

Kinematic Analysis and Dimensional Synthesis of a Slider-Crank Coupled Rocker Mechanism for Automated Carton Flap Closing

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Abstract

This paper deals with the dimensional synthesis and kinematic analysis of a six-link planar mechanism for automated carton flap closing of packaging lines. The mechanism is a combination of a slider-crank sub-chain and a rocker linkage, which converts the continuous motion of the crank into a 90° oscillating flapper motion. The constrained systematic screening approach involved evaluating the candidate link configurations for Grashof feasibility, assembly feasibility, transmission angle quality, dead-point clearance and output swing accuracy throughout the 360° crank cycle. The chosen configuration results in a minimum transmission angle of 46° and a dead point margin of 53mm and a full 90° sweep of the flapper. The GIM software performs the kinematic at 30 rpm and has a peak velocity of 1.3 m/s and peak acceleration of 4.5 m/s² at flapper tip for moderate dynamic loading and gentle flap contact. The critical crank position (folding load 150N) was calculated by static FEA. The results validate the validity of the structure as the max von Mises stress of 103 MPa and minimum safety factor of 2.1 are acceptable. Mechanically simple and economical alternative to pneumatically and servo-electrically operated packaging systems for medium size RSC cartons from 30-60 cycles per minute.

Keywords

Carton Flap Closing, Dimensional Synthesis, Slider-Crank Mechanism, Four-Bar Linkage, Kinematic Analysis.

1. Introduction

The automated carton flap folding process is a basic process in high speed packaging lines for food, pharmaceutical and consumer goods industries. When production rates are above 30 cycles per minute (cpm), the folding mechanism needs to provide accurate and repeatable motion without the inconsistencies of pneumatic actuators or the capital cost and control complexity of servo-electric actuators. There are currently three main families of technology that control this application: pneumatic actuators, cam driven systems and servo-electric rotary tuckers. The problem with pneumatic systems is that they are compressible, which means that they can only practically achieve throughputs of less than 20 cpm (Groover 2019). Cam-driven mechanisms have deterministic motion profiles, but are subject to precision ground surfaces and have fixed timing that is hard to reconfigure (Norton 2012). Servo-electric assemblies offer the fastest speed, but require specialist control programming, high investment, and expose the network (Biegaj et al. 2024).

A tempting alternative is linkage-based actuation, which allows for single degree of freedom motion using a single induction motor, thus eliminating the compressibility delays and servo programming overhead, while still providing

mechanically determinate output trajectories. A number of researchers have studied the linkage mechanisms for carton-folding applications. Wang and Kou (2010) modeled the motion of a crank-rocker arc-type flap folder and set up design guidelines for throughput scaling. Dai and Jones (2002) created a kinematics and mobility analysis framework for carton folds based on the assumption that the fold lines are represented as revolute joints, which is used as a systematic basis for mechanism equivalent modelling. Yao et al. (2011) presented a reconfigurable robotic folding system and emphasized the importance of the kinematic adaptability in multi-format packaging lines. Sirkett et al., (2006) used knowledge-based engineering techniques to enhance the design of the packaging machinery, where the two main kinematic quality criteria were force transmission and dead-point avoidance. Recently, Xin et al. (2023) developed and optimized a flip shaping sub-system for carton packaging based on a rigid-flexible coupled dynamic model and genetic algorithm, which can greatly reduce the peak driving torque. Wang et al. (2022) studied a particular architecture of folding mechanism for regular slotted cartons, and designed relations between link proportions and output motion quality of the folding mechanism. Although the body of work, there are few published formulations that synthesize these two sub-chains (slider-crank and rocker) into a closed form synthesis that satisfies simultaneous dead-point and transmission angle constraints.

1.1 Objectives

The aim of this paper is three fold: (i) to develop and use a constrained systematic screening synthesis for a six-link slider-crank coupled rocker mechanism, obtaining closed form expressions for all the kinematic feasibility and performance constraints; (ii) complete position, velocity and acceleration analyses of the synthesized mechanism geometry using GIM software; and (iii) to check structural integrity using finite element analysis at critical crank position. This is a proven and manufacturable mechanism that is able to close standard RSC carton flaps at 30-60 cpm.

2. Literature Review

There are some different ways to link up synthesis for packaging machinery. Classical methods for four-bar and slider-crank mechanisms are well known and documented by Norton (2012), Myszka (2012) and Uicker et al. (2011), and these are the basis of the position, velocity and acceleration analysis of these mechanisms, using the vector-loop and matrix approach. All these references used the Grashof condition and the transmission angle as the two quality criteria for mechanism synthesis, which are also used in the present work.

These methods have been fine tuned for carton-folding applications. Dai and Jones (2002) proposed a method of representing carton fold patterns by a mechanism with crease lines represented as revolute joints and the mobility calculated with Grübler's criterion in the folded configuration. This is to allow the kinematic modelling of the carton as well as the mechanism analysis. Wang & Kou (2010) determined velocity and acceleration envelopes of the arc type flap folders by motion simulation, and correlated peak flapper acceleration to acceptable contact forces on corrugated board panels. Sirkett et al. (2006) used rule-based design techniques to packaging machinery and found that the two most common constraints that were being violated in industrial flap folder designs were dead-point avoidance and minimum transmission angle.

More recent work has focused on the effects of dynamics and flexible bodies. Xin et al. (2023) have combined a genetic algorithm and a rigid-flexible dynamic model in ADAMS/Flex and have shown that the contact time is greatly influenced by elastic deformation of the long output arms when the speed is higher than 60 cpm. This finding has a direct relevance for the current design with a flapper arm of 402 mm which will be tackled in future dynamic extension. Biegaj et al. (2024) presented a review of the servo drive architectures used in automated packaging lines and presented the performance and cost benchmarks with which the proposed linkage mechanism is compared. The multi-format adaptability was considered by Yao et al. (2011), who demonstrated that the reconfigurable linkage systems can be used to cope with the variation of carton size without retooling, which is very important in flexible manufacturing environments. The output swing angle and the transmission quality are the most important parameters for slider-driven rocker mechanisms, and Wang et al. (2022) showed that the link-length ratios and pivot placement is the most important parameters affecting them, which directly inspired the systematic parameter screening approach used in this paper.

The literature as a whole reveals a lack of a published synthesis procedure that considers the assembly feasibility, Grashof condition, transmission angle, dead-point clearance and output swing simultaneously in a unified closed form set of constraints. The present work fills this gap.

3. Methods

3.1 Mechanism Configuration

The proposed mechanism is a six-link, single-DOF planar linkage decomposed into two serial sub-chains sharing a prismatic joint at point **B**. The slider-crank sub-mechanism (**O₁–A–B**) converts continuous crank rotation into reciprocating slider translation. The slider-driven rocker sub-mechanism (**B–C–O₄**) converts slider displacement into oscillating rotation of a ternary rocker, whose extended arm **CD** is the folding flapper. Grübler's criterion confirms $DOF = 1$: $F = 3(n - 1) - 2j_1 = 3(5) - 2(7) = 1$, with $n = 6$ links and $j_1 = 7$ full joints. A similar six-bar linkage architecture for repetitive cyclic tasks was previously analyzed by Gazo-Hanna et al. (2024), providing a methodological basis for the present synthesis.

The principal design requirements are: input speed 30 rpm; flapper swing $\approx 90^\circ$; minimum transmission angle $\geq 40^\circ$; dead-point-free operation; slider stroke ≤ 200 mm; factor of safety ≥ 2.0 under a 150 N folding load (Figure 1).

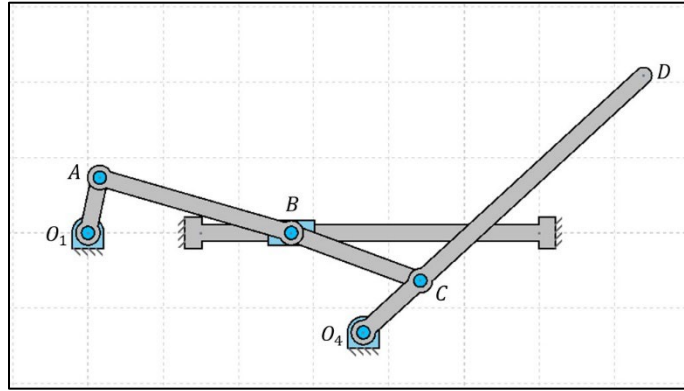


Figure 1. Six-link hybrid mechanism layout developed in GIM

3.2 Dimensional Synthesis

Design variables:

With the origin at O_1 and the slider axis along x , the design vector is $\mathbf{x} = \{r_2, r_3, x_{O_4}, y_{O_4}, r_5, r_6\}$, where r_2 is the crank length (O_1A), r_3 the coupler length (AB), (x_{O_4}, y_{O_4}) the rocker pivot coordinates, r_5 the connecting-rod length (BC), and r_6 the rocker length (O_4C). The ground scalar $r_1 = \sqrt{x_{O_4}^2 + y_{O_4}^2}$ and angle $\theta_1 = \text{atan2}(y_{O_4}, x_{O_4})$ are derived quantities.

Sub-mechanism I: slider-crank.

The vector-loop closure equation $r_2 e^{i\theta_2} + r_3 e^{i\theta_3} = d$ yields, after separation into scalar components:

$$\theta_3 = \arcsin\left(-\frac{r_2}{r_3} \sin\theta_2\right), \quad d(\theta_2) = r_2 \cos\theta_2 + r_3 \cos\theta_3 \quad (1)$$

Continuous crank rotation requires $r_2 < r_3$ (Grashof condition, C_1), giving stroke $\Delta d = 2r_2$.

Sub-mechanism II: slider-driven rocker.

The second loop $r_1 e^{i\theta_1} + r_6 e^{i\theta_6} + r_5 e^{i\theta_5} = d$ is solved by introducing $P = d - r_1 \cos\theta_1$ and $Q = -r_1 \sin\theta_1$ (constant), eliminating θ_5 , and applying the Weierstrass substitution $t = \tan(\theta_6/2)$ to produce the closed-form discriminant:

$$\mathcal{D} = A^2 + B^2 - C^2, \quad A = -2Pr_6, \quad B = -2Qr_6, \quad C = r_5^2 - P^2 - Q^2 - r_6^2 \quad (2)$$

Assembly feasibility (C_2): $\mathcal{D}(\theta_2) \geq 0$ for all θ_2 . The open-circuit rocker angle is:

$$\theta_6 = 2 \arctan\left(\frac{B + \sqrt{\mathcal{D}}}{C + A}\right) \quad (3)$$

Performance constraints:

The transmission angle at C :

$$\mu(\theta_2) = \min(|\theta_5 - \theta_6|, \pi - |\theta_5 - \theta_6|) \quad (4)$$

must satisfy $\mu_{\min} \geq 40^\circ$ (C_3). Dead-point clearance $\gamma_{dp} = \min(\gamma_{ext}, \gamma_{fld}) \geq 20$ mm (C_4), where $\gamma_{ext} = (r_5 + r_6) - \max \rho$ and $\gamma_{fld} = \min \rho - |r_5 - r_6|$ with $\rho(\theta_2) = \sqrt{(d - x_{O_4})^2 + y_{O_4}^2}$. Output swing: $|\Delta\theta_6 - 90^\circ| \leq 2^\circ$ (C_5).

Screening algorithm:

Candidate configurations are evaluated over a discrete grid of x at 1° crank-angle increments. Constraints C_1 – C_5 are applied sequentially; the first violation immediately discards the candidate. Among feasible configurations, the design maximizing $(\mu_{\min} - 40^\circ)$ is selected.

3.3 Kinematic Analysis

Differentiating the two vector-loop equations with respect to time gives the velocity system. For the slider-crank loop:

$$\omega_3 = -\frac{r_2\omega_2\cos\theta_2}{r_3\cos\theta_3}, \quad \dot{d} = -r_2\omega_2\sin\theta_2 - r_3\omega_3\sin\theta_3 \quad (5)$$

For the rocker sub-chain, angular velocities ω_5 and ω_6 are obtained from:

$$\begin{bmatrix} -r_5\sin\theta_5 & -r_6\sin\theta_6 \\ r_5\cos\theta_5 & r_6\cos\theta_6 \end{bmatrix} \begin{bmatrix} \omega_5 \\ \omega_6 \end{bmatrix} = \begin{bmatrix} \dot{d} \\ 0 \end{bmatrix} \quad (6)$$

whose determinant $\Delta = r_5r_6\sin(\theta_6 - \theta_5)$ is non-zero whenever C_4 is satisfied. Accelerations follow by a second differentiation. All quantities were evaluated numerically in GIM software (Macho et al. 2021; Petuya et al. 2011) at $\omega_2 = \pi$ rad/s (30 rpm).

3.4 Finite Element Analysis

A static FEA was conducted in SolidWorks Simulation at the critical crank position $\theta_2 = 75^\circ$, where the flapper first contacts the carton flap. Links were modelled in Aluminium Alloy 5052-H34 ($E = 70$ GPa, $\sigma_y = 215$ MPa, $\rho = 2680$ kg/m³). Fixed supports were applied at O_1 and O_4 ; the slider was constrained to translate along its guide axis; and a 150 N load (representing a $1.5\times$ factor on the estimated 100 N crease-fold resistance of a standard C-flute RSC carton) was applied at flapper tip D perpendicular to the arm. Solid tetrahedral elements with local refinement at joint interfaces were used.

4. Results and Discussion

4.1 Selected Configuration and Synthesis Validation

The final dimensions of the links are given in Table 1. Table 2 shows all five constraints are met with a comfortable margin.

Table 1. Final selected link dimensions and pivot coordinates

Parameter	Symbol	Value
Crank length	r_2	75 mm
Coupler length	r_3	265 mm
Ground-link length	r_1	388.1 mm
Ground-link angle	θ_1	-19.88°
Rocker pivot x -coordinate	x_{O_4}	365 mm
Rocker pivot y -coordinate	y_{O_4}	-132 mm
Connecting-rod length	r_5	183 mm
Rocker length	r_6	102 mm
Flapper arm length	r_{6D}	402 mm

Table 2. Synthesis constraint validation

Constraint	Threshold	Achieved	Status
Grashof ratio r_2/r_3	< 1	0.283	Pass
Assembly feasibility $\mathcal{D}_{\min}$	≥ 0	4218 mm ⁴	Pass
Minimum transmission angle $\mu_{\min}$	$\geq 40^\circ$	46°	Pass
Dead-point margin (folded) γ_{fld}	≥ 20 mm	53 mm	Pass
Dead-point margin (extended) γ_{ext}	≥ 20 mm	66 mm	Pass
Output swing $\Delta\theta_6$	88°–92°	$\approx 90^\circ$	Pass

4.2 Kinematic Analysis Results

The transmission angle μ is plotted for one complete crank revolution in Figure 2. The minimum value of 46° at $\theta_2 \approx 0^\circ$ and 360° indicates a satisfactory quality of force transmission for all positions and allows for some manufacturing tolerances (6° above the 40° limit).

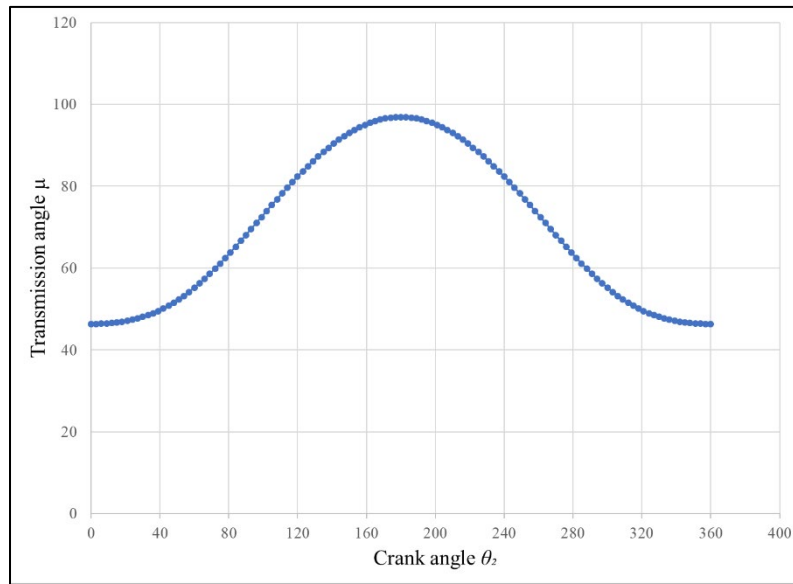


Figure 2. Transmission angle μ at joint BCO₄ versus input crank angle θ_2

Figure 3 shows the linear velocity of flapper tip **D**. The peak of approximately 1.3 m/s occurs near ($\theta_2 \approx 75^\circ$) and is low enough to prevent panel buckling or score-line damage on standard corrugated board during contact.

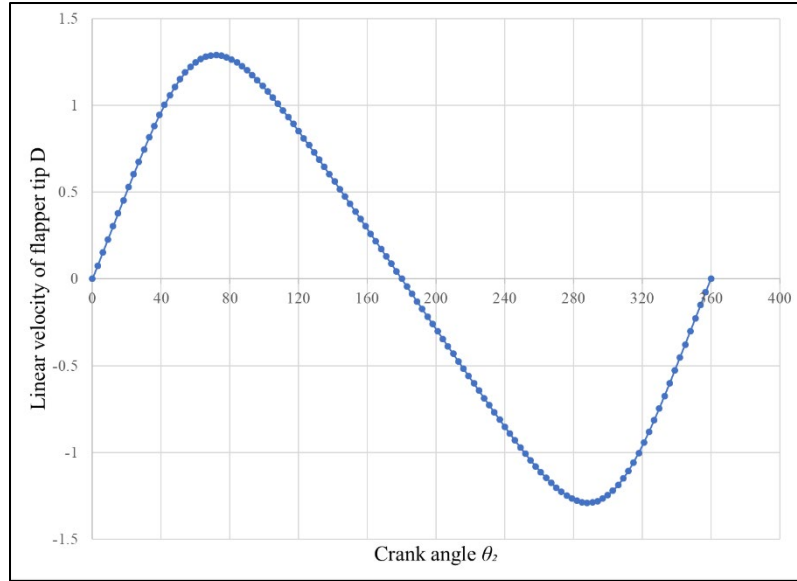


Figure 3. Linear velocity of flapper tip D versus input crank angle θ_2 at 30 rpm

The linear acceleration of point D is given in Figure 4. The maximum of about 4.5 m/s² is the confirmation that the inertial loading of the carton flap during the contact is moderate. An increase in the input speed to 60 rpm (double production rate) would raise the peak acceleration to about 18 m/s² which would need to be analyzed dynamically and for fatigue prior to use.

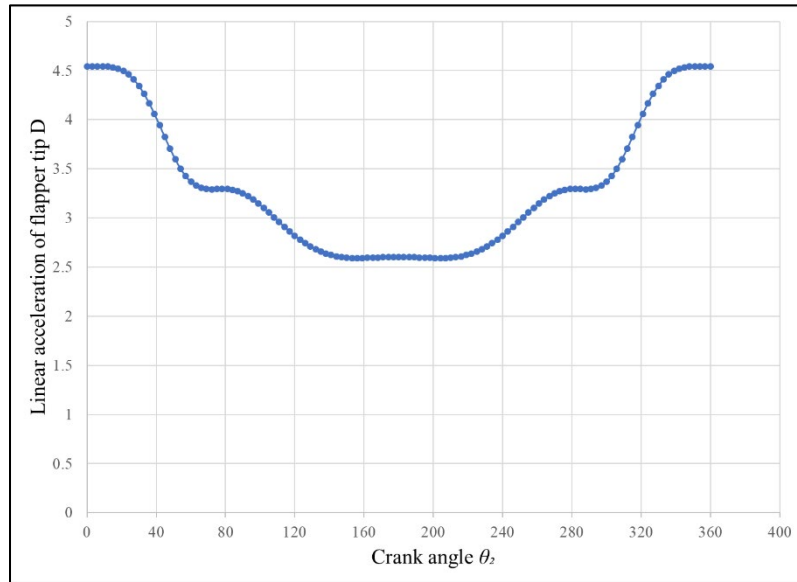


Figure 4. Linear acceleration of flapper tip D versus input crank angle θ_2 at 30 rpm

Table 3 summarizes the kinematic performance of the selected mechanism.

Table 3. Kinematic performance summary at 30 rpm

Metric	Symbol	Value
Slider stroke	Δd	150 mm
Rocker swing	$\Delta\theta_6$	$\approx 90^\circ$
Minimum transmission angle	$\mu_{\min}$	46°
Dead-point margin (governing)	γ_{fld}	53 mm
Peak flapper-tip velocity	$v_{D,\max}$	1.3 m/s
Peak flapper-tip acceleration	$a_{D,\max}$	4.5 m/s ²
Cycle time at 30 rpm	T	2.0 s

4.3 Structural Analysis Results

The FEA results are summarized in Table 4. Figure 5 shows the von Mises stress distribution at the critical crank position.

Table 4. Finite element analysis results ($\theta_2 = 75^\circ$, 150 N load, Al 5052-H34)

FEA Quantity	Value	Reference
Maximum von Mises stress	103 MPa	$\sigma_y = 215 \text{ MPa}$
Stress utilization ratio	47.9%	—
Maximum resultant displacement	6.31 mm	—
Minimum factor of safety	2.1	Required: ≥ 2.0

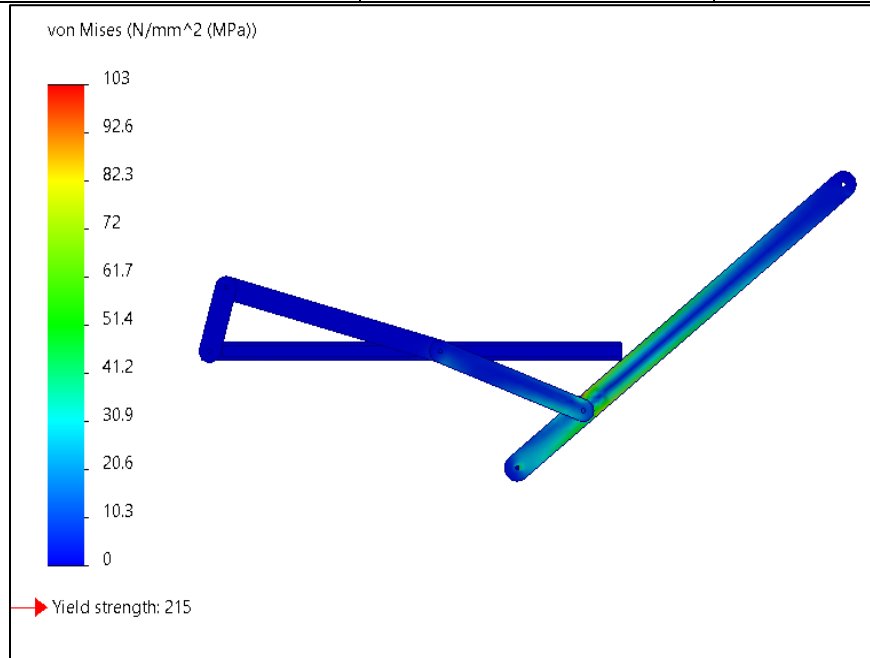


Figure 5. Von Mises stress distribution under 150 N folding load at $\theta_2 = 75^\circ$

The highest von Mises stress is 103MPa that occurs at the flapper arm to rocker body junction where the bending moment is largest. This is 52% below the material yield strength and indicates that the material has a fully elastic behavior under the design load. The factor of safety of 2.1 is within 5% of the prescribed factor of safety of 2.0. The maximum elastic displacement of 6.31 mm at the flapper tip is an inherent result of the 402 mm cantilever arm, and

should be considered in the final design of the contact geometry to assure the proper closed position of the tip under load.

When compared to other packaging systems, it is unique for its throughput of 30-60 cpm, which is higher than passive plow folders and pneumatic systems (4 –20 cpm), but lower than servo-electric rotary tuckers (50–80 cpm); it also has lower capital and control costs than the rotary tucker. The proposed design does not contain any sliding cam-follower contacts, but only revolute and prismatic joints, which will increase the life expectancy of the system in terms of maintenance from 2000 – 4000 hours to more than 8000 hours.

5. Conclusion

A six-link slider-crank coupled rocker mechanism for automated carton flap closing was dimensionally synthesized, kinematically analyzed and structurally validated in this paper. The following conclusions are listed.

A constrained systematic screening synthesis was developed that satisfied five simultaneous constraint equations (Grashof condition, assembly feasibility, transmission angle, dead-point clearance and output swing). Within the search limits that were set, the synthesis found a unique optimal setting, which satisfied all constraints: minimum transmission angle of 46°, dead-point margin of 53 mm, and output swing of 90°. These results support the first objective.

The smooth and continuous displacement, velocity and acceleration profiles throughout the entire crank cycle at 30 rpm were verified using kinematic analysis in GIM software. The second objective was confirmed with the moderate values of peak flapper-tip velocity (1.3 m/s) and acceleration (4.5 m/s²) which result in gentle carton contact without risk of panel damage.

The third objective was achieved by performing finite element analysis at the critical crank position, which showed structural adequacy under a conservative 150 N folding load, with a maximum von Mises stress of 103 MPa (48% of yield) and a factor of safety of 2.1.

This is a proposed mechanism for medium size RSC carton (400 × 300 × 300 mm) at 30–60 cpm as an alternative to the servo-electric and cam driven systems which are mechanically complex and costly. Future research will include a complete dynamic force and fatigue analysis, counterweight balancing to minimize shaking forces, prototype construction, and experimental testing of the kinematic and force prediction.

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