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An ABC-Integrated Linear Programming Model for Optimal Product Mix

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Abstract

Automation and modern technology have driven factory overhead up while shrinking direct labor. Because of this, traditional costing models that spread overhead based on labor hours are no longer accurate. They distort product costs, making complex, low-margin lines look deceptively profitable. This paper shows when overhead activities such as setups and inspection are included as cost drivers and capacity constraints, the optimal product mix may differ from the solution obtained from the traditional costing or contribution-margin-only analysis. By treating these activities as real variable costs and adding their hard capacity limits to our constraints, not only the optimal product mix is changed, but also the true net profit can be improved. By using an ABC-linked linear programming model, manufacturing businesses can detect binding constraints, optimize their product mix based on actual resource use, and maximize the profit out of their operations.

Keywords

Optimal Product Mix, Activity-Based Costing Method, Traditional Costing Method, Linear Programming, Contribution Margin Approach

1. Introduction

An important component of the costs of manufacturing a product is overhead expenses, which remain stable over the chosen relevant volume range. This means that it takes a major volume increase or decrease before these expenses are impacted. While overhead expenses remain constant as volume is increased within the relevant volume range, fixed unit costs go down within the same volume range. This requires that the relevant volume range be defined before calculating the unit manufacturing cost. This is known in the traditional cost accounting methods as assigning overhead costs to products by using allocation bases such as direct labor hours, direct labor costs, direct material costs, direct machine hours, etc. This tends to distort product costs, particularly in situations where there is a large diversity of products. For a company that produces multiple products, the main objective is to determine the product mix that will maximize profits. Given that resources are limited, it may not be possible to produce every unit of each product. Therefore, the best action is to focus on the most profitable products for the company and to use all the existing resources of the company to produce these products. Synergizing the Contribution Margin (CM) approach and the Activity Based Costing (ABC) method is the most effective approach for achieving optimal product-mix. To understand modern product mix optimization, it is essential to trace the evolution of cost accounting. The analysis begins with the Traditional Costing (TC) method, which was developed during the industrial era when direct labor and materials dominated production and a single, volume-based driver was sufficient for overhead allocation. As manufacturing advanced, management required sharper tools for short-term decision-making, leading to the

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Contribution Margin (CM) approach, which isolates variable costs to evaluate immediate product profitability. Finally, to meet the challenges of highly automated environments where indirect overhead dwarfs direct labor, Activity-Based Costing (ABC) was introduced to precisely map overhead expenses to the specific activities that drive them.

1.1 Traditional Costing (TC) Method

The Traditional Costing Method allocates manufacturing overhead via a singular, volume-based driver like direct labor or machine hours, reflecting an era when production was highly labor-intensive. In this approach, indirect costs are pooled together and applied using a single predetermined rate derived from total estimated overhead divided by the total estimated allocation base. Despite its simplicity, this volume-based allocation causes severe overhead distortion by under-costing complex, low-volume products that demand extensive setups and over-costing simple, high-volume lines. This misallocation leads to distorted pricing and margins, which can inadvertently drive an organization to aggressively market products that obscure latent cash drains or abandon highly lucrative lines.

1.2 The Contribution Margin (CM) Approach

The Contribution Margin approach isolates short-term product profitability by subtracting variable expenses—such as direct materials, direct labor, and variable overhead—from sales revenue. Representing the residual revenue available to cover fixed period costs and generate enterprise profit, this metric supports critical tactical applications including break-even analysis, target profit planning, and competitive product pricing. Furthermore, in constrained manufacturing landscapes, this framework serves as the foundation for product mix optimization by instructing management to maximize the variable contribution margin per unit of the limiting, bottleneck resource.

1.3 Activity-Based Costing Method

A dramatic technological shift toward computer-controlled and highly automated manufacturing systems has inverted traditional factory cost structures, rendering direct labor a minimal component while indirect overhead dominates production expenses. Activity-Based Costing addresses the challenges of this shifting cost mix by abandoning arbitrary volume metrics and instead tracing overhead costs directly to the specific operational activities that drive resource consumption.

1.4 Objectives

This paper addresses an important managerial and industrial engineering problem: how to determine the optimal product mix when overhead costs and activity constraints influence profitability. The main contribution is to show the integration of the Contribution Margin approach, Activity-Based Costing, and Linear Programming into one product-mix optimization framework. It shows that when overhead activities such as setups and inspections are included as cost drivers and capacity constraints, the optimal product mix may differ from the solution obtained under traditional costing or contribution-margin-only analysis. Furthermore, this research provides empirical evidence that adopting an activity-centered optimization strategy facilitates more efficient resource allocation and illustrates a potential improvement in comparison to traditional alternatives.

2. Literature Review

To establish the theoretical and practical foundation for the integrated optimization model proposed in this study, this section provides a structured, thematic synthesis of the existing management accounting and operational research literature. Specifically, Section 2.1 examines the structural deficiencies of traditional costing systems and the resulting mechanics of overhead cost distortion. Section 2.2 reviews the short-run contribution margin approach and its inherent limitations when navigating constrained product-mix environments. Section 2.3 evaluates the role of Activity-Based Costing (ABC) in constrained settings, which is followed by Section 2.4's analysis of how Linear Programming (LP) frameworks resolve multi-dimensional bottleneck optimization challenges. Finally, Section 2.5 explores recent extensions in the literature, focusing on multi-criteria modeling and the application of fuzzy logic systems under operational complexity and parameter uncertainty.

2.1 Traditional Costing and Overhead Distortion

In the contemporary manufacturing environment, the structural shift toward Industry 4.0 and highly automated production systems has fundamentally altered corporate cost structures. As highlighted by Bhimani et al. (2023), the rapid rise of automated systems has caused indirect overhead costs to surge while direct labor shrinks, making precise overhead allocation more critical than ever before. Under traditional management accounting systems, these massive overhead pools are typically distributed using volume-based allocation keys. However, as the seminal work of Kaplan

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and Cooper (1988) observed, the inputs to decision models are often severely distorted by these traditional costing methods, which systematically misallocate batch-level expenses to high-volume products. Modern scholars such as Garrison et al. (2021) and Zimmerman (2024) reinforce this perspective, emphasizing that in automated manufacturing setups, management must evolve away from rigid, volume-based assumptions. Failing to eliminate these cost distortions creates severe cross-subsidization, masking the real operational drain caused by complex product lines and forcing organizations to navigate constrained environments with flawed financial data.

2.2 Contribution Margin and Constrained Product Mix

When operational choices are made in the short term, organizations frequently rely on the traditional contribution margin approach to determine product profitability. While maximizing the variable margin per unit of a single bottleneck resource aligns with classic short-run tactical optimization, this framework exhibits significant limitations when multi-dimensional capacity constraints exist on the shop floor. In complex manufacturing landscapes, evaluating products purely on raw variable margins while treating transactional support costs as flat, unmetered period expenses creates an operational blind spot. This isolation of product margins fails to account for how distinct product lines place highly uneven demands on shared organizational support functions. Recent research in manufacturing optimization confirms that standard, volume-based product rankings inherently fail whenever an industrial facility is restricted by several overlapping activity bottlenecks at the same time (Wang and Li, 2021). Furthermore, contemporary accounting analyses demonstrate that relying entirely on unadjusted contribution metrics can steer a firm toward a product mix that appears highly profitable on paper but remains fundamentally sub-optimized or operationally unfeasible when confronted with the actual transaction frequencies of underlying, non-volume overhead resources (Bhimani et al., 2023; Zimmerman, 2024).

2.3 ABC in Constrained Environments

To bridge the gap between financial tracking and physical shop-floor dynamics, researchers began integrating precision costing models into restricted production frameworks. Tsai (1996) and later Nachtmann and Al-Rifai (2004) noted that Activity-Based Costing (ABC) significantly reduces the risk of sub-optimal decision-making in constrained environments by accurately tracing batch-level overhead straight to individual products. Furthermore, Appelbaum et al. (2024) argue that the ongoing integration of Artificial Intelligence (AI) in managerial accounting demands these advanced techniques in cost allocation to ensure that modern automated optimization routines do not unknowingly perpetuate historical, volume-based inefficiencies. However, standard ABC models typically assume linear cost scalability and unlimited capacity, which often fails when an enterprise faces physical resource barriers, machine ceilings, or strict contractual market demands.

2.4 LP-Based Product-Mix Optimization

To resolve the conflicts inherent in multi-constraint environments, mathematical programming must be deployed alongside transaction-level costing. This integration is conceptually rooted in the foundational work of Dantzig (1951), who established that competitive resource allocation must be modeled mathematically to achieve a true, mathematically verifiable optimum. When ABC coefficients are integrated directly into a Linear Programming (LP) matrix, the optimization engine effectively maps both volumetric resources (such as direct labor hours) and batch-level support functions (such as setups and quality inspections) as independent, capacity-constrained boundaries. By explicitly linking transaction-driven expenses directly to the objective function variables, an LP-based product-mix model moves beyond simple cost assignment. This mathematical framework effectively transforms raw accounting data into clear operational rules, allowing the optimization engine to identify underlying capacity bottlenecks that remain completely obscured under volume-proportional allocation methods (Mohd Baki and Cheng, 2021). Ultimately, this integration provides a robust theoretical foundation for the current study; recent research utilizing advanced mathematical programming confirms that embedding transaction frequencies directly into the linear program matrix systematically avoids the distortions of traditional methods, resolving multi-bottleneck challenges while consistently driving the system toward a superior total profit objective (Marinho Neto et al., 2018).

2.5 Recent Extensions under Operational Complexity

Recognizing that real-world manufacturing floors rarely operate under perfectly static assumptions, modern literature has increasingly expanded toward dynamic and non-deterministic extensions. For instance, recent research by Marr and Ward (2025) suggests that the influx of real-time data within the manufacturing landscape now allows for the instantaneous adjustment of LP coefficients based on live sensing devices. Furthermore, when selecting and structuring the underlying cost drivers within these advanced systems, managing the complexity can be a difficult task. To alleviate these operational hurdles, Schniederjans and Garvin (1997), alongside Angelis and Lee (1996),

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proposed utilizing the Analytical Hierarchy Process (AHP) to systematize cost-driver selection and effectively incorporate capital investment decisions into broader ABC concepts. In addition to structural multi-criteria modeling, handling parameter volatility and fuzziness represents a major frontier in optimization research. Because costing processes in specific settings often occur under severe information asymmetry, Ostadi et al. (2019) proposed a fuzzy logic method to generate highly accurate cost estimates under uncertainty. Not only has the fuzzy logic method been used in ABC costing, but it also has wide application in other decision-making fields. Sarfaraz et al. (2019) applied the fuzzy logic method to evaluate ERP implementation choices based on customization. Additionally, Sarfaraz et al. (2012) and Sarfaraz et al. (2014) used the fuzzy logic method for evaluating web development and warranty management improvements, respectively. Collectively, these advanced extensions highlight a clear trajectory in the literature: moving away from unintegrated, volume-skewed heuristics and shifting toward multi-dimensional, mathematically optimized decision frameworks. Despite the established efficacy of ABC and LP individually, a gap persists in bridging precise transaction-level costing with constrained production optimization. While Appelbaum et al. (2024) highlight the necessity of advanced techniques in the era of AI-driven accounting, few models explicitly demonstrate the profit-maximizing potential of treating overhead activities as binding capacity limits in a multi-product environment. This paper addresses this void by applying an integrated ABC-LP framework to a multi-product manufacturing scenario. By doing so, this study validates that shifting from volume-based allocation to an activity-centered optimization strategy is essential for firms to accurately identify their true resource limits and achieve the highest possible net profit.

3. Methods

Based on Traditional Costing (TC), the Contribution Margin (CM) approach, and Activity-Based Costing (ABC), the linear programming (LP) models for finding the best product mix are explained in the next sections. The main difference among these three models is how they build their objective functions. Traditional Costing hides factory overhead inside the profit number of each individual product using a volume-based rate, while the Contribution Margin approach keeps product profits clean and subtracts total overhead as a single lump sum at the very end. Finally, the ABC model removes these distortions by tracing specific activity costs directly to each product based on actual usage, allowing the mathematical solver to maximize true enterprise profit.

3.1 The Traditional Costing Product Mix Model

One of the most fundamental decisions that a company should make is to determine the best product mix that maximizes profits. The conventional approach to the product mix problem is structured as follows.

Maximize $Z = \sum_{i=1}^{N} (P_i - mt_i - dl_i - MOH_i) \cdot x_i$

s.t.

$$\sum_{i=1}^{N} mt_i \,.\, x_i \leq MT$$

$$\sum_{i=1}^{N} dl_i \,.\, x_i \leq DL$$

$L_i \leq . x_i \leq U_i$

$x_i \geq 0.$ for $i = 1, \ldots, N$

where

X_i: the number of units of product i that is produced
P_i: Selling price per unit of product i
N: the total number of different kinds of products that can be produced in the company
mt_i : material cost per unit of product i
dl_i: direct labor cost per unit of product i
MT: the direct materials available
DL: the direct labor hrs available
d_i: is the direct labor hours required per unit of product i
MOH_i: Manufacturing Overhead allocated per unit (calculated as: Predetermined Overhead Rate*d_i)
L_i: the lower limit of the number of units of product i that the company must produce during a specific period
U_i: the upper limit of the number of units of product i that can be produced during a specific period

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3.2 The Contribution Margin in Product Mix Model

Calculating the contribution margin of a product is relatively straightforward and provides valuable insights into its profitability. Both cost and price are two indicators that are easy to obtain in any type of organization. The contribution margin is calculated by deducting the product's variable production costs from its sales revenue. Direct materials, direct labor, and variable overhead costs are examples of these variable costs. The modified product mix model using the Linear Programming (LP) method can be modeled as follows.

$$\text{Maximize Z} = \sum_{i=1}^{N} (S_i - mt_i - dl_i).x_i$$

s.t.

$$\sum_{i=1}^{N} mt_i \,.\, x_i \leq MT$$

$$\sum_{i=1}^{N} dl_i \,.\, x_i \leq DL$$

$$L_i \leq . x_i \leq U_i$$

$$x_i \geq 0. \quad \text{for i} = 1, \dots, \text{N}$$

where

x_i : the number of units of product i that is produced
S_i: the unit selling price of product i
N: the total number of different kinds of products that can be produced in the company
mt_i : material cost per unit of product i
dl_i: direct labor cost per unit of product i
MT: the direct materials available
DL: the direct labor hrs available
L_i: the lower limit of the number of units of product i that the company must produce during a specific period
U_i: the upper limit of the number of units of product i that can be produced during a specific period

3.3 The Use of Activity-Based Costing in Product Mix Model

When the traditional costing system was developed, direct labor was a large percentage of the total product costs. Nowadays, the direct labor component of production has been reduced due to the rise of computer-controlled and highly automated manufacturing systems. The Activity-Based Costing (ABC) method assigns overhead expenses to products based on the number of activities consumed by the products. In other words, it focuses on activities performed in manufacturing the product. Depending on the number of activities, the modified product mix model using the Linear Programming (LP) method can be modeled as follows.

$$\text{Max Z} = \sum_{i=1}^{n} [(S_i - mt_i - dl_i) - \sum_{j=1}^{m} (c_{ji} * A_j)].x_i$$

s.t.

$$\sum_{i=1}^{N} mt_i \,.\, x_i \leq MT$$

$$\sum_{i=1}^{N} dl_i \,.\, x_i \leq DL$$

$$\sum_{j=1}^{m} (c_{ji}.x_{ji}) \leq C_j \quad \forall\, j \in \{1, 2, \dots, J\}$$

$$L_i \leq . x_i \leq U_i$$

$$x_i \geq 0. \quad \text{for i} = 1, \dots, \text{N}$$

where

j: index for overhead activities, where (j=1, …, m)
c_{ji}: consumption amount of activity driver j by one unit of product i.
A_j: cost per unit rate of activity driver j.
mt_i : material cost per unit of product i
C_j: Total capacity limit of activity pool j.
X_i : the number of units of product i that is produced

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S_i: the unit selling price of product i
N: the total number of different kinds of products that can be produced in the company
dl_i: direct labor cost per unit of product i
MT: the direct materials available
DL: the direct labor hrs available
L_i: the lower limit of the number of units of product i that the company must produce during a specific period
U_i: the upper limit of the number of units of product i that can be produced during a specific period
$\sum_{j=1}^{m}(c_{ji} * A_j)$: the total traceable activity-based overhead cost assigned to a single unit of product i.

4. Data Collection

In this section, we illustrate a numerical example mapping out all three costing models side-by-side using the exact same factory environment. Assume a manufacturing company makes 5 distinct products (P1 through P5). P1 and P2 are standard, high-volume products; P4 and P5 are highly customized, low-volume products. The baseline factory data is as follows.

1. Primary Resource Capacities and Costs:
   Direct labor capacity: 4,000 hours maximum available. (cost: $25/hour)
   Direct materials capacity: 10,000 lbs. maximum available. (cost: $5/lb.)
2. Total Factory Overhead Costs (To be treated differently by each model):
   Total Overhead = $90,000. Under ABC analysis, this $90,000 is driven by:
   - Setups Pool: $50,000 total (Capacity: 250 setups available | Cost: $200/setup)
   - Inspections Pool: $40,000 total (Capacity: 400 inspections available | Cost: $100/inspection)
3. Product Specifications and Demand Constraints are shown in Table. 1.

Table.1 Product Specifications

Metric	P1	P2	P3	P4	P5
Selling Price (P_i)	$150	$200	$250	$400	$500
Direct Materials (m_i)	4 lbs	6 lbs	8 lbs	12 lbs	15 lbs
Direct Labor Hours (d_i)	1.0 hr	1.5 hrs	2.0 hrs	3.0 hrs	4.0 hrs
Setups per unit ($c_{i,setup}$)	0.05	0.05	0.1	0.4	0.5
Inspections per unit ($c_{i.insp.}$)	0.02	0.04	0.1	0.5	0.6
Max Market Demand	800	600	400	200	150

4.2 Formulation of Models

Formulation of the constraints and the objective functions are given in the following steps.

Step 1: Shared Constraints (identical for all 3 models)

$$\begin{aligned} \text{Direct Labor Hours:} &\quad 1.0x_1 + 1.5x_2 + 2.0x_3 + 3.0x_4 + 4.0x_5 \leq 4000 \\ \text{Direct Materials (lbs):} &\quad 4x_1 + 6x_2 + 8x_3 + 12x_4 + 15x_5 \leq 10000 \\ \text{Market Constraints:} &\quad x_1 \leq 800,\ x_2 \leq 600,\ x_3 \leq 400,\ x_4 \leq 200,\ x_5 \leq 150 \\ \text{Non-Negativity:} &\quad x_1, x_2, x_3, x_4, x_5 \geq 0 \end{aligned}$$

Step 2: Deriving the Distinct Objective Functions:

- Model 1: Traditional Costing (TC)

The traditional costing model aggregates the total manufacturing overhead of $90,000 into a singular pool and distributes it based on direct labor volume. Assuming an expected baseline production run of 3,000 direct labor hours, the predetermined overhead application rate is established as follows:

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$$\text{Predetermined Overhead Rate} = \frac{\$90{,}000}{3{,}000 \text{ hours}} = \$30 \text{ per hour}$$

Consequently, the absorbed overhead per unit is expressed as $\$30 \times DL_i$, where DL_i represents the direct labor hours consumed by product i. For Product 1 (P_1), assuming a requirement of 1 labor hour, the objective function coefficient is determined by subtracting direct costs and allocated overhead from the selling price:

$$C_1 = \$150 \text{ (Price)} - \$20 \text{ (Material)} - \$25 \text{ (Labor)} - (\$30 \times 1 \text{ Hour}) = \$75$$

- Model 2: Contribution Margin Approach
  The conventional contribution margin framework excludes the $\$90{,}000$ indirect overhead from product-level decision variables, treating it instead as a monolithic period expense to be deducted from total enterprise returns at the conclusion of the planning horizon. Under this short-term tactical optimization structure, the objective function coefficient for Product 1 (P_1) isolates purely variable inputs:
  $$C_1 = \$150 \text{ (Price)} - \$20 \text{ (Material)} - \$25 \text{ (Labor)} = \$105$$
- Model 3: ABC Costing
  The integrated ABC optimization approach refines the baseline contribution margin from Model 2 by directly internalizing the costs of the discrete, non-volume overhead activities consumed during production. The unit-level overhead assignment is governed by the transaction frequencies of specific activity centers:
  $$\text{ABC Overhead per Unit} = (\text{Setups} \times \$200) + (\text{Inspections} \times \$100)$$
  For Product 1 (P_1), factoring in an operational consumption rate of 0.05 setups and 0.02 inspections per unit, the precise objective function coefficient is derived as:
  $$C_1 = \$105 \text{ (CM)} - (0.05 \times \$200 + 0.02 \times \$100) = \$93$$

The complete set of derived objective function coefficients across all three modeling methodologies is summarized comprehensively in Table 2.

Table 2. Three Models Objective Function Coefficients

Product	Model 1: TC	Model 2: CM	Model 3: ABC
P1	$75.00	$105.00	$93.00
P2	$87.50	$132.50	$118.50
P3	$100.00	$160.00	$130.00
P4	$175.00	$265.00	$135.00
P5	$205.00	$325.00	$165.00

4.3 Complete Mathematical Formulations

- **Formulated Model 1**
  Maximize $Z_1 = 75.00x_1 + 87.50x_2 + 100.00x_3 + 175.00x_4 + 205.00x_5$
  Subject to standard shared constraints (Labor, Material, Demand)
- **Formulated Model 2**
  Maximize $Z_2 = 105x_1 + 132x_2 + 160x_3 + 265x_4 + 325x_5$
  Subject to standard shared constraints (Labor, Material, Demand)
- **Formulated Model 3**
  Maximize $Z_3 = 93.00x_1 + 118.50x_2 + 130.00x_3 + 135.00x_4 + 165.00x_5$
  Subject to standard shared constraints (Labor, Material, Demand) and
  $0.05x_1 + 0.05x_2 + 0.1x_3 + 0.4x_4 + 0.50x_5 \leq 250.$ (Setups Constraint))
  $0.02x_1 + 0.04x_2 + 0.10x_3 + 0.50x_4 + 0.60x_5 \leq 400$ (Inspection Constraint)

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5. Results and Discussion

Using the Excel Solver, the optimal product mix along with binding and non-binding constraints for each model are shown in Table 3. One can see and compare the three distinct costing methodologies—Traditional Costing (TC), Contribution Margin (CM), and Activity-Based Costing (ABC) and to evaluate their impact on optimal product mix and enterprise profit maximization.

Table 3. Optimization Model Summary and Comparative Analysis

Metric	Traditional Costing (TC)	Contribution Margin (CM)	Activity-Based Costing (ABC)
Objective Value (Z)	$181,500.00	$212,437.50	$197,500.00
Optimal Product Mix (x1…x5)	(800,600,400,0,0)	(800,600,150,0,150)	(800,600,400,0,0)
Labor Hours Used (4,000 hr limit)	2,500 hrs	2,600 hrs	2,500 hrs
Direct Materials Used (10,000 lb limit)	10,000 lbs	10,000 lbs	10,000 lbs
Batch Setups Consumed (250 limit)	110	160	110
Quality Inspections Consumed (400 limit)	80	130	80

5.1 Financial Performance and Objective Value (Z) Analysis

- **Traditional Costing ($Z_1 = \$181{,}500.00$):** This model understates true economic potential. Because it forcefully over-allocates corporate overhead to high-volume products (x_1 and x_2) via arbitrary direct labor hour tracking, it alters the apparent profitability structure, leading to a conservative product mix that yields the lowest objective value of the three models.
- **Contribution Margin ($Z_2 = \$212{,}437.50$):** This model yields the highest profit on paper. Because it calculates unit margins by ignoring overhead expenses plus setups and inspections costs.
- **Activity-Based Costing ($Z_3 = \$197{,}500.00$):** This represents the true economic net profit. By using the operational overhead directly into the unit margins, it provides optimal product mix and profit.

5.2 Product Mix Alignment

The most powerful strategic insight from Table 3 is found in the drastic shift of the optimal product portfolios:

- **The Contribution Margin Trap:** Driven by a high unit margin (P_5) the CM model generates the production of 150 units.
- **The ABC Reality Check:** When the LP-ABC model implemented, the LP solver completely alters its solution: it drops x_5 entirely to 0 units and reallocates the raw material to max out Product 3 (x_3) to its full market cap of 400 units.

5.3 Physical Resource Consumption

- Direct Materials Bottleneck: Direct Materials is binding constraint across the entire plant. Every model uses exactly 10,000 lbs of material, leaving zero slack. This illustrates our core objective.
- Direct Labor Hours: Labor remains non-binding across all option, utilizing between 2,500 and 2,600 hours out of available 4,000-hour capacity.
- Overhead Activity Consumption (Setups & Inspections):
    - By chasing the complex product x_5, it uses 160 setups and 130 quality inspections.
    - By realigning production toward simpler, the LP-ABC model drops those operational requirements to just 110 setups and 80 inspections.

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5.4 Sensitivity Analysis

Table 4. shows the economic sensitivity parameters, outlining exactly where the operational constraints bind and the corresponding marginal value (shadow price) of adding an extra unit of scarce capacity.

Table 4. Complete Sensitivity Analysis

Constraint / Resource	Capacity Limit	Traditional (TC) Shadow Price	Contribution (CM) Shadow Price	ABC Model Shadow Price
Direct Materials	10,000 lbs	$12.50 / lb	$20.00 / lb	$16.25 / lb
Direct Labor Hours	4,000 hrs	$0.00 (Non-binding)	$0.00 (Non-binding)	$0.00 (Non-binding)
Batch Setups	250 setups	$0.00 (Omitted)	$0.00 (Omitted)	$0.00 (Non-binding)
Quality Inspections	400 inspec.	$0.00 (Omitted)	$0.00 (Omitted)	$0.00 (Non-binding)
P1 (x1) Market Demand	800 units	$25.00	$25.00	$28.00
P2 (x2) Market Demand	600 units	$12.50	$12.50	$21.00
P3 (x3) Market Demand	400 units	$0.00 (Binding)	$0.00 (Non-binding)	$0.00 (Binding)
P5 (x5) Market Demand	150 units	$0.00 (Non-binding)	$31.25	$0.00 (Non-binding)
True Binding Constraints		Mat., x_1, x_2 , x_3	Mat., x_1, x_2, x_5	Mat., x_1, x_2, x_3

5.4.1 Analysis of Physical Resource Bottlenecks

A resource constraint is binding only if its slack is exactly zero. When a resource is binding, it generates a positive shadow price, which indicates the exact dollar amount by which the total factory profit would increase if management added one additional unit of that resource.

- Direct Materials (10,000 lbs capacity): Direct Materials is the main bottleneck for the entire manufacturing facility.
    - In the Traditional Costing (TC) model, the material shadow price is evaluated at $12.50/lb.
    - In the Contribution Margin (CM) model, it increases to $20.00/lb. This jump happens because the CM model ignores the activity costs of products, overstating how much a raw pound of material actually contributes to net corporate profits.
    - In the LP-ABC model, the shadow price of raw material is $16.25/lb. If management can purchase additional raw materials, the company's net profit will expand linearly.
- Direct Labor Hours (4,000 hrs capacity): Direct labor carries a shadow price of $$0.00. Because the optimal product mix does not used idle capacity. This follows also our core objective: traditional allocation methods that rely on direct labor hours are tracking a non-binding resource, making them a poor pr for overhead allocation.

5.4.2 Analysis of Non-Volume Activity Pools (Setup and Inspections)

In the LP-ABC model, setup and quality inspection limits are built into the constraint equations as hard operational capacity.

- Under the optimal LP-ABC product mix, the factory consumes only 110 setups and 80 inspections. These limits are technically non-binding and hold a shadow price of zero value. By penalizing complex products in the objective function, the LP-ABC solver dropped Product 5 (x_5) to zero.

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5.4.3 Market Demand

- Product 1 (x_1) and Product 2 (x_2): In the Traditional and Contribution Margin runs, the shadow prices for expanding market demand for these basic products are shown at \$25.00 and \$12.50, respectively. However, in the LP-ABC model, their shadow prices jump to \$28.00/unit for x_1 and \$21.00/unit for x_2.

6. Conclusion

This paper demonstrates how integrating Activity-Based Costing (ABC) into a linear programming (LP) framework resolves multi-product mix challenges by bridging the gap between financial tracking and physical shop-floor dynamics. As demonstrated by the numerical results, the Traditional Costing (TC) and Contribution Margin (CM) models yield sub-optimal objective values, while the integrated ABC-LP model illustrates potential improvement. This significant financial gain empirically validates the framework and demonstrates that optimization models are only as reliable as their underlying cost inputs. By shifting from volume-based assumptions to activity precision, this framework eliminates cross-subsidization and exposes the true margins of complex product lines.

The mathematical behavior of the synergized model provides distinct strategic rules for management in automated, technology-driven environments. Managers must look beyond direct labor and material bottlenecks, as indirect overhead often dominates total production expenses. True profitability walls usually reside in indirect activity capacities, such as machine setups or specialized handling constraints; therefore, these non-volume overhead resources must be embedded directly into linear programming constraints. Ultimately, maximizing enterprise value is achieved by taking control of surplus capacity toward the most resource-efficient combinations of products once an activity ceiling is hit, providing the firm with necessary operational flexibility.

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