the combination of adaptive grid and refinement mechanisms produces more evenly distributed and representative solution sets without increasing computational time.

## **Keywords**

Facility Location, Multi-Objective Optimization, Sustainable Supply Chain

## **Biographies**

**Özen Ergezer** is a PhD candidate in Industrial Engineering at Gazi University. She holds both her BS and MS degrees from Middle East Technical University. Her research focuses on optimization theory and algorithms, with applications in sustainable supply chain and logistics systems. Her areas of expertise include mixed-integer programming, multi-

objective optimization, and the design of efficient solution algorithms. She has experience applying these methods to real-world problems, particularly in sustainable logistics, healthcare service operations, and the automotive industry.

**Assistant. Prof. Dr. Gonca Yıldırım** is an Assistant Professor of Industrial Engineering at Gazi University, Türkiye. She received her BS and MS degrees in Industrial Engineering from Ihsan Dogramaci Bilkent University and her PhD in Industrial Engineering from University of Florida, USA. Her research focuses on optimization theory and methods, particularly in the areas of inventory systems and supply chain management. Her work emphasizes the development of analytical and mathematical models to support decision-making in complex and uncertain environments. She is actively involved in teaching and research in operations research, optimization, and logistics systems, contributing to both theoretical advancements and practical applications.

**Prof. Dr. Fulya Altıparmak Baykoç** is a Professor of Industrial Engineering at Gazi University, Türkiye. She earned her BS, MS, and PhD degrees in Industrial Engineering from Gazi University, Türkiye. Her research interests include stochastic modeling, multi-objective optimization, heuristic methods, and simulation, with strong applications in service systems, reliability and maintenance, and supply chain and logistics management. She has led and contributed to numerous academic and applied research projects, particularly in the modeling and analysis of complex service and production systems. She has published extensively in international journals and conferences and has supervised many graduate students. Her academic work bridges theoretical modeling with real-world industrial applications.