

Optimization of Parallel Machine Scheduling to Minimize Sequence-Dependent Setup Times: An Application in the Defense Industry

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Abstract

This study addresses production planning inefficiencies and delivery delays in a prominent defense industry manufacturing company. The research addresses the printed circuit board (PCB) production area as the critical operational stage where improper sequencing significantly escalates setup times. To mitigate these delays, a parallel machine scheduling model is developed, specifically designed to optimize job sequences and minimize sequence-dependent setup times on the production lines. The results demonstrate that the proposed model substantially reduces setup durations and leads to a marked improvement in on-time delivery performance. Ultimately, this study contributes to production planning literature by providing an applicable and efficient solution for minimizing scheduling-related delays in modern manufacturing environments.

Keywords

Parallel Machine Scheduling, Sequence-Dependent Setup Times, Defense Industry, Production Planning.

1. Introduction

For companies in the sector that are well-known for their comprehensive technological infrastructure and high production volumes, achieving sustainable and domestically-based production that meets quality standards is directly linked to the effective management of available resources.

Within the company, it provides technological solutions both nationally and globally by offering Electronic Manufacturing Services (EMS) and producing base station infrastructure for Nokia, as well as operating across a wide range of sectors including defense technologies, automotive, and IoT. When examining trend charts for the company's areas of operation (Figures 1, 2, 3, 4, and 5), while the EMS, Nokia, automotive, and IoT sectors show a stagnant or

declining trend, the production volume and strategic importance of the defense industry sector are seen to be steadily increasing.

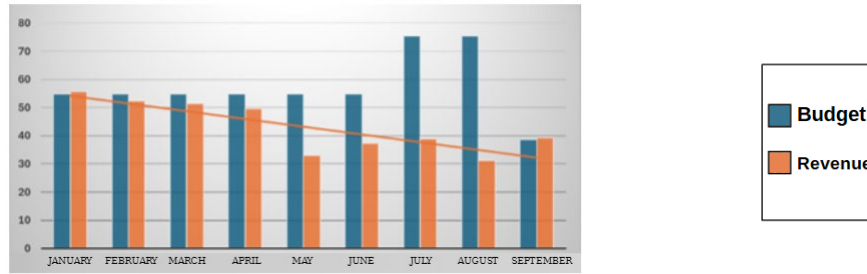


Figure 1. EMS Trend Chart

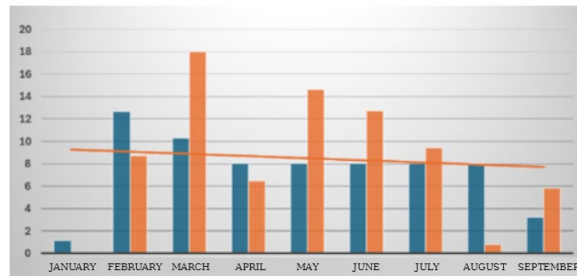


Figure 2: NOKIA Trend Chart

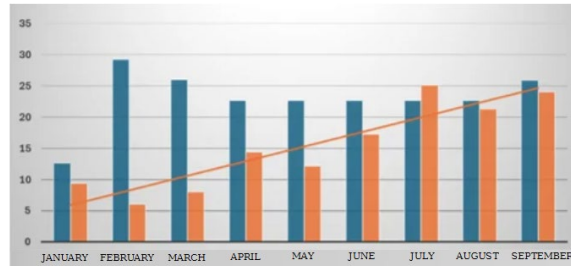


Figure 3: Defense Techonology Trend Chart

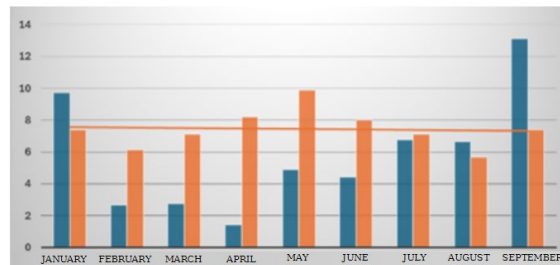


Figure 4: Otomotive Trend Chart

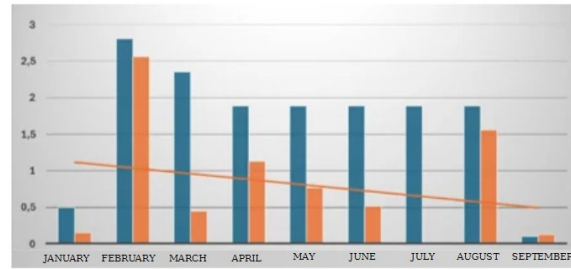


Figure 5: IOT Trend Chart

The company's strategic shift toward the defense industry has led to an increase in the volume and diversity of production in this sector. This growth has also brought new challenges to production planning processes; field investigations have determined that the primary source of problems in production planning lies specifically in these expanding defense production processes.

For companies operating in the defense industry, achieving sustainable and domestically produced output that meets quality standards is directly linked to the effective use of available resources. In this study conducted in the defense production sector, shortcomings in current production planning processes were analyzed. The identified workload imbalances and delivery delays make it imperative to improve planning processes in this area.

While defining the scope of the study, the entire defense production line was observed in detail, and the critical importance of the automated circuit board production area—the first stage of production—was identified. This area manufactures the Printed Circuit Boards (PCBs) for all products entering the defense production line and serves as a strategic hub that supports the entire production process in the defense industry. Consequently, disruptions in this unit directly impact production performance, delivery times, and customer satisfaction.

Upon examining the current system, it was observed that production plans struggled to align with realistic on-site implementation conditions, particularly due to the failure to account for the sequence-dependent high setup times of parallel automatic assembly machines. This situation disrupts the continuity of the production line.

The primary objective of this study is to develop methods that optimize sequence-dependent setup times in order to resolve these operational issues in the field of automated circuit board production. The approach devised within the scope of this study aims to contribute to making the defense industry production line more fluid, efficient, and predictable by improving job sequencing processes.

1.1 Objectives

In today's manufacturing environments, where industry competition is intensifying and delivery times have become a critical success factor, maintaining the efficiency of operational processes is a strategic necessity for companies. Prepared in this context, this study aims to analyze the root causes of delivery delays in the field of automated circuit board production and to develop a realistic production scheduling approach to reduce these delays. Based on field observations and analyses, it was determined that in the field of automated card production, the long setup times of seven automated sorting machines—which operate in parallel—and the failure to adequately account for these times during the planning process negatively impacted production performance.

In this context, the objectives of the study are structured as follows:

- **Model Development Using Real Data:** Instead of using hypothetical data from the literature, a dataset was created using real operational data collected directly from the company's automated card production line, ensuring an exact representation of the production line.
- **Development of a Realistic Scheduling Model:** A mathematical model was designed to reflect the reality of the automated card production process on the shop floor, taking into account seven parallel machines and sequence-dependent setup times.

- **The Contribution of Integrated Heuristic Approaches:** To overcome the computational burden of exact solution methods (CPLEX) in large-scale and complex production problems, a problem-specific solution approach was developed by integrating Random Search, Random Search and Simulated Annealing, Variable Neighborhood Search (VNS), and Variable Neighborhood Search and Simulated Annealing (SA).
- **Optimization and Performance Improvement:** Through the developed mathematical model and heuristic methods, the goal is to optimize the scheduling of parallel machines with sequence-dependent setup times and to minimize the total completion time ($C_{\max}$) in the system, thereby improving delivery performance.
- **Statistical Performance Validation:** The developed heuristic approaches have been proven to deliver superior solution quality, processing speed, and consistent results compared to traditional methods, and their applicability has been demonstrated through the use of defense industry data.
- **Design of a Decision Support Mechanism:** Instead of existing planning methods, a decision support framework that takes on-site realities into account, is suited to the dynamic nature of operations in the defense industry, and is practical has been implemented for the company.
- **Contribution of the Study to Sustainability Goals:** This work is not merely a technical optimization process but is directly linked to the United Nations Sustainable Development Goals (SDGs). The sustainability goals to which this study contributes are SDG 8,9, 12, and 17.

The contribution of this research lies in presenting an integrated meta-heuristic framework for a multidimensional scheduling problem that accounts for sequence-dependent setup times and in transforming this solution into a decision support framework based on real-world data from a critical sector such as the defense industry, thereby directly serving a vision of efficient and responsible production aligned with the United Nations Sustainable Development Goals.

2. Literature Review

In this section, the current literature in the field of production planning and scheduling was reviewed; in particular, studies focusing on minimizing setup times in systems with parallel machines and sequence-dependent setup times were evaluated. As part of the literature review, CPLEX-based models that generate optimal solutions using Mixed-Integer Linear Programming were analyzed, along with heuristic and meta-heuristic approaches for cases where the problem size increases. In this context, the study focused on Random Search, Simulated Annealing, and the Variable Neighborhood Search (VNS) method, which is one of the effective approaches in the literature.

Agnetis et al. (2025) provide a comprehensive review of academic research conducted in the field of scheduling over the past fifty years, outlining the theoretical foundations, complexity analyses, and fundamental deterministic models of machine scheduling from a chronological perspective. This study defines the mathematical framework of scheduling models, including single-machine, parallel-machine, and flow-type models. By addressing this broad spectrum, it provides a fundamental reference framework that summarizes current optimization trends and multi-objective decision-making processes. Ying et al. (2025), systematically maps the evolution of sequence-dependent setup times—which have been recognized as a strategic variable in production planning processes from the late 1960s to the present—within the literature on short-term production scheduling. By analyzing articles published between 1986 and 2024 using objective methods such as Main Path Analysis and Cluster Analysis, the researchers have revealed how the body of knowledge in the field has evolved and which technological advancements served as turning points. By documenting the critical impact of scheduling problems that account for setup times on production efficiency using academic data, this study provides a comprehensive foundation that solidifies the place of the model you developed in the literature and strengthens your problem definition. Xanthopoulos et al. (2016) examined the performance of rules developed for optimizing machine assignments and sequence-dependent setup times in dynamic and stochastic production environments. In the study, in addition to standard rules such as EDD and SPT, setup-oriented rules such as SPSU and SPTNS were also tested, and it was statistically proven that these rules provide improvements, particularly under high-load conditions. Furthermore, it was observed that different rules demonstrated superiority depending on the magnitude of setup times; while the SPSU rule stood out in overall performance, rules such as MMS and SPTNS were found to provide more stable results, particularly in delay metrics. These findings support the notion that dynamic and reactive

scheduling policies are more applicable than static methods in real production environments. Ceylan et al. (2019) examined the effects of sequence-dependent setup times on production performance in the paint shop of a company operating in the home appliance sector. This study, which demonstrated that setup times resulting from product color changes are a determining factor in the completion time of the last job ($C_{\max}$) and the total delay time, evaluated the industrial applicability of heuristic priority rules such as SPT, LPT, EDD, and FCFS, in addition to exact solution methods, due to the NP-hard nature of the problem. By comparing the results of mathematical models with those of heuristic rules, this study validates the critical role of setup-time scheduling models in improving operational efficiency using real-world data. In their comprehensive study examining the effects of setup times and costs on scheduling, Allahverdi et al. (2008) classified these problems according to the machine environment and the nature of the setup time (sequence-dependent or independent). The researchers noted that neglecting setup times—particularly in modern production systems—leads to bottlenecks and productivity losses, and emphasized the necessity of explicitly incorporating these times into scheduling models. This study establishes, within an academic framework, the strategic importance of optimizing sequence-dependent setup times in environments requiring high product variety and changeovers—such as PCB assembly lines—for enhancing operational flexibility and capacity utilization. Bektur (2021) proposed a multi-objective framework—including a speed-adjustment strategy—to improve energy efficiency in a single-machine scheduling problem with sequence-dependent setup times. Due to the problem's complexity, the study developed a hybrid meta-heuristic algorithm based on Simulated Annealing and statistically validated the algorithm's performance against the well-established NSGA-II method in the literature. It has been demonstrated that, particularly in large-scale problems, hybrid methods offer more flexible and faster solutions than exact methods when optimizing conflicting objectives (such as lead time and energy cost). This research supports the rationale for preferring the hybrid meta-heuristic model used in our study in the context of more efficient search of the solution space and support for operational decision-making. Bektur (2017) detailed the complexities encountered in independent parallel machine scheduling problems and the role of meta-heuristic methods in solving these problems.

The study highlights the advantages of meta-heuristic algorithms in terms of solution quality and speed, particularly in cases where the computation time of exact solution methods (MILP) increases exponentially as the problem size grows. This approach serves as a fundamental methodological reference for the scheduling of multi-machine, sequence-dependent production environments with setup times in PCB assembly lines, ensuring that the hybrid heuristic model we developed produces both fast and practical results. Tan et al. (2000) conducted a comparative analysis of branch-and-bound, genetic search, simulated annealing, and random-start binary search methods to minimize total delay time in single-machine environments with sequence-dependent setup times. The study demonstrated that, despite the success of exact solution methods for small-scale problems, meta-heuristic techniques such as simulated annealing offer much more stable and practical solutions in complex production environments where problem size increases. These findings support the methodological validity of preferring hybrid meta-heuristic algorithms for solving multidimensional production constraints with sequence-dependent setup times on our PCB assembly line. Rocha et al. (2007) proposed the Variable Neighborhood Search (VNS) algorithm for solving scheduling problems involving sequence-dependent setup times on parallel machines. The study demonstrated that, thanks to its systematically varying neighborhood structures, VNS scans the solution space without getting stuck in local optima and, particularly for large-scale problems, yields faster and higher-quality results compared to traditional methods such as GRASP. This approach supports the use of VNS as a fundamental methodological reference for the complex scheduling requirements of the multi-machine PCB assembly line in our study. Kılıç and Organ (2026) developed a new neighborhood-based Variable Neighborhood Search (VNS) algorithm for an NP-hard scheduling problem focused on minimizing sequence-dependent setup times on independent parallel machines. The study highlights the inefficiency of traditional methods in large-scale problems and demonstrates that the VNS-based algorithm scans the solution space much more effectively and optimizes the ($C_{\max}$) value.

This research proves that the hybrid heuristic model we selected to meet the requirements of the PCB lines in our study is fully aligned with VNS-based approaches in the current literature and constitutes one of the most effective solution strategies for industrial-scale problems.

3. Methods

This study is structured around the model developed to minimize the total completion time ($C_{\max}$) based on the results of the literature review and analyses, as well as the heuristic approaches integrated into scenarios where the problem size (8, 12, and 16 products) is expanded. A holistic solution approach has been developed to eliminate time losses on the production line and ensure a smoother flow. The most significant aspects of the problem observed on the production floor are the long setup times associated with product changes and the uncertainties in the production sequence. The

developed solution methods offer two distinct approaches to minimize these uncertainties: a Mathematical Model that seeks the optimal solution and a Heuristic Method tailored to the implementation speed on the shop floor. These two approaches support both achieving the best results in production planning processes and enabling rapid decision-making under dynamic production conditions.

3.1. Mathematical Modeling Approach

As part of this study, a mathematical model based on Mixed-Integer Linear Programming was developed to reduce delivery delays. The model was designed to reflect the constraints of the current production environment and setup times dependent on sequencing. The primary objective of the model is to minimize the system's C_{max} (Total Completion Time) value by minimizing the total setup times on the production line. The mathematical model is as follows:

Sets:

J : Job set

N : Machine Set

K : Sequence Sey

Indices:

$i, j \in J$: jobs

$m \in M$: machines

$k \in K$: job sequences

Parameters:

S_{ijm} : Setup time required to switch from job i to job j on machine m

P_{mj} : Processing time for job j on machine m

M : Big M

Objective Values:

X_{mkj} : Assignment of job j to sequence k on machine m

C_{mk} : Completion time of the job in sequence k on machine m

C_{max} : Completion time of the last job in the system

Objective Fonctions:

$$Enk = C_{max} \quad (1)$$

Constraints:

$$1) \quad \sum_{m=1}^M \sum_{k=1}^K X_{mkj} = 1 \quad \forall j \in J \quad (2)$$

$$2) \quad \sum_{m=1}^M X_{mkj} \leq 1 \quad \forall j \in J, \quad (3)$$

$$3) \quad \sum_{m=1}^M X_{mkj} - \sum_{j=1}^J X_{mk-1j} \geq 0 \quad \forall m, k > 1 \quad (4)$$

$$4) \quad C_{mk} \geq h_j + p_{mj} \times X_{mkj} \quad M(1 - X_{mkj}) \quad \forall m, j, k = 1 \quad (5)$$

$$5) \quad C_{mk} \geq C_{mk-1} + S_{ij} \times X_{mkj} + p_{mj} \times X_{mkj} - M(2 - X_{mkj} - X_{mk-1j}) \quad \forall m, i, j, k > 1 \quad (6)$$

$$6) \quad C_{max} \geq C_{mk} \quad \forall m, k \quad (7)$$

$$7) \quad X_{mkj} \in \{0,1\} \quad \forall m, k, j \quad (8)$$

$$8) \quad C_{max}, C_{mk} \geq 0 \quad \forall m, k \quad (9)$$

The model aims to minimize the total completion time (C_{max}) of the system, which is the key performance indicator of the scheduling problem in the objective function. It guarantees that each job (j) is assigned to a suitable machine and a specific position on that machine among all machine and sequence alternatives. (1) It prevents conflicts by specifying that a machine cannot process more than one job simultaneously during any given operation (k th sequence). (2) It ensures the continuity of the processing sequence on a machine; that is, it requires that the ($k-1$)th slot be occupied or completed before a job can begin in the k th slot. (3) It guarantees that the completion time of a product to be processed in the first slot ($k=1$) cannot be less than the sum of the product's initial setup time and processing time. (4) It manages the transition between consecutive jobs ($k > 1$); it relates a product's completion time to the sum of the previous product's finish time, the inter-job setup time, and the current product's processing time. (5) It defines the completion time of the last job in the system (C_{max}) as being greater than or equal to the completion times of all jobs on all machines and in all queues. (6) It specifies that the job assignment variable is a binary structure that can take only the values "0" or "1." (7) It guarantees that completion times and the total production time (C_{max}) cannot take on negative values. (8)

The model was run in CPLEX with a one-hour (3,600-second) time limit for simulations with 8, 12, and 16 products. The results obtained at the end of one hour were saved to files in .txt format.

In the CPLEX solution, as the number of products (n) increases, the solution space grows rapidly. On a 7-machine system, the probability of sorting increases exponentially as n rises (7^n). Therefore, although the mathematical model is closer to finding the optimal solution in small-scale trials, it becomes insufficient for a one-hour run as the number of products increases. In such cases, heuristic algorithms have been employed in the literature. Excel VBA was used to implement the heuristic methods. The application and results of the heuristic method will be evaluated in the next section of this study.

3.2 Heuristic Algorithms

For the same test set, the “Random Search” button in Excel triggers the heuristic algorithm to run and generate a solution. The second button, “VNS,” performs Variable Neighborhood Search (VNS), and the third button, “VNS + Simulated Annealing,” integrates Variable Neighborhood Search (VNS) with Simulated Annealing.

In all heuristic approaches, the processing times of the products, initial preparation times, and component disassembly and assembly times are read from the Excel sheet.

Random Search Approach:

An experimental design consisting of 15 independent replicates was implemented. Each replicate runs in a loop of 500 iterations and analyzes a total of 7,500 solution scenarios.

Steps of the Algorithm:

In each iteration, 12 products are randomly assigned to a machine (1–7), creating a new scheduling scenario (candidate solution) to scan the solution space. This “Random Search” approach helps avoid local optima in the problem’s complex solution space and quickly scan different job-machine assignment combinations.

Using the “CmaxCalculate_12” function, the total completion time is calculated for each candidate solution generated. At the end of each iteration, the $C_{\max}$ value obtained is compared with the best value recorded so far. If the new solution has a smaller $C_{\max}$ value, it is updated. This approach ensures that the algorithm retains the best solution at every stage. Once all replications and iterations are complete, the global best solution and its scheduling data are exported to Excel via the “Write the best solution” block.

VNS (Variable Neighborhood Search):

The developed VNS algorithm was tested using an experimental design consisting of 15 independent replications. Each replication scans the solution space using local neighborhood searches and attempts to reach the minimum $C_{\max}$ value. The performance under different initial conditions was analyzed using the 15 distinct results obtained at the end of the experimental design.

Algorithm Steps:

The algorithm performs 100 different initial trials. In the first 30 trials, random assignments are made using the “Rnd()” function, while trials 61–100 follow a runtime-sensitive assignment strategy to focus on better solutions in the solution space.

The best candidate obtained from the initial phase is improved using two operators over 500 iterations. In the first replacement ($k \pmod{2} = 0$), a machine is randomly selected for a job; in the second ($k \pmod{2} = 1$), the machine assignments of the first job are swapped with each other.

In each iteration, the newly calculated $C_{\max}$ value is compared with the current best value. If the new solution is smaller, the solution is updated.

Integration of VNS (Variable Neighborhood Search) and Simulated Annealing:

An optimization framework has been created by combining the local search power of the VNS algorithm with the Simulated Annealing mechanism.

Steps of the Algorithm:

- 1) Initially, a high-exploration phase is initiated with $T=1000$. Throughout the iterations, the temperature is cooled using the coefficient α ($\alpha=0.9999$), allowing the algorithm to converge to the optimal solution over time.
- 2) A product is selected at random, and a neighboring solution is generated by changing the machine assignment. Unlike VNS, this integrated model does not simply accept better solutions. Using its formula, it applies an acceptance criterion with a certain probability and may also accept worse solutions.

$$AcceptProbability = \text{Exp}\left(-\frac{(newCmax - Cmax)}{T}\right) \quad (10)$$

- 3) Local Optima: This mechanism prevents the algorithm from getting stuck in local optima, allowing it to scan a broader portion of the solution space.
- 4) At the end of each iteration, the system is cooled using the operation $T = T * \alpha$. After 100,000 iterations, the best solution is recorded.

4. Data Collection

Production data for the company's 18 different card types—including processing times, initial setup times, and removal and insertion times—were compiled. The probability distributions of these four basic data groups were analyzed using the EasyFit software. The suitability of the distribution models was determined using the Anderson-Darling goodness-of-fit test, and the graphs for the selected distributions are presented as an example in Figure 6.

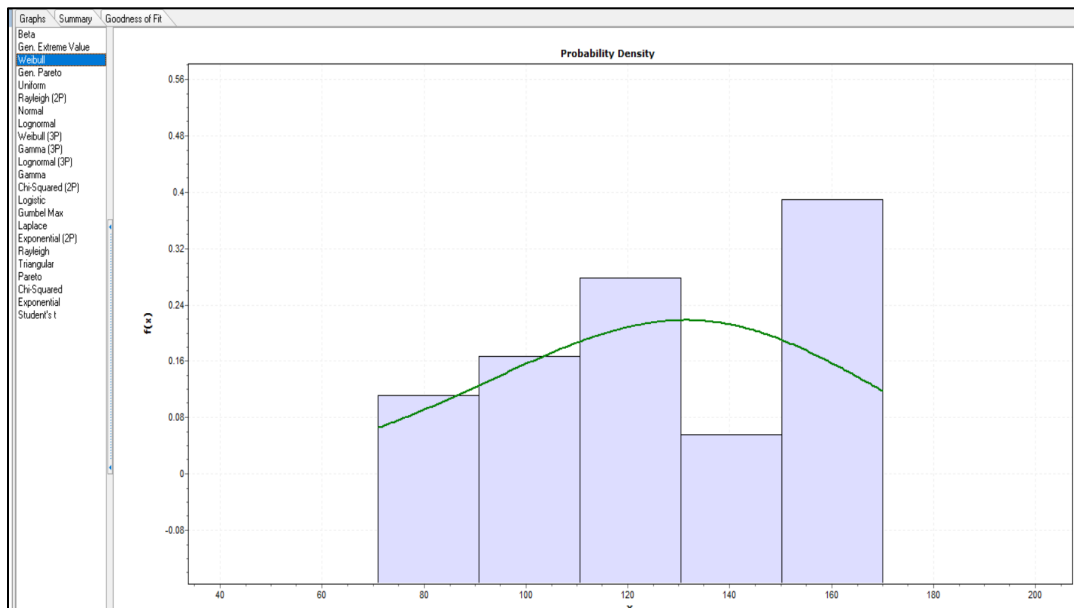


Figure 6. The Anderson-Darling goodness-of-fit test result example

Based on the determined distributions, the data were generated in Excel using VBA for use in mathematical models and heuristic algorithms. Test sets were generated for 8, 12, and 16 products and made ready for use. The code screens for the generation of the 8-product test set are shown in Figures 7-9.

```

For i = 1 To n
    rndDeger = Rnd()
    result = 140.72 * ((-Log(rndDeger)) ^ (1 / 4.0792))

    ws.Cells(ProcessingTimesRow + i, 1).Value = Products(i - 1)
    ws.Cells(ProcessingTimesRow + i, 2).Value = result
Next i

```

Figure 7. Processing Times Production Code Part - VBA

Data was generated using the parameters $\alpha = 4.0792$ and $\beta = 140.72$ for transaction times that follow a Weibull distribution. The code snippet is highlighted in dark red.

```

For i = 1 To n
    rndDeger = Rnd()
    result = WorksheetFunction.LogNorm_Inv(rndDeger, 4.336, 0.98336)

    ws.Cells(initialsetupRow + i, initialsetupCol).Value = Products(i - 1)
    ws.Cells(initialsetuptRow + i, iinitialsetupCol + 1).Value = result
Next i

```

Figure 8. Initial Preparation Times Code Part - VBA

Data was generated using the parameters $\sigma = 0.98336$ and $\mu = 4.336$ for the initial lead times, which follow a log-normal distribution. The code snippet is highlighted in dark red.

```

ws.Cells(rowNo, DisassemblyTimesCol).Value = WorksheetFunction.LogNorm_Inv(Rnd(), 3.6336, 0.98812)
ws.Cells(rowNo, AssemblyTimesCol).Value = WorksheetFunction.LogNorm_Inv(Rnd(), 4.3249, 0.98651)

```

Figure 9. Component Disassembly and Assembly Times Code Part – VBA

Data was generated using the parameters $\sigma = 0.9812$ and $\mu = 3.6336$ for removal times, and $\sigma = 0.98651$ and $\mu = 4.3249$ for insertion times, which conform to the log-normal distribution. The code snippet is highlighted in dark red. The generated data, along with the model constraints and the generated time data for mathematical model integration, were transferred to the C programming language. Excel VBA was used for the heuristic algorithms.

5. Results and Discussion

5.1 Numerical Results

Analysis of Production Status and Performance Analysis for 7 Machines and 8 Products

Under a production scenario involving 7 machines and 8 products, CPLEX—an exact solution method—can quickly reach optimal results because the solution space is small. In this study, the performance of CPLEX, an exact solution method, and the Random Search method, a basic heuristic approach, was compared based on the results of 15 trials. The results are presented in Table 1 below.

Table 1. The solution results of 15 trials

Replication	Iteration	Min Cmax
1	500	413.6645266
2	500	390.7085824
3	500	390.7085824
4	500	413.6645266
5	500	390.7085824
6	500	390.7085824
7	500	390.7085824
8	500	390.7085824
9	500	390.7085824
10	500	390.7085824
11	500	390.7085824
12	500	390.7085824
13	500	411.3573219
14	500	390.7085824
15	500	390.7085824
Mean		395.1459576
Std. Deviation		9.200026056

CPLEX found the $C_{\max}$ value to be 391.84 minutes in 1.95 seconds. The average $C_{\max}$ value for the Random Search method was 395.15 minutes. This result shows that CPLEX produces better solutions in terms of average performance. Additionally, the standard deviation for the Random Search method was low at 9.200. This indicates that CPLEX performed slightly better than the Random Search method. Looking at the minimum and maximum values, the results of the Random Search method ranged from 390.71 to 413.66 minutes.

Analysis of Production Conditions and Performance Analysis for a 7-Machine, 12-Product System

In this study, a production scenario involving 7 machines and 12 products was examined, and the solution performance of the CPLEX, Random Search, and Random Search-based Simulated Annealing methods was analyzed comparatively. In this context, solutions were obtained using the CPLEX, Random Search, Random Search-based Simulated Annealing, and Variable Neighbor Search-based Simulated Annealing methods. The $C_{\max}$ values obtained from Random Search are presented in Table 2.

Table 2. Solution results of the random search method

Replication	Iteration	Min Cmax
1	500	563.7886484
2	500	520.8902328
3	500	554.7317268
4	500	463.4675485
5	500	563.7886484
6	500	443.8366101
7	500	527.1963782
8	500	563.7886484
9	500	433.0938921
10	500	563.7886484
11	500	563.7886484
12	500	512.318548
13	500	491.0813983
14	500	582.2740342
15	500	563.7886484
Mean		527.441484
Std. Deviation		48.73815573

The Random Search method was found to produce high $C_{\max}$ values when compared to the CPLEX result of 392.28 minutes. The average $C_{\max}$ value of the Random Search method was 527.38 minutes, which was insufficient. Accordingly, a decision was made to improve the heuristic algorithm; the Random Search results were accepted as the initial solution, and the Simulated Annealing algorithm was applied. The $C_{\max}$ value yielded the same result across 15 replications, and the average was reduced to 418.36, resulting in a 20.67% improvement. Simulated Annealing reduced the standard deviation from 48.74 to 0. The results are presented in Table 3.

Table 3. Solution results of the Simulated Annealing method

Replication	Min Cmax
1	418.3645981
2	418.3645981
3	418.3645981
4	418.3645981
5	418.3645981
6	418.3645981
7	418.3645981
8	418.3645981
9	418.3645981
10	418.3645981
11	418.3645981
12	418.3645981
13	418.3645981
14	418.3645981
15	418.3645981
Mean	418.3645981
Std. Deviation	0

Investigation and Performance Analysis of a Production Scenario with 7 Machines and 16 Products

The proposed hybrid approach yields results that are closer to CPLEX's best solution and much more reliable (a standard deviation of 0) compared to the classical Random Search method. Although these improvements have contributed to the method's solution quality, the fact that the results obtained are far from the best value produced by CPLEX highlights the need for a more effective search strategy. It has been determined that a basic Random Search-based improvement is limited in terms of scanning the solution space and escaping local optima. Accordingly, to further improve solution quality and enrich the search process with a systematic neighborhood structure, it was decided to use the Variable Neighborhood Search (VNS) algorithm instead of the random search method. It produced the same Cmax values as the simulated annealing results but in a different order.

In this section, the scale of the production scheduling problem was expanded to 7 machines and 16 products to examine the behavior of the developed heuristic approaches in a high-dimensional problem space. As the number of products increased, the solution space grew exponentially; this not only increased the computational load on CPLEX, the exact solution method, but also made the solution quality and stability of the heuristic methods more critical. CPLEX took 499 minutes to complete the calculation but remained at a high Gap value of 55.78%. In the 16-product scenario, the algorithms' convergence success rates to the optimal solution and their performance distributions, based on statistical data, are presented in Figure 10 below using Minitab.

Descriptive Statistics: Random Search; VNS; VNS + Simulated Annealing

Statistics

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median
Random Search	15	0	747,954	9,74974	37,7606	684,939	721,730	747,547
VNS	15	0	569,654	3,18459	12,3339	555,048	558,759	571,481
VNS + Simulated Annealing	15	0	558,616	1,49990	5,80910	547,416	555,048	558,759

Variable	Q3	Maximum
Random Search	784,657	797,235
VNS	571,531	599,830
VNS + Simulated Annealing	563,817	571,481

Figure 10. The comparative analysis results for random search, VNS, and VNS+Simulated Annealing methods

5.3 Proposed Improvements

The performance results of the different solution approaches implemented in this study are detailed in Figure 11.

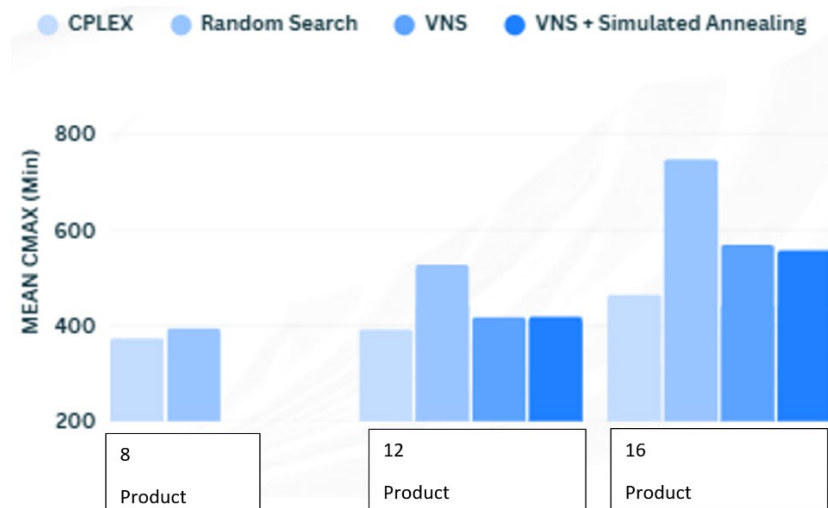


Figure 11. Comparative analysis of the heuristic methods' results

The graph shows that CPLEX produces the lowest $C_{\max}$ value (i.e., the fastest solution) for all product counts. Since CPLEX is a deterministic and exact solution method, it represents the “ideal” or “closest” performance benchmark that heuristic methods aim to achieve.

As the number of products increases (from 8 to 16), the $C_{\max}$ values for all methods rise. This confirms that the difficulty level of the scheduling problem increases exponentially with the number of products (NP-hard nature). In particular, for 12 and 16 products, Random Search has significantly higher $C_{\max}$ values than all other methods. This demonstrates that random assignments lead to inefficiency in complex production lines.

VNS and VNS + Simulated Annealing (Success): The developed VNS and VNS + Simulated Annealing methods are much more successful than Random Search for 12 and 16 products and produce results close to those achieved by CPLEX.

While VNS and the hybrid model yield nearly identical results for 12 products, the VNS + Simulated Annealing method shows a slight improvement for 16 products, converging even closer to CPLEX’s results.

6. Conclusion

The conclusions from our proposed study are as follows:

- Based on data from 15 replications, this study discussed the average $C_{\max}$ values calculated for different product scenarios and methods.
- In the 12-product scenario, the VNS+Annealing hybrid approach produced lower and more stable $C_{\max}$ values compared to the pure VNS and Random Search methods. These findings demonstrate that the Variable Neighborhood Search–Annealing algorithm exhibits successful performance results in the planning processes of the production scenarios considered, compared to other methods.
- The developed heuristic approaches have not remained merely optimization methods; they have also been integrated into a Decision Support System (DSS) module that can be implemented within the company.
- Operational Contribution: This system facilitates the transition from a “heuristic” planning approach on the shop floor to a “data-driven and optimized” production vision. Thanks to the DSS to be developed in the future based on the proposed study’s results, production managers can leverage the power of the theoretical model to make more predictable decisions regarding machine assignments and job sequencing, thereby minimizing delivery delays.

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