Proceedings of the International Conference on Industrial Engineering and Operations Management

Publisher: IEOM Society International, USA DOI: 10.46254/AN16.20260026
Published: June 23, 2026

A Better Algorithm for Two-Machine Flowshop to Minimize Total Tardiness with Interval Setup Times

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Abstract

This paper examines a two-machine flowshop scheduling problem with the objective of minimizing total tardiness in an environment where setup times are treated separately from processing times and are subject to uncertainty. Setup times are modeled as interval parameters, where only their lower and upper bounds are known, reflecting practical manufacturing conditions in which setup durations may vary due to operational factors. The total tardiness performance measure is of particular importance in manufacturing systems because it is closely associated with due-date adherence and customer satisfaction. This problem has been previously studied in the literature, and several algorithms have been proposed. Building on these studies, the present paper addresses the same problem and introduces a new algorithm that explicitly incorporates bounded setup-time information into the sequencing decision. Extensive computational experiments are conducted over a wide range of problem sizes and parameter settings, and the results demonstrate that the proposed algorithm consistently outperforms the best-known existing algorithm for this problem. On average, the new approach achieves a meaningful reduction in total tardiness, and therefore the algorithm proposed in this paper is recommended for two-machine flowshop scheduling problems with uncertain setup times.

Keywords
Flowshop, total tardiness, uncertain setup times, algorithm

1. Introduction
Customer satisfaction in manufacturing systems is closely linked to the ability to deliver jobs on time. As a result, scheduling objectives that penalize late deliveries play a critical role in evaluating system performance. Among these objectives, total tardiness is widely used and has been adopted as a key managerial performance indicator in several industries, including automotive and semiconductor manufacturing (Seo et al., 2005).

A large portion of the scheduling literature that explicitly distinguishes setup times from processing times assumes that setup durations are known with certainty (see, for example, Aqil, 2024). In practice, however, this assumption may be unrealistic. In many production environments, setup operations are influenced by factors such as operator experience and crew proficiency, which can lead to substantial variability in setup durations. Empirical evidence and prior studies suggest that setup times are often subject to uncertainty, and that ignoring this uncertainty may result in schedules that perform poorly in real-world settings (Kim and Bobrowski, 1997; Ertem et al., 2019).

This study focuses on a two-machine flowshop scheduling problem in which setup times are not deterministic but instead lie within known lower and upper bounds. Early research on flowshop scheduling with bounded setup times was conducted by Allahverdi (2005), who derived several dominance properties for this class of problems. Subsequent work extended these results to other performance measures. In particular, Aydilek et al. (2013) developed solution procedures for minimizing makespan, while Allahverdi (2006a, 2006b) established dominance relations for maximum lateness and total completion time. Further extensions to three-machine flowshop environments were later provided in a series of studies by Allahverdi (2007, 2008, 2009).

More recently, the total tardiness criterion has received attention in the context of two-machine flowshops with setup considerations. Allahverdi (2023) examined this problem under the assumption of deterministic setup times and proposed several algorithms. Since minimizing total tardiness is already known to be NP-hard even in single-machine scheduling problems (Du et al., 1990), the introduction of a second machine and uncertain setup times preserves this computational complexity. To address this challenging setting, Allahverdi and Allahverdi (2025) investigated the two-machine flowshop problem with bounded setup times and total tardiness as the objective, proposing multiple algorithms and demonstrating that one approach consistently outperforms the others.

Building on this line of research, the present paper revisits the same problem and introduces a new algorithmic solution. Through extensive computational experiments, we demonstrate that the proposed algorithm significantly outperforms the best-performing method reported by Allahverdi and Allahverdi (2025), offering improved solution quality for minimizing total tardiness in two-machine flowshop environments with uncertain setup times.

2. Proposed Algorithm (PA)

This section describes the proposed algorithm for the two-machine flowshop scheduling problem with uncertain setup times and the objective of minimizing total tardiness. The algorithm constructs a job sequence by employing a priority index that integrates due dates, processing times, and bounded setup times on both machines. Uncertainty in setup durations is incorporated through their lower and upper bounds.

Step 1: Specify the lower and upper bounds of the setup times for each job on both machines.

Step 2: For each job, generate setup times on both machines such that the generated values lie within the bounds specified in Step 1.

Step 3: Generate the processing times for all jobs on both machines.

Step 4: Select a combination of the job tightness parameter T and the job range parameter R.

Step 5: For the selected value of T and R, compute a priority index for each job as follows

$I_j = d_j + 0.5LBs_{j1} + 0.5UBs_{j1} + UBs_{j2} + p_{j1} + p_{j2}$

where d_j denotes the due date of job j; LBs_{j1} and UBs_{j1} represent the lower and upper bounds of the setup time of job j on machine 1, respectively; UBs_{j2} denotes the lower bound of the setup time on machine 2; and p_{j1} and p_{j2} are the processing times on machines 1 and 2, respectively. Sequence the jobs in nondecreasing order of I_j.

Step 6: The resulting job sequence obtained in Step 5 is taken as the schedule generated by the proposed algorithm

3. Computational Experiments

A series of computational experiments was carried out to evaluate the effectiveness of the proposed algorithm (PA) in comparison with the best-performing algorithm reported by Allahverdi and Allahverdi (2025), hereafter referred to as algorithm AA. The objective of these experiments was to assess the relative performance of the two algorithms under a wide range of problem settings and parameter combinations.

For each test instance, processing times for all jobs on both machines were independently generated from a uniform distribution over the interval [1, 100]. Similarly, the upper bounds of the setup times on both machines were randomly generated using a uniform distribution with the same range [1, 100]. Given the upper bounds, the corresponding lower bounds of the setup times were generated from a uniform distribution defined over the interval [upper bounds on setup times – 50, upper bounds on setup times]. I Whenever this procedure produced a negative value, the lower bound was adjusted and set equal to 1 to ensure feasibility.

To examine the impact of due date characteristics on algorithm performance, three levels of the job tightness parameter T and the job range parameter R were considered, namely 0.25, 0.50, and 0.75. For each combination of T and R, job due dates were generated from the uniform distribution with the ranges of (lower setup time plus processing time)*(1-T-R/2) and (lower setup time plus processing time)*(1-T+R/2). This due-date generation mechanism allows for systematic variation in both the tightness and dispersion of due dates across the test instances.

To facilitate a normalized comparison between the two algorithms, a relative error measure was employed. The error associated with algorithm K, where K e {AA, PA}, was computed as

$$\text{Error (K)} = \frac{TT(K) - \min(\text{TT(AA)},\text{TT(PA)})}{(\max(\text{TT(AA)},\text{TT(PA)}) - \min(\text{TT(AA)},\text{TT(PA)}))} \times 100,$$

where TT*(*AA*)* denotes total tardiness of algorithm AA while TT(PA) denotes total tardiness of algorithm PA. Under this definition, an error value of 0 corresponds to the better-performing algorithm for a given instance, while an error value of 100 corresponds to the worse-performing one.

Setup times within their respective bounds were generated using a uniform distribution. This choice reflects the assumption that, in the absence of additional information, any setup duration within the specified interval is equally likely to occur. Problem sizes of 100, 200, 300, 400, and 500 jobs were examined in order to evaluate scalability. For each problem size, 50 independent replications were generated. In total, nine combinations of the parameters T and R were tested, resulting in a comprehensive experimental design.

The aggregate results of the computational experiments are reported in Table 1. The overall average error values obtained for algorithms AA and PA are 11.752 and 10.686, respectively. These results indicate that the proposed algorithm PA consistently outperforms algorithm AA across the tested scenarios. On average, PA achieves an improvement of approximately 10% in terms of the error measure when compared to the best-known algorithm. Based on these findings, the proposed algorithm PA is recommended for minimizing total tardiness in two-machine flowshop scheduling problems with uncertain setup times.

Table 1. Overall average errors

Algorithm	n=100	n=200	n=300	n=400	n=500	Average
AA	12.12	11.81	11.59	11.66	11.58	11.752
PA	10.86	10.83	10.49	10.66	10.59	10.686

4. Conclusion

This study examined the two-machine flowshop scheduling problem with the objective of minimizing total tardiness in an environment where setup times are explicitly distinguished from processing times. Unlike many classical models, setup times were not assumed to be deterministic; instead, they were treated as uncertain parameters that may take any value within known lower and upper bounds. This modeling approach more accurately reflects practical manufacturing settings in which setup durations are influenced by factors such as operator skills and operational variability.

The considered problem has been previously investigated in the literature, and earlier studies identified algorithm AA as the most effective solution method among existing approaches. Building upon these results, this paper introduced a new heuristic algorithm, referred to as PA, which incorporates information on bounded setup times and processing requirements into its sequencing rule. The performance of the proposed algorithm was evaluated through extensive computational experiments and directly compared with that of algorithm AA.

The numerical results demonstrate that the proposed algorithm PA consistently outperforms the best-known existing algorithm. On average, PA reduces the relative error by approximately 10% when compared to algorithm AA across a wide range of problem sizes and parameter settings. These findings indicate that explicitly accounting for setup-time uncertainty within the scheduling rule can lead to meaningful improvements in solution quality.

In conclusion, the proposed algorithm PA represents an effective and computationally efficient approach for minimizing total tardiness in two-machine flowshop environments with uncertain setup times. Given its superior performance relative to existing methods, PA is recommended for use in practice and as a benchmark for future research. Further extensions of this work may consider additional machines, alternative uncertainty representations, or other performance measures.

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