

Identical Parallel Machines Scheduling and Rescheduling Problem Solutions Based on Dispatching Rules

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Abstract

This report was prepared on a personal interest subject and applies theoretical dispatching rules to a practical scenario. For this reason, there may be places where the relevant rules and patterns are not followed. In this report, scheduling and rescheduling on identical parallel machine systems are discussed. The principles on which the scheduling will be made at the beginning of the process and how the currently used rule will be applied by rescheduling in case of problems that arise are discussed. In addition, the algorithms are shared for relevant dispatching rules. After explaining the dispatching rules used in the report, which are the basis of production planning, a problem will be discussed following in the report. The solution stages of the problem were done via Excel usage. All kinds of graphics, tables and diagrams used were prepared via Excel. In the conclusion section, the methods used are compared. The best in the options and the most efficient answer will be selected according to different priorities for the problem.

Keywords

Identical Parallel Machines, Scheduling and Rescheduling, Dispatching Rules, Production Planning, Process Optimization

1. Introduction

Since most of the studies and researches on scheduling are focused on single-machine systems, parallel-machine systems researches are limited. That's why, This report focused on scheduling problems for identical parallel machine systems.

Following the short definitions and explanations, an example is shared to further concretize the study. Scheduling and rescheduling on identical parallel machines are discussed in the problem section. A small-scale sample was chosen to simplify the example and convey the message in the best way.

1.1. Objectives

Critical Ratio (CR), Minimum Slack (MS) and Weighted Shortest Processing Time (WSPT) were chosen from the dispatching rules to be used for problem solutions. In addition to solution steps, algorithms are shared in the relevant section.

Scheduling and rescheduling were performed with the defined path and the results were compared. Because of the existing of reworks, rescheduling process is necessary. The main purpose of the report is to select the best method in the options for live assignments in instant situations in the case of rework and rescheduling needed. The effect of the rules give more chance to decide which one has the shorter production process rather than improvement rates.

In addition, efficiency calculation has become possible by using the initial scheduling. The results were evaluated according to the delivery time and total processing time criteria.

2. Literature Review

The problem of scheduling jobs on identical parallel machines has been a central theme in operations research for decades. As defined by Graham et al. (1979) in their seminal tripartite notation α, β, γ , the P_m environment represents a system where any of the n jobs can be processed by any of the m available processors with identical speeds. While solving for global optimality in parallel systems is often computationally intensive (NP-hard for most performance measures), the use of priority dispatching rules remains the most prevalent approach in real-time industrial applications (Pinedo, 2012).

The Weighted Shortest Processing Time (WSPT) rule is widely recognized for its efficiency in minimizing the weighted sum of completion times. According to Pinedo (2012), WSPT provides an optimal solution for single-machine environments and serves as a highly effective heuristic in parallel machine configurations. Research by Smith (1956) initially established that prioritizing jobs with the highest ratio of importance (weight) to processing time effectively reduces work-in-process (WIP) levels and inventory holding costs. In P_m systems, WSPT is preferred when the primary objective is internal economic efficiency and throughput maximization (Baker & Trietsch, 2009).

When scheduling objectives shift toward customer satisfaction and on-time delivery, due-date-based rules such as Minimum Slack (MS) and Critical Ratio (CR) become essential. MS focuses on the absolute safety buffer remaining for a job. Blackstone et al. (1982) emphasize that MS is particularly robust in reducing maximum tardiness, as it prevents jobs with narrow margins from being delayed by longer, less urgent tasks. Unlike static rules, CR is a dynamic index that integrates both the remaining work and the time remaining until the deadline. Vepsäläinen and Morton (1987) argue that CR provides a more sophisticated monitoring mechanism in complex job shops because it normalizes urgency. A CR value less than one serves as a critical signal for potential lateness, allowing for proactive intervention in parallel machine allocation.

The literature consistently highlights a fundamental trade-off between flow-time-based rules (WSPT) and due-date-based rules (MS, CR). In a comprehensive survey, Haupt (1989) noted that while WSPT excels in reducing average lateness, it may lead to high variance in tardiness, potentially "starving" low-priority jobs with long processing times. Conversely, MS and CR reduce the variance of tardiness but often at the cost of higher average flow times and WIP levels (Blackstone et al., 1982).

Furthermore, the performance of these rules in parallel machine environments is highly dependent on the "saturation level" of the system. Research indicates that as machine utilization approaches 100%, the discriminatory power of CR and MS diminishes, often requiring a hybrid approach or a temporary shift to Shortest Processing Time (SPT) logic to clear system congestion (Baker & Trietsch, 2009; Pinedo, 2012).

There are more than 100 of these rules, and the only and comprehensive source where all of them are together is "Survey of Dispatching Rules" wrote by S.S. Panwalkar and Wafik Iskander.

3. Methods

Initial point is definition of parallel and identical machines, which is the subject of the report. Machines are the unit element that carries out production in a production system. This element can sometimes be an equipment that produces product as an output in response to inputs, or sometimes it can be a human.

Identical meaning is that production equipment has exactly the same features and functions (Tighazoui et al., 2020). In manpower-based production, people who are identical to the person performing the production are taken into account in terms of technical knowledge, producing the same number of end products in the same unit of time, finishing the same end product in the same time, etc.

Parallel machine scheduling refers to a production environment where multiple machines are available in parallel to process a set of job, allowing for simultaneous task execution to improve system throughput and reduce makespan (Tighazoui et al., 2021).

Some dispatching rules will be considered in solving the our problem. A dispatching rule is a strategy or procedure designed to manage and prioritize tasks or jobs in dynamic and complex settings, such as automobile production, textile manufacturing, and glass container fabrication (Panwalkar & Iskander, 1977).

Which of these rules should be applied may vary depending on the sector and expectations. For this reason, the appropriate rule should be selected by taking into account the type of production, the suitability of the available data and expectations.

There are implementation requirements and linked results such as;

Data Accuracy: MS, CR and WSPT rules require precise processing time estimates. MS and CR further require strict due-date management.

Computational Overhead: WSPT is static; the sequence is determined once at the start. MS and CR are dynamic; the priority must be recalculated every time a machine becomes available, especially in environments with stochastic job arrivals (Haupt, 1989).

The Specificity of Parallel Machine: Unlike single-machine environments, identical parallel machines allow for "load balancing." While WSPT minimizes total holding costs, it does not guarantee balanced utilization across machines if job sizes are highly disparate. In such cases, combining WSPT with Longest Processing Time (LPT) logic for initial batching can improve makespan (Vepsäläinen & Morton, 1987).

The "Tardy" Threshold: In practice, CR is superior when the primary goal is customer satisfaction. However, if multiple jobs already have a $CR < 0$ (already late), the rule loses its discriminatory power, and the scheduler should switch to Shortest Processing Time (SPT) to clear the backlog quickly (Baker & Trietsch, 2009).

Weight Sensitivity: WSPT is highly sensitive to how "weights" are assigned. These weights should represent holding costs, penalty costs or strategic customer importance to yield meaningful economic results. These feature make comparative analysis and strategic differentiation impossible;

Temporal Dynamics: WSPT is a static rule in most contexts; once processing times and weights are known, the priority sequence remains constant regardless of the passage of time. MS and CR are dynamic; the priority of a job increases as the current time approaches the due date. In a parallel machine environment, this means the "best" job for an available machine may change even if no new jobs have arrived.

Mathematical Sensitivity: CR is a ratio-based logic, meaning it normalizes the urgency relative to the work remaining. MS is a linear subtraction logic, focusing on the absolute safety margin (Petrovic & Duenas, 2006).

Robustness to Processing Time Variance: WSPT is highly sensitive to estimation errors. If processing time is underestimated, a job that should have been processed quickly might block a machine, negating the flow-time benefits. CR and MS, by incorporating due dates, offer a slightly more robust buffer against minor processing fluctuations (Ulucak & Gökçen, 2025).

Computational Complexity in Parallel Machine Systems: In a system with "m" parallel machines, the overhead of recalculating CR and MS every time a machine becomes free can be significant if the job pool is large. WSPT requires only a single sort at the beginning, making it computationally "cheaper" for real-time execution.

The "Lateness" Saturation Point: A critical divergence point occurs when the system is overloaded. When all jobs are already late, MS and CR lose their effectiveness. In such "saturated" states, switching to SPT logic is often the only way to recover system stability (Turkcan et al., 2009).

3.1. Critical Ratio (CR)

The formula is applied to all available jobs and the jobs are sorted into the machines from smallest to largest according to the results. The formula used in critical ratio rule calculations is " $(Due\ Date - Current\ Day) / Operation\ Duration$ ". The due date in the formula indicates the day the job will be delivered. The current day represents the day which is today. The process on one machine took 10 days. After the process is completed, the day that machine is found is referred to as the 10th day. Operation duration is the estimated number of days required to complete the job before starting production. But the actual realized processing time may be longer or shorter. In the ranking of jobs that have

the same value as a result of the formula calculation, the weight factor is taken into account and the larger one is selected.

The CR algorithm is;

Notation & Parameters:

J: Set of all jobs, $J = \{1, 2, \dots, n\}$

U: Set of unscheduled jobs

M: Set of identical parallel machines, $M = \{1, 2, \dots, m\}$

p_j : Processing time of job $j \in J$

d_j : Due date of job $j \in J$

t: Current system time (Event clock)

A_k : Available time of machine $k \in M$

$CR_j(t)$: Critical Ratio of job j at time t

j^* : Selected job to be scheduled

k^* : Selected machine for scheduling

Step 0: Initialization

$U \leftarrow J$

$A_k \leftarrow 0, \forall k \in M$

$t \leftarrow 0$

Step 1: Machine Selection and Time Advancement

While $U \neq \emptyset$ **do:**

Find the earliest available machine:

$k^* \leftarrow \operatorname{argmin}_{k \in M} (A_k)$ (If multiple machines are available at the same time, select the one with the smallest index k .)

Advance system time to the available time of the selected machine:

$t \leftarrow A_{k^*}$

Step 2: Priority Evaluation (CR Calculation)

For each job $j \in U$ **do:**

If $p_j = 0$ **then**

$CR_j(t) \leftarrow -\infty$ (Safeguard against division by zero.)

Else

$CR_j(t) = (d_j - t) / p_j$

End If

End For

Step 3: Job Selection and Tie-Breaking

Define subset of jobs with the minimum CR value:

$U'_{CR} \leftarrow \{j \in U \mid CR_j(t) = \min_{i \in U} CR_i(t)\}$

If $|U'_{CR}| = 1$ **then**

$j^* \leftarrow$ the single job in U'_{CR}

Else (Tie-breaking logic for identical CR values.)

a. Apply Earliest Due Date (EDD) rule:

$U''_{EDD} \leftarrow \{j \in U'_{CR} \mid d_j = \min_{i \in U'_{CR}} d_i\}$

b. **If** $|U''_{EDD}| = 1$ **then**

$j^* \leftarrow$ the single job in U''_{EDD}

c. **Else** (Secondary tie-breaking.) Apply Shortest Processing Time (SPT) rule:

$j^* \leftarrow \operatorname{argmin}_{j \in U''_{EDD}} (p_j)$ (If ties still exist, pick the job with the smallest index j .)

End If

Step 4: Scheduling and State Update

Assign job j^* to machine k^* starting at time t .

Update the available time of the selected machine: $A_{k^*} \leftarrow t + p_{j^*}$

Remove the scheduled job from the unscheduled set: $U \leftarrow U \setminus \{j^*\}$

End While

Step 5: Output

Return the scheduling sequence and completion times (C_j) for all $j \in J$.

3.2. Minimum Slack (MS)

The minimum slack rule applies the principle of completing a job with the least delay without affecting the total project process (Ulucak & Gökçen, 2025). Slack refers to the amount of time a task can be postponed, and this rule ranks tasks with the least delay time first. The formula used for this rule is expressed as “*Due Date-(Current Day-Operation Duration)*”.

Definitions of formula elements are mentioned in the Critical Ratio section. In the equation resulting from the formula calculation, weight is taken into consideration and the largest is selected and continued.

The MS algorithm is;

Notation & Parameters:

n : Total number of jobs to be scheduled.

m : Total number of identical parallel machines.

J : Set of unscheduled jobs, $J = \{1, 2, \dots, n\}$

M : Set of identical parallel machines, $M = \{1, 2, \dots, m\}$

p_i : Processing time required for job $i \in J$

d_j : Due date of job $j \in J$

t : Current system time

A_k : Available time of machine $k \in M$

$S_i(t)$: Dynamic slack time of job i evaluated at time t .

Q : The ordered set of the final schedule.

Step 0: Initialization

$A_k = 0, \forall k \in M$

$J = \{1, 2, \dots, n\}$

$Q = \emptyset$

Step 1: Determination of Decision Epoch and Machine Selection

$t = \min_{k \in M} (A_k)$

$M^* = \{k \in M \mid A_k = t\}$

$k^* = \min \{k \mid k \in M^*\}$

Step 2: Dynamic Slack Evaluation

$S_i(t) = d_i - p_i - t, \forall i \in J$

Step 3: Job Prioritization and Tie-Breaking Formulation

$J_{MS} = \{i \in J \mid S_i(t) = \min S_j(t)\}$

Step 4: Assignment and State Update

$A_{k^*} = t + p_{i^*}$

$J = J \setminus \{i^*\}$

Step 5: Termination Condition

If $J = \emptyset$, then **STOP**

If $J \neq \emptyset$, then **GO TO Step 1**

3.3. Weighted Shortest Processing Time (WSPT)

There is a special requirement in the application of this rule. This particular requirement is the importance of jobs. This level of importance can be determined by the person doing the job. According to special criteria, the ones with a

high level of importance and a short processing time are prioritized to be processed (Ulucak & Gökçen, 2025). The formula required for this rule is expressed as “*Operation Duration/Weight*”.

In addition to the operation duration defined previously, weight represents an operation importance level and will be considered in the equality of formula calculation.

The WSPT algorithm is;

Notation & Parameters:

N: Set of jobs, where $j \in \{1, 2, \dots, n\}$

M: Set of identical parallel machines, where $k \in \{1, 2, \dots, m\}$

p_j : Processing time of job j

w_j : Weight (priority/penalty) of job j

I_j : Priority index of job j

A_k : Available time of machine k

S_j : Start time of job j

C_j : Completion time of job j

Q_k : Ordered sequence of jobs assigned to machine k

Step 1: Initialization

$A_k = 0, \quad \forall k \in M$

$Q_k = \emptyset, \quad \forall k \in M$

Step 2: Index Calculation

$I_j = w_j / p_j$

Step 3: Sorting and Sequence Generation

$I_{j[1]} > I_{j[2]} > \dots > I_{j[n]}$

Tie-breaking rules;

If $I_{j[u]} = I_{j[v]}$, then prioritize the job with the smaller processing time.

If $p_{j[u]} > p_{j[v]}$, then prioritize the job with the smaller job index.

Step 4: Machine Assignment (Load Balancing)

For $i = 1$ to n :

1. Let current job be $j = j_{[i]}$
2. Find the earliest available machine $k^* \in M$ such that:
 $A_{k^*} = \min\{A_k\}$

Tie-breaking rule: If multiple machines are available at the same time, select the machine with the smallest index k .

3. Assign job j to machine k^* :
 $S_j = A_{k^*}$
 $C_j = S_j + p_j$
4. Update the availability of machine k^* :
 $A_{k^*} = C_j$
5. Append job j to the sequence Q_{k^*} :
 $Q_{k^*} \leftarrow Q_{k^*} \cup \{j\}$

Step 5: Termination Condition

If $N = \emptyset$, then **STOP**

If $N \neq \emptyset$, then **GO TO Step 4**

4. Data Collection

The problem is given in an hypothetical small scale for a better understanding of the subject. The problem, which is designed to be seen in daily life, will be solved with 3 different methods and the most appropriate result will be tried to be found. There are two identical parallel machines and some tasks are ready to assign these machines. The machines are completely identical which means they have the same features. They finish the same job in the same amount of time with same inputs.

There are 20 jobs are waiting to be assigned. For each job, release date, due date, operation duration, realized operation time and weight information are given in Table 1. Also, some jobs require a rework process which is given in Table 1. To check the jobs that will be subject to rework operations, 3 units of time are required. Release date is the day the job arrives at the facility to operate, due date is the last day the job should be being operated, operation duration is the completion time of that job based on previous information and calculations before assignment, realized operation time is the actual realization time of that job which is different from the planned one, weight gives the importance of that job depending on external factors. Requiring extra rework are given for related jobs. Rework indicates situations where extra work is required after the initial process is completed. Accordingly, how many extra days of rework are required for which job is expressed in the same table. In light of the given data, 20 of our jobs and jobs which require rework will be assigned to 2 identical machines. During the assignment process, Critical Ratio (CR), Minimum Slack (MS) and Weighted Shortest Processing Time (WSPT) methods will be examined. These methods will be used for both initial schedule and reschedule and all rankings will be given in detail. All processes and graphics are prepared with Excel. In selection these methods, the aim was to use all available data and the selection was made accordingly.

Table 1. Job's Information

Card No	Release Date	Due Date	Operation Duration	Weight	Realized Operation Time	Rework (3 units time for checking)
K1-1	0	30	15	4	13	K5-R 3
K2-1	0	35	20	2	21	K9-R 5
K3-1	0	30	18	3	18	K12-R 10
K4-1	0	35	13	4	15	K15-R 5
K5-1	10	40	5	1	5	K18-R 5
K6-1	10	50	25	4	23	
K7-1	10	40	10	2	15	
K8-1	10	50	15	3	15	
K9-1	15	65	12	1	10	
K10-1	15	75	21	4	20	
K11-1	15	75	15	2	15	
K12-1	15	80	10	3	12	
K13-1	20	90	10	4	8	
K14-1	20	95	15	3	15	
K15-1	20	95	20	3	25	
K16-1	20	110	7	2	5	
K17-1	25	100	4	3	5	
K18-1	25	120	20	4	18	
K19-1	25	130	15	1	17	
K20-1	25	130	10	3	15	

5. Results and Discussion

5.1. Critical Ratio (CR) Calculations

How to perform the process steps with formula for this method is specified in the definitions section. Firstly, CR method is used to create initial scheduling according to the planned details. While making these calculations, rework requirements and realized operation times are ignored. According to the results, Machine-1 has 144 units of operation time while Machine-2 has 136 units of operation time which is showed in Figure 1.

													PLANNED
M1	K3	K1	K6	K9	K11	K13	K14	K17	K18	K20			144 Units
M2	K2	K4	K7	K5	K8	K10	K12	K15	K16	K19			136 Units

Figure 1. Initial Scheduling With CR Method

Total operation time is 280 units without idle time. The difference between the operating times of the machines is 8 units. Initial scheduling, which is formed by ordering the jobs into machines thanks to the CR method, is given in Figure 1.

Secondly, a reschedule is created, taking into account the rework jobs and realized operation times. After the calculations are made in the same way, a new assignment order is created. The results of the all and rework calculations and assignments are given in Table 2. Taking all the results into account, the rescheduling assignment orders to the machines and the total operation times are given in Figure 2.

Card No	Realized Operation Time	(DD-0)/OD Day 0	(DD-18)/OD Day 18	(DD-21)/OD Day 21	(DD-31)/OD Day 31	(DD-36)/OD Day 36	(DD-51)/OD Day 51	(DD-54)/OD DAY 54	(DD-56)/OD Day 56	(DD-66)/OD Day 66	(DD-69)/OD Day 69	(DD-74)/OD Day 74	(DD-89)/OD Day 89	(DD-97)/OD Day 97	(DD-101)/OD Day 101	(DD-112)/OD Day 112	(DD-122)/OD Day 122	(DD-126)/OD Day 126	(DD-127)/OD Day 127	(DD-131)/OD Day 131	(DD-136)/OD Day 136	(DD-145)/OD Day 145
K3-1	13	2	0,8																			
K2-1	21	1,75																				
K3-1	18	1,6666667																				
K4-1	15	2,6923077	1,307692308	1,076923077																		
K5-1	5		4,4	3,8	1,8	0,8	-2,2															
K6-1	23		1,28	1,16	0,76																	
K7-1	15		2,2	1,9	0,9	0,4																
K8-1	15		2,133333333	1,933333333	1,266666667	0,933333333	-0,066666667	-0,266666667														
K9-1	10		3,916666667	3,666666667	2,833333333	2,416666667	1,166666667	0,916666667	0,75													
K10-1	20		2,714285714	2,571428571	2,095238095	1,857142857	1,142857143	1	0,904761905	0,428571429	0,285714286											
K11-1	15		3,8	3,6	2,933333333	2,6	1,6	1,4	1,266666667	0,6	0,4	0,066666667										
K12-1	12		6,2	5,9	4,9	4,4	2,9	2,6	2,4	1,4	1,1	0,6	-0,9									
K13-1	8			6,9	5,9	5,4	3,9	3,6	3,4	2,4	2,1	1,6	0,1									
K14-1	15			4,933333333	4,266666667	3,933333333	2,933333333	2,733333333	2,6	1,933333333	1,733333333	1,4	0,4	-0,133333333								
K15-1	25			3,7	3,2	2,95	2,2	2,05	1,95	1,45	1,3	1,05	0,3	0,1	-0,8							
K16-1	5			12,71428571	11,28571429	10,57142857	8,428571429	8	7,714285714	6,285714286	5,857142857	5,142857143	3	1,857142857	1,285714286	-0,285714286	-1,714285714	-2,285714286				
K17-1	5				17,25	16	12,25	11,5	11	8,5	7,75	6,5	2,75	0,75	-0,25	-3	-5,3					
K18-1	18			4,45	4,2	3,45	3,3	3,2	2,7	2,55	2,3	1,55	1,15	0,95	0,4	-0,1	-0,3	-0,35				
K19-1	17			6,6	6,266666667	5,266666667	5,066666667	4,933333333	4,266666667	4,066666667	3,733333333	2,733333333	2,2	1,933333333	1,2	0,533333333	0,266666667	0,2	-0,066666667	-0,4	-1	
K20-1	15			9,9	9,4	7,9	7,6	7,4	6,4	6,1	5,6	4,1	3,3	2,9	1,8	0,8	0,4	0,3	-0,1	-0,6		

Card No	Realized Operation Time	(DD-0)/OD Day 0	(DD-18)/OD Day 18	(DD-21)/OD Day 21	(DD-31)/OD Day 31	(DD-36)/OD Day 36	(DD-51)/OD Day 51	(DD-54)/OD DAY 54	(DD-56)/OD Day 56	(DD-66)/OD Day 66	(DD-69)/OD Day 69
K1-1	13	2	0,8								
K2-1	21	1,75									
K3-1	18	1,6666667									
K4-1	15	2,6923077	1,307692308	1,076923077							
K5-1	5		4,4	3,8	1,8	0,8	-2,2				
K6-1	23		1,28	1,16	0,76						
K7-1	15		2,2	1,9	0,9	0,4					
K8-1	15		2,133333333	1,933333333	1,266666667	0,933333333	-0,066666667	-0,266666667			
K9-1	10		3,916666667	3,666666667	2,833333333	2,416666667	1,166666667	0,916666667	0,75		
K10-1	20		2,714285714	2,571428571	2,095238095	1,857142857	1,142857143	1	0,904761905	0,428571429	0,285714286
K11-1	15		3,8	3,6	2,933333333	2,6	1,6	1,4	1,266666667	0,6	0,4
K12-1	12		6,2	5,9	4,9	4,4	2,9	2,6	2,4	1,4	1,1
K13-1	8			6,9	5,9	5,4	3,9	3,6	3,4	2,4	2,1
K14-1	15			4,933333333	4,266666667	3,933333333	2,933333333	2,733333333	2,6	1,933333333	1,733333333
K15-1	25			3,7	3,2	2,95	2,2	2,05	1,95	1,45	1,3
K16-1	5			12,71428571	11,28571429	10,57142857	8,428571429	8	7,714285714	6,285714286	5,857142857
K17-1	5				17,25	16	12,25	11,5	11	8,5	7,75
K18-1	18			4,45	4,2	3,45	3,3	3,2	2,7	2,55	2,3
K19-1	17			6,6	6,266666667	5,266666667	5,066666667	4,933333333	4,266666667	4,066666667	3,733333333
K20-1	15			9,9	9,4	7,9	7,6	7,4	6,4	6,1	5,6

Card No	Realized Operation Time	(DD-74)/OD Day 74	(DD-89)/OD Day 89	(DD-97)/OD Day 97	(DD-101)/OD Day 101	(DD-112)/OD Day 112	(DD-122)/OD Day 122	(DD-126)/OD Day 126	(DD-127)/OD Day 127	(DD-131)/OD Day 131	(DD-136)/OD Day 136	(DD-145)/OD Day 145
K1-1	13											
K2-1	21											
K3-1	18											
K4-1	15											
K5-1	5											
K6-1	23											
K7-1	15											
K8-1	15											
K9-1	10											
K10-1	20											
K11-1	15	0,066666667										
K12-1	12	0,6	-0,9									
K13-1	8	1,6	0,1									
K14-1	15	1,4	0,4	-0,133333333								
K15-1	25	1,05	0,3	-0,1	-0,3							
K16-1	5	5,142857143	3	1,857142857	1,285714286	-0,285714286	-1,714285714	-2,285714286				
K17-1	5	6,5	2,75	0,75	-0,25	-3	-5,5					
K18-1	18	2,3	1,55	1,15	0,95	0,4	-0,1	-0,3	-0,35			
K19-1	17	3,733333333	2,733333333	2,2	1,933333333	1,2	0,533333333	0,266666667	0,2	-0,066666667	-0,4	-1
K20-1	15	5,6	4,1	3,3	2,9	1,8	0,8	0,4	0,3	-0,1	-0,6	

Rework (3 units time for checking)	(DD-66)/OD Day 66	(DD-69)/OD Day 69	(DD-112)/OD Day 112	(DD-131)/OD Day 131	(DD-151)/OD Day 151
K5-R	3	-8,6666667			
K9-R	5	-0,2	-0,8		
K12-R	10		-3,2		
K15-R	5			-7,2	

													REALIZED				
M1	K3		K1	K6		K8		K9-R	K11	K12	K15	K16	K15-R	K20		K18-R	156 Unit Time
M2	K2		K4		K7	K5	K9	K5-R	K10	K13	K14	K12-R	K17	K18		K19	162 Unit Time

Table 2. CR Calculations and Selection

According to Figure 2, the first machine performs a 156-hour process while the second machine performs a 162-hour process. No idle time was observed in either machine during the operation process. Only the second machine has a 6 units time difference from the other. The total operation time was found to be 318 units time.

5.2. Minimum Slack (MS) Calculations

How to perform the process steps for this method is specified in the definitions section. Initial scheduling, which was first created with the MS method, is given in Figure 3. As a result of the calculations, the difference between the two machines is calculated as 14 units and the assignments are given in detail in the relevant table. Since these calculations are based on initial planning, rework and realized operation times are ignored.

												PLANNED
M1	K3	K4	K6	K5	K10	K12	K13	K14	K18	K20		147 Units
M2	K1	K2	K7	K8	K9	K11	K15	K17	K16	K19		133 Units

Figure 3. Initial Scheduling With MS Method

The results of the calculations and assignments with consideration of rework and realized operation duration data are given in Table 3. Taking all the results into account, the assignment orders to the machines and the total operation times are given in Figure 4.

Table 3. MS Calculations and Selection

Card No	Realized Operation Time	DD-0-OD Day 0	DD-13-OD Day 13	DD-18-OD Day 18	DD-33-OD Day 33	DD-34-OD Day 34	DD-49-OD Day 49	DD-56-OD Day 56	DD-61-OD Day 61	DD-64-OD Day 64	DD-67-OD Day 67	DD-71-OD Day 71	DD-86-OD Day 86	DD-87-OD Day 87	DD-91-OD Day 91	DD-99-OD Day 99	DD-107-OD Day 107	DD-116-OD Day 116	DD-117-OD Day 117	DD-122-OD Day 122	DD-127-OD Day 127	DD-131-OD Day 131	DD-136-OD Day 136	DD-145-OD Day 145
K1-1	13	15																						
K2-1	21	15	2																					
K3-1	18	12																						
K4-1	15	22	9	4																				
K5-1	5		22	17	2	1	-14	-21																
K6-1	23		12	7	-8																			
K7-1	15		17	12	-3	-4																		
K8-1	15		22	17	2	1	-54																	
K9-1	10			35	20	19	4	-3	-8															
K10-1	20			36	21	20	5	-2	-7	-10	-13													
K11-1	15			42	27	26	11	4	-1	-4	-7	-11												
K12-1	12			52	37	36	21	14	9	6	3	-1	-16	-17										
K13-1	8				47	46	31	24	19	16	13	9	-6	-7	-11	-19								
K14-1	15				47	46	31	24	19	16	13	9	-6	-7	-11	-19	-27	-36						
K15-1	25				42	41	26	19	14	11	8	4	-11	-12	-16									
K16-1	5				70	69	54	47	42	39	36	32	17	16	12	4	-4	-13	-14	-19	-24	-28		
K17-1	5				63	62	47	40	35	32	29	25	10	9	5	-3	-11	-20	-21					
K18-1	18				67	66	51	44	39	36	33	29	14	13	9	1	-7	-16	-17	-22	-29			
K19-1	17				82	81	66	59	54	51	48	44	29	28	24	16	8	-1	-2	-7	-12	-16	-21	
K20-1	15				87	86	71	64	59	56	53	49	34	33	29	21	13	4	3	-2	-7	-11	-16	-25

	Realized	DD-0-OD	DD-13-OD	DD-18-OD	DD-33-OD	DD-34-OD	DD-49-OD	DD-56-OD	DD-61-OD	DD-64-OD	DD-67-OD	DD-71-OD
Card No	Operation Time	Day 0	Day 13	Day 18	Day 33	Day 34	Day 49	Day 56	Day 61	Day 64	Day 67	Day 71
K1-1	13	15										
K2-1	21	15	2									
K3-1	18	12										
K4-1	15	22	9	4								
K5-1	5		22	17	2	1	-14	-21				
K6-1	23		12	7	-8							
K7-1	15		17	12	-3	-4						
K8-1	15		22	17	2	1	-14					
K9-1	10			35	20	19	4	-3	-8			
K10-1	20			36	21	20	5	-2	-7	-10	-13	
K11-1	15			42	27	26	11	4	-1	-4	-7	-11
K12-1	12			52	37	36	21	14	9	6	3	-1
K13-1	8				47	46	31	24	19	16	13	9
K14-1	15				47	46	31	24	19	16	13	9
K15-1	25				42	41	26	19	14	11	8	4
K16-1	5				70	69	54	47	42	39	36	32
K17-1	5				63	62	47	40	35	32	29	25
K18-1	18				67	66	51	44	39	36	33	29
K19-1	17				82	81	66	59	54	51	48	44
K20-1	15				87	86	71	64	59	56	53	49

DD-86-OD Day 86	DD-87-OD Day 87	DD-91-OD Day 91	DD-99-OD Day 99	DD-107-OD Day 107	DD-116-OD Day 116	DD-117-OD Day 117	DD-122-OD Day 122	DD-127-OD Day 127	DD-131-OD Day 131	DD-136-OD Day 136	DD-145-OD Day 145
-16	-17										
-6	-7	-11	-19								
-6	-7	-11	-19	-27	-36						
-11	-12	-16									
17	16	12	4	-4	-13	-14	-19	-24	-28		
10	9	5	-3	-11	-20	-21					
14	13	9	1	-7	-16	-17	-22	-27			
29	28	24	16	8	-1	-2	-7	-12	-16	-21	
34	33	29	21	13	4	3	-2	-7	-11	-16	-25

Rework (3 units time for checking)		DD-64-OD Day 64	DD-86-OD Day 86	DD-107-OD Day 107	DD-122-OD Day 122	DD-153-OD Day 153
K5-R	3	-27				
K9-R	5		-26			
K12-R	10			-37		
K15-R	5				-32	
K18-R	5					-38

According to Figure 4, the first machine performs a 158-hour process while the second machine performs a 160-hour process. No idle time was observed in either machine during the operation process. Only the second machine has

	Machine													Realized	
M1	K3-1	K4-1	K6-1		K5-1	K9-1	K11-1	K9-R	K15-1		K14-1	K16-1	K19-1	K18-R	158 TIMES
M2	K1-1	K2-1	K7-1	K8-1	K5-R	K10-1	K12-1	K13-1	K12-R	K17-1	K15-R	K18-1	K20-1		160 TIMES

Figure 4. Rescheduling With MS Method

5.3. Weighted Shortest Processing Time (WSPT) Calculations

How to perform the process steps for this method is specified in the definitions section. Initial scheduling rework and realized operating times are disregarded and shared in Figure 5. According to this assignment, the difference between the total process time of the two machines was 12 units.

												PLANNED
M1	K4	K8	K13	K16	K14	K5	K10	K6	K2	K19		146 UNITS
M2	K1	K12	K17	K20	K18	K7	K3	K15	K11	K9		134 UNITS

Figure 5. Initial Scheduling With WSPT Method

The results of the calculations and assignments with consideration of rework and realized operation duration data are given in Table 4. Taking all the results into account, the assignment orders to the machines and the total operation times are given in Figure 6.

Table 4. WSPT Calculations and Selection

Card No	Realized Operation Time	OD/W Day 0	OD/W Day 13	OD/W Day 15	OD/W Day 18	OD/W Day 27 and later	Order of Operations
K1-1	13	3,75					
K2-1	21	10	10	10	10	10	16
K3-1	18	6	6	6	6	6	11
K4-1	15	3,25					
K5-1	5		5				
K6-1	23		6,25	6,25	6,25	6,25	13
K7-1	15		5	5	5		
K8-1	15		5	5	5	5	7
K9-1	10			12	12	12	18
K10-1	20			5,25	5,25	5,25	10
K11-1	15			7,5	7,5	7,5	15
K12-1	12			3,3333			
K13-1	8					2,5	2
K14-1	15					5	8
K15-1	25					6,666666667	14
K16-1	5					3,5	6
K17-1	5					1,333333333	1
K18-1	18					5	9
K19-1	17					15	19
K20-1	15					3,333333333	5

Order of Operations	Rework (3 units time for checking)	OD/W Day 27	OD/W Day 32	OD/W Day 33	OD/W Day 36	OD/W Day 90	OD/W Day 133	OD/W Day 150
3	K5-R	3	3	3				
20	K9-R	5						5
4	K12-R	10	3,333333	3,333333	3,333333			

				4	K12-R		10		3,333333		3,333333		3,333333				REALIZED					
M1	K4-1		K12-1		K17-1	K13-1	K20-1		K14-1		K10-1		K18-R	K6-1		K11-1		K15-R	K9-1	IDLE TIME	K9-R	153 TIMES
M2	K1-1		K5-1	K7-1	K5-R		K12-R	K16-1	K8-1	K18-1		K3-1		K15-1		K2-1	K19-1				165 TIMES	

Figure 6. Rescheduling With WSPT Method

According to Figure 6, the first machine performs a 153-hour working process while the second machine performs a 165-hour process. In addition, 3 units time idle time was observed in machine 1 during the operation process. The second machine has a 12 units time difference from the other and the total operation time was found to be 318 units time and 3 units idle time.

5.4. Summary of Calculations

According to the initial scheduling results, with the application of CR, MS and WSPT rules, the production times of the first and second machines were found to be 144-136, 147-133, 146-134 unit times, respectively. Total production times were found to be 280 unit times in all rules and the differences in production times between the two machines were found to be 8, 14, 12, respectively. According to these data, the least production time difference between the machines was found to be 8 unit times in the CR method. When rework and realized operation times were taken into consideration, the total times after rescheduling were found to be 156-162, 158-160, 153-165 unit times, respectively. The total production time was found to be 318, 318, 321 (3 idle time units) and the production time differences between the two machines were found to be 6, 2, 12, respectively. According to the second data, the least production time difference between the machines was the MS method with 2 units of time.

6. Conclusion

The job assignment process for identical parallel machines was carried out under 3 methods. All calculations were made using Excel via dispatching rules of the relevant methods. All definitions are explained in detail and formulas and graphics are presented for the application of the rules. Initial schedules and reschedules are needed to make efficiency and optimum evaluations in the extraction phase. All necessary data and tables are readily available in the report.

The values that will be considered as criteria in the efficiency calculation will be initial scheduling process times and reschedule process times. The machine production time differences formed from these data are our primary calculation variables. The change between the total process times under each rule will be used as secondary data. Accordingly, the production difference efficiency between the machines has shown a decrease of 25% - 85.7% - 0% after the application of the rules. The development is a good thing for the simultaneous operation of the machines and shift arrangements. In addition, the simultaneous completion of the jobs is also a targeted phenomenon.

On the other hand, when the total process times are considered, the difference is found to be 38 - 38 - 41, respectively. When the two types of data found are evaluated together for three method, the most efficient rule is found to be Minimum Slack (MS).

When all the results are considered, a total of 318, 318 and 321 units of time are required for the Critical Ratio (CR), Minimum Slack (MS) and Weighted Shortest Processing Time (WSPT) methods respectively. 3 hours of idle time was observed in the WSPT method. Within the framework of the results of this example, the best job assignment was determined as the MS method. These calculations do not say the optimal but these methods decrease the decision time and give the best option in appropriate expectation for local. One of these three methods can be preferred in the light of the operating facility's own characteristics and requirements.

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Biography

Melih Osman UZUN is a 2026 Industrial Engineering (English) graduate from Gazi University. With a vision focused on optimizing industrial systems and sustainability, he has gained professional experience across diverse sectors including the defense industry, logistics, and FMCG. His research interests include production planning, supply chain management, decision support systems, process optimization, and AI-based applications. His areas of expertise encompass lean manufacturing, data analysis, inventory management, operations research, and advanced integrated logistics support systems.