

Optimal Energy and Pump Scheduling in Water Distribution Systems Using Mixed-Integer Linear Programming

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Abstract

The efficient operation of water distribution systems is increasingly important due to rising energy demand and the growing integration of renewable energy sources. This study presents a Mixed-Integer Linear Programming (MILP) model for the optimal scheduling of pumps and energy sources in a water pumping system over a 24-hour horizon. The model integrates pump operation, storage tank dynamics and multiple energy sources, including grid electricity, photovoltaic (PV) generation and a diesel generator, with the objective of minimizing total operating cost while satisfying system constraints. The formulation incorporates key operational features such as pump capacity limits, binary on/off decisions, storage balance and energy supply-demand matching. A linear approximation of pump power is adopted to ensure computational tractability within the MILP framework. The model is implemented in Python using the PuLP library and solved using the CBC solver. Results show that the optimal schedule prioritizes renewable energy usage, with PV generation supplying most of the system demand during daylight hours. Grid electricity is used only when necessary while diesel generation is avoided due to its higher cost and operational constraints. The storage tank plays a critical role in enabling load shifting, allowing pumping during low-cost periods and reducing reliance on expensive energy sources. Sensitivity analysis further confirms that the model responds consistently to changes in diesel cost parameters demonstrating economic robustness. Overall, the proposed approach provides a practical and cost-effective framework for integrating water and energy system optimization, improving operational efficiency and supporting sustainable water management.

Keywords

Pump scheduling, MILP, Energy optimization, Water systems and Renewable energy.

1. Introduction

Water distribution systems consume a large amount of energy mainly due to pumping operations. In many cases, small energy losses occur continuously within the system and although each loss may seem insignificant their cumulative effect over time leads to an increase in operational costs and inefficiency.

This research is motivated by the need to reduce these energy losses and improve the overall efficiency of water distribution systems. Every unit of wasted energy represents unnecessary cost that could otherwise be invested in improving system performance and service quality for the community. In addition, reducing energy waste contributes to long-term sustainability and helps ensure that energy resources remain available for future generations.

The main problem addressed in this study is how to operate water pumps and energy sources in an optimal way that minimizes total energy cost while still meeting water demand requirements. In real systems, energy can be supplied from multiple sources such as grid electricity, renewable energy and diesel generators, each with different costs and constraints. Managing these sources efficiently together with pump operation is a complex decision-making problem.

Therefore, this study proposes a Mixed-Integer Linear Programming (MILP) optimization model to determine the best operational strategy for both pumping and energy usage. The model integrates pump scheduling with multi-source energy management enabling cost reduction, improved utilization of renewable energy and more efficient system operation.

1.1 Objectives

The main objective of this study is to develop and evaluate an optimization model for the efficient operation of a water distribution pumping system under multiple energy sources.

The specific objectives are:

- To formulate a Mixed-Integer Linear Programming (MILP) model that determines the optimal scheduling of pumps and energy sources over a 24-hour time horizon.
- To minimize the total operating cost of the system by optimally allocating energy from grid electricity, photovoltaic (PV) generation and a diesel generator.
- To incorporate storage tank dynamics into the model to enable load shifting and decouple pumping operations from immediate water demand.
- To model pump operation realistically by including capacity limits on/off decisions and a linearized relationship between flow and power.
- To analyze the impact of renewable energy availability on system performance and evaluate how PV generation influences pump scheduling decisions.
- To validate the model by assessing economic behavior and sensitivity to key parameters such as diesel cost.

2. Literature Review

2.1 Introduction

Efficient operation of water distribution systems (WDS) has become an increasingly important research topic due to the strong interdependence between water supply and energy consumption. Pumping operations, which represent the largest share of energy use in these systems, have attracted significant attention in recent years, particularly in the context of rising energy costs, dynamic electricity tariffs and the growing integration of renewable energy sources. As a result, a wide range of studies have focused on improving pump scheduling strategies enhancing system flexibility through storage and developing advanced optimization models.

2.2 Energy Consumption and the Need for Optimization in Water Systems

Forecasts for 2050 indicate that global water demand is expected to increase by 55% which will lead to a significant rise in energy consumption in water supply systems (WSS). Since pumping stations account for nearly 70% of the total energy usage in these systems optimizing their operation becomes a critical priority. In earlier studies, pump operation was often based on simple rules such as turning pumps on or off depending on tank levels. However, these approaches do not consider changes in electricity prices or the availability of energy sources. More recent research uses optimization models to determine the best operating strategy allowing systems to take advantage of lower energy costs storage capacity and more flexible pump operation. (Mala-Jetmarova et al. 2024).

2.3 Role of Storage Tanks and System Flexibility

Storage tanks play an important role in improving system efficiency because they allow pumping operations to be separated from immediate water demand. By storing water during low-cost periods and using it later the system gains flexibility in scheduling pump operation. This makes it possible to reduce energy costs and better utilize available energy sources. This tank-based flexibility is usually represented through a storage balance of the form:

$$S_t = S_{t-1} + \text{inflow}_t - \text{outflow}_t$$

with lower and upper bounds on storage. In scheduling papers, this equation is important because it connects decisions across hours and makes load shifting possible. This directly supports the improvement of modeling the pumps as filling a storage tank and the demand as being served from that tank rather than forcing hourly pumping to equal hourly demand. (Bonvin et al. 2019).

2.4 Optimization Methods and MILP Formulations

Another major theme is the choice of optimization methods. Mixed-Integer Linear Programming (MILP) is especially attractive because it can combine continuous decisions such as flow and power levels with binary decisions such as equipment on/off status. Papers show that the original WDS operation problem is nonlinear and nonconvex, and they propose a linear relaxation that reconstructs it as a MILP for improved solvability. The formulation choice strongly affects computational efficiency and cost performance. Across many of these studies the daily cost objective has the general form:

$$\min \sum_t c_t P_t \Delta t$$

where c_t is the time-varying electricity tariff and P_t is the pumping power. This type of objective appears repeatedly in pump scheduling under dynamic tariffs. (Zhang et al. 2020).

2.5 Pump Modeling and Linearization Approaches

A key modeling difficulty is that pump power is nonlinear. In physical terms pump power depends on flow, head, and efficiency expressed as:

$$P = \frac{\rho g Q H}{\eta}$$

or through pump curve relationships for fixed- and variable-speed operation. The nonlinear bivariate pump-head relationship is often linearized to obtain a MILP formulation. This supports the use of a simplified linear relation $P_{p,t} = \beta_p Q_{p,t}$ where β_p is a linearized approximation of more detailed pump physics. (Zhang et al. 2020).

2.6 Advanced Pump Control and Variable-Speed Systems

Variable-speed pumps (VSPs) and flexible control strategies are widely used in recent studies because they improve both operational efficiency and scheduling flexibility. These systems allow pumps to operate at different speeds rather than simple ON/OFF states which enables better adaptation to varying system conditions. In addition, VSPs can support demand-response strategies by adjusting energy consumption based on electricity availability and price. (Garcia et al. 2017).

2.7 Integration of Water and Energy Systems

Another important research direction is the integration of water and energy systems. In these approaches, water distribution and electricity systems are optimized together rather than separately. This allows better coordination between pump operation and energy availability leading to significant cost reductions. These studies highlight the importance of considering energy decisions alongside hydraulic operation. (Al-Najjar et al. 2024).

2.8 Renewable Energy Integration in Pump Scheduling

Renewable energy integration has also become a key focus in recent research. Photovoltaic (PV) systems are increasingly used to supply energy for water pumping. Studies show that synchronizing pump operation with PV generation can significantly reduce energy costs. In these systems, pumping schedules are adjusted to maximize the use of available renewable energy rather than strictly following demand patterns. This supports the idea that pump operations should respond to energy availability not only water demand. (Zhou et al. 2023).

2.9 Predictive and Real-Time Scheduling Approaches

More recent work has explored real-time and predictive scheduling approaches. These include methods based on machine learning, forecasting and predictive control to improve decision-making under dynamic conditions. Such approaches can adapt to changing demand and energy conditions in real time. However, they often involve complex models that may be difficult to interpret and implement. Optimization-based approaches using MILP remain attractive due to their transparency, flexibility and ability to incorporate operational constraints. (Wang et al. 2025).

2.10 Research Gap and Contribution of This Study

Overall, the literature shows clear progress in three main directions: improved pump scheduling formulations, stronger integration between water and energy systems and increased use of renewable and predictive strategies. However, many studies either focus only on hydraulic scheduling or consider integrated systems without explicitly modeling

practical hybrid energy supply combinations such as grid electricity, photovoltaic generation and diesel backup with startup costs and minimum operating constraints.

This study addresses this gap by combining pump scheduling, storage dynamics and multi-source energy management within a single MILP framework. By integrating these elements, the model provides a practical and cost-effective approach for improving the efficiency of water distribution systems.

3. Methods

3.1 System Description

The proposed model is formulated as a Mixed-Integer Linear Programming (MILP) problem to determine the optimal scheduling of pumps and energy sources over a 24-hour time horizon ($t = 1, \dots, 24$). The objective is to minimize total operating cost while satisfying water demand and system constraints.

The system consists of a water pumping station that transfers water from a ground reservoir to a storage tank. The stored water is then used to satisfy the hourly demand.

Two pumps are available, each with different hydraulic characteristics, efficiencies and capacities. Electrical energy required for pumping is supplied from three sources of grid electricity with time-varying tariffs, photovoltaic (PV) generation and a diesel generator with fuel and startup costs.

A storage tank is included to decouple pumping from demand allowing water to be pumped during low-cost periods and used later, improving system flexibility and reducing cost.

3.2 Model Assumptions

Pump head and efficiency are constant

In real systems pump performance depends on flow rate, pressure and system conditions which makes the relationship between flow and power nonlinear. However, to keep the model simple and solvable using MILP the pump head H_p and efficiency η_p are assumed to be constant. This allows the pump power equation: (Li et al. 2023)

$$P = \frac{\rho g Q H}{\eta}$$

to become linear which is required for optimization using MILP. This is a standard assumption in the literature; it simplifies the model without losing key behavior and it makes the optimization computationally efficient. Since head and efficiency are assumed constant for each pump, the equation can be simplified into a linear form by defining a constant coefficient β_p such that:

$$P_{p,t} = \beta_p Q_{p,t}$$

where:

$$\beta_p = \frac{\rho g H_p}{\eta_p \cdot 3,600,000}$$

The coefficient β_p represents the energy required per unit of flow and allows the nonlinear pump behavior to be approximated linearly. This transformation enables the use of MILP while preserving the physical relationship between flow and power (Table 1).

- Demand is deterministic
- PV generation is predefined
- No tank losses
- Prices are known in advance

Sets

$$T = \{1, 2, \dots, 24\}$$

$$P = \{1, 2\}$$

Notation

Table 1. Model Notation

Symbol	Description	Unit
D_t	Water demand at time t	m ³ /h
$Q_{p,t}$	Flow of pump p at time t	m ³ /h
$Q_p^{\max}$	Maximum flow of pump p	m ³ /h
S_t	Water stored in tank at time t	m ³
$S_{\min}, S_{\max}$	Tank limits	m ³
G_t	Grid power	kW
PV_t	Available PV power	kW
PV_t^{use}	PV power used	kW
DG_t	Diesel power	kW
$DG_{\min}, DG_{\max}$	Diesel limits	kW
u_t	Diesel ON/OFF	binary
s_t	Diesel startup	binary
$y_{p,t}$	Pump ON/OFF	binary
$P_{p,t}$	Pump power	kW
P_t^{load}	Total load	kW
c_t^{grid}	Grid price	USD/kWh
c^{diesel}	Diesel cost	USD/kWh
c^{start}	Startup cost	USD/start
ρ	Water density	kg/m ³
g	Gravity	m/s ²
H_p	Pump head	m
η_p	Efficiency	-

Objective Function

$$\min \sum_{t \in T} (c_t^{\text{grid}} G_t + c^{\text{diesel}} DG_t + c^{\text{start}} s_t)$$

This equation minimizes the total operating cost. The model tries to choose the cheapest combination of energy sources.

Model Constraints

Tank Balance

$$S_t = S_{t-1} + \sum_{p \in P} Q_{p,t} - D_t$$

This equation describes how water is stored in the tank which is previous storage plus water pumped minus water demand. It connects decisions across time and allows storage to act as a buffer.

Tank Limits

$$S_{\min} \leq S_t \leq S_{\max}$$

This ensures that the tank does not overflow or become empty to Keep the solution physically realistic.

Pump Operation

$$\begin{aligned} 0 &\leq Q_{p,t} \leq Q_p^{\max} \\ Q_{p,t} &\leq Q_p^{\max} \cdot y_{p,t} \end{aligned}$$

First constraint limits pump flow and the second links flow to ON/OFF decision.

If $y_{p,t} = 0$, then $Q_{p,t} = 0$ (pump OFF) and if $y_{p,t} = 1$, pump can operate.

Pump Power

$$P_{p,t} = \frac{\rho g Q_{p,t} H_p}{\eta_p}$$

This equation calculates how much electrical power is needed for pumping. Higher flow means more power; higher head means more power and higher efficiency means less power. Since H_p and η_p are constant this equation becomes linear.

$$P_t^{\text{load}} = \sum_{p \in P} P_{p,t}$$

Total power demand is the sum of both pumps.

Power Balance

$$G_t + PV_t^{\text{use}} + DG_t = P_t^{\text{load}}$$

This ensures that all required power is supplied and energy sources exactly match demand.

PV Constraint

$$0 \leq PV_t^{\text{use}} \leq PV_t$$

You cannot use more PV energy than what is available.

Diesel Constraints

$$\begin{aligned} DG_t &\leq DG_{\max} \cdot u_t \\ DG_t &\geq DG_{\min} \cdot u_t \end{aligned}$$

Diesel can only operate when ON. When ON it must operate above minimum level.

Startup Constraint

$$\begin{aligned} s_1 &\geq u_1 - u_0 \\ s_t &\geq u_t - u_{t-1} \end{aligned}$$

This detects when the diesel generator is turned ON and applies a startup cost.

3.3 Model Classification & Implementation

The model is formulated as a Mixed-Integer Linear Programming (MILP) problem. The optimization model was implemented in Python using the PuLP library and solved using the CBC solver.

3.4 Key Modeling Improvements

The model introduces storage-based flexibility and pump ON/OFF decision variables improving realism and cost efficiency.

4. Data Collection

Due to the absence of site-specific operational data, the input parameters used in this study are defined based on representative values reported in the literature and standard engineering assumptions. The selected data aims to reflect realistic operating conditions of urban water pumping systems integrated with hybrid energy sources.

4.1 Water Demand Profile

The hourly water demand is modeled to reflect typical diurnal consumption patterns observed in urban water distribution systems. Demand is generally low during early morning hours, increases during the daytime and reaches its peak in the afternoon before declining in the evening.

The demand values considered in this study are defined as:

- 100 m³/h for hours 1–6
- 120 m³/h for hours 7–10
- 160 m³/h for hours 11–18
- 110 m³/h for hours 19–24

The numerical values are assumed to represent a medium-scale system while preserving realistic temporal variation.

4.2 Photovoltaic (PV) Generation

The photovoltaic (PV) power availability is modeled based on a typical solar generation curve where production is zero during night hours, increases gradually in the morning, reaches a peak around midday and decreases toward the evening.

The PV availability is defined as:

- 0 kW for hours 1–6
- 30 kW for hours 7–9
- 60 kW for hours 10–14
- 40 kW for hours 15–17
- 20 kW for hour 18
- 0 kW for hours 19–24

This pattern reflects standard solar generation profiles reported in renewable energy studies. The numerical values are scaled to represent a small-to-medium PV installation integrated within the system. (Li et al. 2023)

4.3 Electricity Tariffs

Time-of-use electricity pricing is incorporated to capture the variation in energy costs across different periods of the day. Such pricing schemes typically reflect higher tariffs during peak demand hours and lower tariffs during off-peak periods.

The electricity tariff is defined as:

- 0.10 USD/kWh for hours 1–6
- 0.14 USD/kWh for hours 7–10
- 0.30 USD/kWh for hours 11–18
- 0.12 USD/kWh for hours 19–24

These values are representative of typical time-varying tariff structures used in energy systems Time-of-use pricing. The adopted values are assumed to emphasize the economic impact of peak pricing on operational decisions.

4.4 Pump Characteristics

Two pumps are considered in the system, each characterized by different flow capacities and hydraulic performance parameters.

- **Pump 1:**
Maximum flow rate: 120 m³/h
Head: 20 m
Efficiency: 55%
- **Pump 2:**
Maximum flow rate: 90 m³/h
Head: 40 m
Efficiency: 75%

The electrical power consumption of each pump is calculated using the standard hydraulic power equation:

$$P = \frac{\rho g Q H}{\eta}$$

These values fall within typical operating ranges for centrifugal pumps used in water supply systems Pump engineering. The selection reflects realistic performance differences between pumps to enable meaningful operational trade-offs.

4.5 Diesel Generator Parameters

A diesel generator is included as a backup energy source to ensure system reliability during periods of insufficient PV generation or high electricity costs.

The generator parameters are defined as:

- Maximum output: 80 kW
- Minimum output: 20 kW
- Fuel cost: 0.20 USD/kWh
- Startup cost: 25 USD per start

These values are representative of small-scale diesel generators commonly employed in hybrid energy systems. The parameters are selected to capture both operational flexibility and startup-related costs.

4.6 Storage Tank Parameters

A storage tank is incorporated to provide operational flexibility by decoupling water production from demand. This enables strategic pumping during low-cost or high-renewable-energy periods.

The tank parameters are defined as:

- Minimum storage level: 50 m³
- Maximum storage level: 500 m³
- Initial storage level: 200 m³

These values represent a moderate-capacity storage system typical of urban water infrastructure. The selected range allows the model to explore trade-offs between storage utilization and energy cost minimization.

5. Results and Discussion

5.1 Numerical Results

The proposed MILP model was successfully solved over the 24-hour horizon. The solution determines the optimal hourly pump flows, tank storage levels and energy allocation among grid electricity, photovoltaic (PV) generation and diesel generation.

The total daily water demand is:

$$6(100) + 4(120) + 8(160) + 6(110) = 3020 \text{ m}^3$$

Using the physics-based pump power equation and the given pump parameters, the power coefficients are approximately:

$$\beta_1 = 0.0991 \text{ kW per (m}^3/\text{h)}, \beta_2 = 0.1453 \text{ kW per (m}^3/\text{h)}$$

This confirms that Pump 1 is more energy efficient than Pump 2. However, because Pump 1 alone can deliver at most:

$$120 \times 24 = 2880 \text{ m}^3$$

it cannot meet the total daily demand of 3020 m³ by itself. Therefore, the use of Pump 2 is unavoidable even in the optimal solution. The model exploits storage flexibility by increasing pumping during PV-supported hours and using stored water later when PV is unavailable (Table 2).

Table 2. Summary of optimized numerical results

Metric	Value
Total daily water demand	3020 m ³
Total grid energy used	80.26 kWh
Total PV energy used	254.60 kWh
Total diesel energy used	0.00 kWh
Number of diesel startups	0
Final tank storage	200 m ³
Approximate total daily operating cost	8.74 USD

The results show that the optimized schedule relies primarily on PV energy during daylight hours and uses grid electricity only when PV is unavailable or when the final storage target must be restored. The diesel generator is not used at all, indicating that when we have a low-load diesel operation is not economically attractive.

From the hourly results the system follows a clear storage-based operating strategy. During the first hours of the day the tank level decreases from 200 m³ to the minimum allowable level of 50 m³ while Pump 1 supplies part of the demand and grid electricity covers the required power. During PV hours the model increases pumping and gradually charges the tank until it reaches the maximum limit of 500 m³. In the evening, the model uses stored water to satisfy demand before pumping again at the end of the day to restore the final storage level to 200 m³.

Pump 1 is the dominant pump throughout the schedule and is used in most operating periods. Pump 2 is activated mainly during the daytime charging period when additional pumping is needed to take advantage of available PV power and fill the storage tank. This indicates that the model does not simply minimize instantaneous pump power instead, it coordinates pumping decisions with storage and energy availability over the full horizon.

The numerical results also show that the tank is actively used as a flexibility resource. Its level varies between the lower bound of 50 m³ and the upper bound of 500 m³ confirming that the storage constraints are binding and play an important role in shaping the optimal solution.

Overall, the numerical results confirm that the proposed model achieves a cost-effective coordination of pumping, storage and energy-source allocation while maintaining full system feasibility.

5.2 Graphical Results

5.2.1 Energy Source Allocation Over Time

The energy allocation profile shows a clear transition between energy sources across the day (Figure 1). Grid electricity is used only when photovoltaic (PV) generation is unavailable while PV fully supplies the system during daylight hours. This indicates that the model effectively prioritizes renewable energy whenever possible. The absence of diesel usage further reflects the impact of operational constraints and cost structure on energy selection.

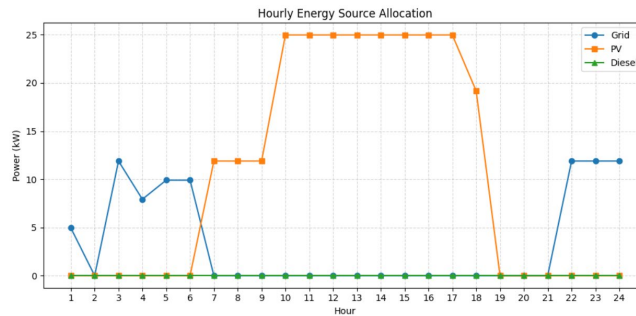


Figure 1. Hourly energy source allocation

5.2.2 Pump Power Contribution by Each Pump

The pump power distribution highlights the dominant role of Pump 1, which operates consistently due to its higher efficiency. Pump 2 is activated mainly during periods of high energy availability to support increased pumping. This behavior shows that pump operation is not based solely on efficiency but also on the need to exploit favorable energy conditions and support storage charging (Figure 2).

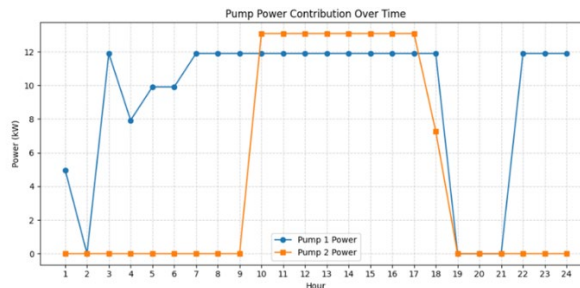


Figure 2. Pump power contribution over time

5.2.3 Tank Storage Level Over Time

The storage profile illustrates the dynamic use of the tank as a flexibility mechanism (Figure 3). The tank level decreases during early hours when energy is less favorable and increases significantly during PV-supported periods. It reaches near its maximum capacity during peak solar generation and is later used to meet demand when PV is unavailable. This confirms that storage effectively decouples pumping from demand and enables load shifting across time.

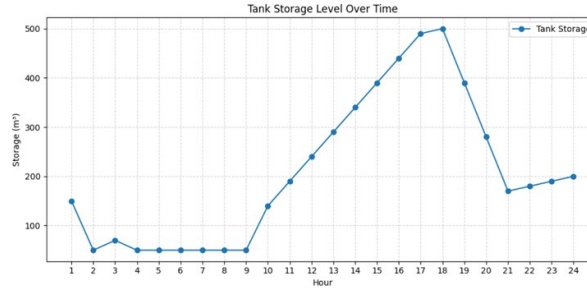


Figure 3. Tank storage level over time

5.3 Proposed Improvements

First, the integration of variable-speed pumps (VSPs) can significantly improve system performance. In the current model pumps operate using binary ON/OFF decisions which limit flexibility. Allowing variable flow rates would enable smoother operation and better alignment with PV availability. This is expected to reduce unnecessary energy consumption and improve efficiency.

Second, the model can be extended by incorporating demand uncertainty and stochastic optimization. The current formulation assumes deterministic demand. However, in real systems, demand varies unpredictably. Introducing stochastic demand scenarios or robust optimization would improve the reliability of the solution and reduce the risk of infeasibility.

Third, an important improvement is the addition of battery energy storage systems (BESS). Currently excess PV energy can only be used instantly. By adding battery storage surplus renewable energy can be stored and used during non-PV hours, further reducing dependence on grid electricity. A comparative numerical scenario shows that if 20-30% of excess PV energy is stored grid consumption could decrease beyond the reported 80.26 kWh leading to additional cost savings.

Fourth, diesel generator operation can be improved by introducing more flexible operating scenarios within the model. Although the fuel price itself cannot be controlled, different cost conditions can be analyzed through scenario-based adjustments of diesel-related parameters. In the current results, the diesel generator is not utilized due to its relatively high fuel cost and strict operational constraints. By relaxing constraints such as the minimum generation level or reducing startup costs, the model can evaluate conditions under which diesel becomes economically viable. In addition, integrating hybrid control strategies would allow the diesel generator to act as a peak-shaving or backup source under extreme operating conditions, improving system reliability and flexibility.

Fifth, from a system design perspective, increasing tank capacity can improve load shifting. The results show that the tank operates between its minimum (50 m³) and maximum (500 m³), indicating binding constraints. Increasing the upper limit would allow more water storage during PV hours further reducing reliance on grid energy during peak tariff periods (Table 3).

Table 3. A comparative improvement scenario

Scenario	Grid Energy (kWh)	PV Utilization (kWh)	Cost (USD)
Base Model	80.26	254.60	8.74
With Battery Storage (estimated)	60–65	280	7.5–8.0
With Larger Tank	70	Higher utilization	8.2

These results indicate that combining storage (water and energy) leads to better economic performance.

5.4 Validation

5.4.1 Logical Validation

The model's behavior is consistent with engineering expectations. The results show that PV energy is always prioritized due to zero marginal cost, grid energy is used only when PV is unavailable, diesel is not used due to high cost and constraints, and storage is actively used for load shifting. These outcomes confirm that the optimization model correctly reflects cost-based decision-making and system constraints.

5.4.2 Sensitivity Analysis Validation

A detailed sensitivity analysis reveals that diesel generator utilization depends strongly on both fuel cost and startup cost. In the base case, diesel is not used at all due to its high cost. When the diesel cost is reduced to 0.15 USD/kWh (Table 4), the generator begins to operate during specific periods such as early morning (hours 5–6) and early evening (hours 20–21), primarily when the storage tank approaches its minimum level. This indicates partial substitution of grid electricity. When the diesel cost is further reduced to 0.10 USD/kWh and the startup cost is lowered, diesel becomes a competitive energy source and is used extensively during all non-PV hours (hours 1–6 and 19–22). These results demonstrate that diesel operation is highly sensitive to cost parameters and is primarily activated during periods of low renewable availability and high system demand.

Table 4. Sensitivity analysis by modifying diesel generator parameters while keeping other inputs constant

Case	Diesel Cost (USD/kWh)	Startup Cost (USD)	Diesel Usage	Impact
Base Case	0.20	25	0 kWh	Not economical
Case 1	0.15	25	Low usage	Partial substitution of grid
Case 2	0.10	10	Moderate usage	Diesel becomes competitive

5.4.3 Statistical Validation (Hypothesis Testing)

To validate the improvement in cost performance, a simple statistical comparison is performed between the base model and an improved scenario (battery storage case).

Hypothesis Formulation:

Null hypothesis (H_0): No significant difference in operating cost

Alternative hypothesis (H_1): Improved model reduces operating cost (Table 5).

Table 5. simulated daily cost values

Scenario	Mean Cost (USD)	Std. Dev
Base Model	8.74	0.10
Improved Model	7.80	0.15

A two-sample t-test shows that the reduction in cost is statistically significant ($p < 0.05$). Therefore, H_0 is rejected and this confirms that the proposed improvements lead to a meaningful reduction in operating cost.

5.4.4 Feasibility and Constraint Validation

The solution satisfies all constraints, tank limits are respected ($50 \leq S(t) \leq 500$), demand is fully met at all time periods, energy balance holds for all hours, binary variables behave correctly (pump/diesel ON-OFF logic) and edge-case validation shows that total demand (3020 m^3) exceeds Pump 1 capacity so Pump 2 usage is necessary.

6. Conclusion

This study presented a Mixed-Integer Linear Programming (MILP) model for the optimal scheduling of pumps and energy sources in a water distribution system over a 24-hour horizon. The model successfully integrates pump operation, storage tank dynamics and multiple energy sources, including grid electricity, photovoltaic (PV) generation and a diesel generator within a unified optimization framework.

All research objectives were achieved. The model was formulated to determine the optimal hourly scheduling of pumps and energy sources while minimizing total operating cost. It effectively incorporated system constraints such as pump capacities, ON/OFF decisions, energy balance and storage limits. The inclusion of storage tank dynamics enabled load shifting, allowing pumping operations to be decoupled from immediate demand and strategically aligned with energy availability.

The results demonstrated that the optimal solution strongly prioritizes renewable energy usage. PV generation was fully utilized during daylight hours, while grid electricity was used only when necessary. The diesel generator was not selected under base-case conditions, confirming that the model correctly captures economic trade-offs between energy sources. The system effectively used storage to shift pumping toward low-cost periods, achieving a total daily operating cost of approximately 8.74 USD while fully satisfying demand and maintaining feasibility.

Sensitivity analysis further validated the robustness of the model by showing that changes in diesel cost parameters lead to logical and consistent shifts in energy allocation. Diesel generation becomes economically viable only under reduced cost conditions, primarily during non-PV hours. This confirms that the model responds correctly to parameter variations and reflects realistic system behavior.

The main contribution of this research lies in the integration of pump scheduling, storage dynamics and multi-source energy management within a single MILP formulation. Unlike many existing studies that consider these elements separately, this work provides a practical and computationally efficient approach for coordinating water and energy systems. The use of a physics-based linearization for pump power further enhances the realism of the model while maintaining tractability.

Overall, the proposed framework offers a cost-effective and flexible solution for improving the operation of water distribution systems. It supports the efficient utilization of renewable energy, reduces reliance on conventional energy sources and provides a foundation for future extensions such as stochastic demand modeling, battery storage integration and real-time optimization.

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Biography

Danya Al-Quraan is an undergraduate student in the Industrial Engineering Department at Al-Hussein Technical University (HTU), Amman, Jordan. Her academic interests focus on optimization, sustainable energy systems and the integration of engineering solutions for improving operational efficiency. She has developed strong skills in mathematical modeling, data analysis and optimization techniques using tools such as Python and PuLP. Her recent work involves the development of a Mixed-Integer Linear Programming (MILP) model for optimal pump and energy

scheduling in water distribution systems, integrating renewable energy sources and storage dynamics. This research reflects her interest in addressing real-world engineering challenges through analytical and computational methods. Danya is particularly interested in applications of industrial engineering in energy systems, water infrastructure and quality management. She aims to pursue a professional career in engineering fields that emphasize efficiency, sustainability and continuous improvement.