A Bi-Objective Model for Locating and Sizing Emergency Stations

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Abstract

In this paper, we want to present a bi-objective model within the integer linear programming framework to determine the location of emergency stations, the regions covered by each of these stations and the allocation of ambulances to these stations. Objectives of this model are minimizing the cost of locating the stations and ambulances as well as the maximum time it takes to serve the demands. In this model, service time is the interval between dispatching the ambulance and transporting the patient to the closest hospital. Thus, the locations of existing hospitals are taken into account in making decisions while this parameter has been ignored in most of the models available in the literature review. Solving this model by using the $\varepsilon-$constraint method, provides us with a set of Pareto optimal solutions, so we can make better decisions. The presented algorithm is able to solve problem instances containing up to 150 regions within a period of less than 12 seconds which is a reasonable time.

Keywords

Emergency station. Location problems. Ambulance allocation. Bi-objective model. $\varepsilon-$Constraint method.

1. Introduction

One of the most important and of course practical applications of operation research is solving the locating problems. The importance of these problems in practice and the diversity and complexity of them has led to the development of several theoretical approaches in this field. Among these, emergency station problems are of particular importance especially when the problem's dimensions are large. Emergency stations are among the most important and vital service centers in a city and play an important role in peoples' health. It is obvious that providing timely services by emergency stations is mostly dependent on locating them in the right positions so that the ambulances be able to reach the patient quickly without being prevented by the urban limits (like heavy traffic) and also with the least interference with other residents' lives.

Several factors are involved in locating the municipal facilities and because of the large amounts of data it is not possible to analyze them completely by using the traditional locating methods. Hence, determining the right place to locate the emergency stations and allocating the appropriate number of ambulances to them, play an important role in increasing the speed of response and giving service to the patients as well as making optimal use of available resources. Increasing the number of ambulances is not solely enough to improve the services given to the patients. The number of ambulances, their distributing among the emergency stations and the location of these stations should be considered altogether.

In this paper, we present a model to determine the location of emergency stations, the number of ambulance allocated to each station and how to allocate the regions to the stations while we consider these two objectives: minimizing the total cost and minimizing the maximum time of service. In this model, service time is the interval between dispatching the ambulance and transporting the patient to the closest hospital. The $\varepsilon-$constraint method is used to solve this bi-objective model [3]. Numerical results show that the presented algorithm can solve problem instances containing up to 150 regions within a period of less than 12 seconds which is a reasonable time.

2. Literature Review

The location problems of Emergency medical services (EMS) facilities are an active research area. Most studies in this area are on model development and solution methods rather than being application-oriented studies. The

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main criteria in these studies are accessibility, adaptability and availability of emergency services. It is suggested that reliability, robustness and adaptability of emergency medical systems will become important in the future. The basic location models that can be applied for emergency services are location set covering (LSCP), maximal covering location (MCLP) and p-median models and their extensions [5]. Badri et al. (1998), presented a multiple criteria modeling approach, via integer goal programming, to the fire-station location problem that involves conflicting objectives [2].

Brotcorne et al. (2003) traced the evolution of ambulance location and relocation models [4]. Alsallouma et al. (2006) extended models for the maximal covering location problem in two ways. First, the usual 0–1 coverage definition is replaced by the probability of covering a demand within the target time. Second, once the locations are determined, the minimum number of vehicles at each location that satisfies the required performance levels is determined [1]. Coskun and Erol (2008) considered full coverage of all regions in such a way that the net present value of installation costs of new stations and cost of running the system over a planning horizon is minimized [5]. Schmid and Doerner (2010) have formulated a mixed integer program which tries to optimize coverage at various points in time simultaneously. They show that it is essential to consider time-dependent variations in travel times and coverage respectively [6].

Contributions of this paper are: (1) Considering the two objectives: minimizing the total cost and minimizing the maximum time of service in the model; (2) Considering the locations of hospitals in decision making; (3) providing a set of Pareto optimal solutions for the decision makers which give them the opportunity to make their preferences, consider the situations and make the best decision from among the available options.

3. Notation

This paper presents a bi-objective model to determine the emergency locations, the regions covered by each of these stations and the allocation of ambulances to them. Population and demand density, distances between the regions, number of times a region needs to be served (coverage) by neighboring stations and positions of existing hospitals are the model's inputs. In this model, all of the demand nodes are covered as we consider the two goals of minimizing the costs and minimizing the maximum service time simultaneously and this characteristic makes this model different from other existing models in the literature review.

Furthermore, service time is defined as the interval between dispatching the ambulance and transporting the patient to the closest hospital. Therefore, the locations of existing hospitals are taken into account in the decision process while this parameter has been ignored in most of the models available in the literature review. In this model, we use two concepts: the first and the second coverage (giving service). We define two acceptable time radii for a station, so that an emergency station should serve a demand in the first or the second coverage within those acceptable time radii. Therefore for each region i we can determine a set of acceptable regions to establish the emergency stations in them in order to give service to the region i.

Finally, with regard to the model objectives and constraints, the location of emergency stations and the number of allocated ambulances should be determined in such a way that all of the regions lie in the first coverage area and some of them lie in the second coverage area of an emergency station if necessary.
The notations used for mathematical modeling of the problem are as follows:
Decision variables:

X_{ij} : A binary variable that equals one if and only if region i is in the first coverage area of station in region j.

X_{ij}^{s} : A binary variable that equals one if and only if region i is in the second coverage area of station in region j.

n_j : Number of ambulances in region j.

Parameters:

N : Number of regions.

δ_i^S : A binary parameter that equals one if and only if region i needs to be covered secondly.

t_{ij} : Average time to travel from region i to region j.

h_i : Average time to travel from region i to closest hospital.

CA: Unit purchasing cost of an ambulance.

CF_j : Cost of installing a station in region j.

C_{ij} : Unit cost of responding a call from region i by a station in region j .

$P_{\min}$: Minimum population that must be covered by one ambulance.

$P_{\max}$: Maximum population that can be covered by one ambulance.

D_i : Demand of region i .

f_s : Assumed second coverage demand as a fraction of first coverage demand.

M : Very large number.

Sets:

S_i^F : Set of regions that an emergency station in them can serve demands in region i , in the first coverage within acceptable response time.

S_i^S : Set of regions that an emergency station in them can serve demands in region i , in the second coverage within acceptable response time.

4. Formulation

Our model is formulated as follows.

$$\min z_1 \tag{1}$$

$$z_1 = \sum_{j=1}^{N} X_{jj} CF_j + \sum_{j=1}^{N} (n_j CA) + \sum_{j=1}^{N}\sum_{i=1}^{N} C_{ij} D_i X_{ij} + \sum_{j=1}^{N}\sum_{i=1}^{N} C_{ij} D_i f_s X_{ij}^s$$

$$\min z_2 = \max_{i,j}\{(t_{ij} + h_i) X_{ij}\} \tag{2}$$

$s.t.$

$$\sum_{j \in S_i^F} X_{ij} = 1 \qquad i = 1,\dots,N \tag{3}$$

$$\sum_{\substack{j \in S_i^S \\ i \neq j}} X_{ij}^S = 1 \qquad i = 1,\dots,N \;\; where \;\; \delta_i^s = 1 \tag{4}$$

$$X_{ij} \le X_{jj} \qquad i = 1,\dots,N\,;\;\; j = 1,\dots,N\,;\; i \neq j \tag{5}$$

$$X_{ij}^s \le X_{jj} \qquad i = 1,\dots,N\,;\;\; j = 1,\dots,N\,;\; i \neq j \tag{6}$$

$$\sum_{i=1}^{N} (X_{ij}) D_i + \sum_{i:\delta_i^S = 1} X_{ij}^s D_i f_s \ge p_{\min}(n_j) \qquad j = 1,\dots,N \tag{7}$$

$$\sum_{i=1}^{N} (X_{ij}) D_i + \sum_{i:\delta_i^S = 1} X_{ij}^s D_i f_s \le p_{\max}(n_j) \qquad j = 1,\dots,N \tag{8}$$

$$X_{jj} \le n_j \qquad j = 1,\dots,N \tag{9}$$

$$n_j \le M X_{jj} \qquad j = 1,\dots,N \tag{10}$$

$$X_{ij} \in \{0,1\} \qquad (\, j \in S_i^F \;,\; i = 1,\dots,N\,) \tag{11}$$

$$X_{ij}^S \in \{0,1\} \qquad (\, j \in S_i^S \;,\; i = 1,\dots,N\,) \tag{12}$$

$$n_j \ge 0 \text{ integer} \quad (\, j = 1,\dots,N\,) \tag{13}$$

In this formulation, Eq. (1) states the first objective function which is composed of the total cost of installing new stations, the total cost of purchasing ambulances and responding demands.

Eq. (2) states the second objective function which is minimizing the maximum time it takes to serve the demands.

Eq. (3) is the first set of constraints for making sure that each region is covered firstly by a station located in a region which satisfies the response time limit.

Eq. (4) guarantees that a pre-set of regions must be covered secondly by exactly one station in a region within the response time limit.

The constraint sets in Eq. (5) ensure that a region can be covered firstly from a different region only if a station is located in that region respectively.

Eq. (6) states similar constraints for second coverage of respective regions.

Eq. (7) ensures that the total population covered firstly and secondly by a station cannot exceed the population capacity limit ($P_{\max}$).

On the other hand, constraints in Eq. (8) prevent population per ambulance at any station from dropping below the minimum limit ($P_{\min}$).

Constraints in Eq. (9) and Eq. (10) prevent any possible contradiction between the number of ambulances at a region and whether or not there is a station located in that region.

Eq. (11), Eq. (12) and Eq. (13) define the decision variables of the problem and their values.

5. Solution Algorithm and Numerical Result

The $\varepsilon-$constraint method is used to solve this model. The algorithm used will be described below. The output of this algorithm is the Pareto optimal set illustrated as F .

Solution Algorithm

[1] Set $i=2$, $j=1$ or $i=1$, $j=2$

[2] Compute the:

$z_i^I = \min_{\vec{z}\in Z} z_i$

$z_j^I = \min_{\vec{z}\in Z} z_j$

$z_i^N = \min_{\vec{z}\in Z} \{z_i : z_j = z_j^I\}$

$z_j^N = \min_{\vec{z}\in Z} \{z_j : z_i = z_i^I\}$

[3] Set $F = \left(z_i^I, z_j^N\right)$ and $\varepsilon_j = z_j^N - \Delta$ $(\Delta = 1)$

[4] While $\varepsilon_j \geq z_j^I$

(a) Solve $P_i(\varepsilon_j) = \min\{z_i : z_j \leq \varepsilon_j\}$ through branch & bound and add the optimal solution value $\left(z_i^*, z_j^*\right)$ to F .

(b) Set $\varepsilon_j = z_j^* - \Delta$.

[5] Remove dominated points from F if required.

Δ can often take a value greater than 1. Selecting Δ is often affected by factors such as the decision maker, the difference between z_j^I, z_j^N and so on. However, it is important to choose appropriate value for Δ because this determines a bound for jth objective in the $P_i(\varepsilon_j)$ problem and is equivalent of considering a weight corresponding to the mentioned objective. Thus it is proposed to select the value of Δ in such a way that the density of resulting Pareto optimal solutions from the above algorithm lie in a better range with regards to the relative weight of objectives.

To evaluate the efficiency of this algorithm at solving the presented model, problem instances ranging from 20 to 150 regions were generated randomly and solved by the algorithm. The results have been shown in the Table 1. The algorithm has been applied to each of these problem instances to find 5 Pareto optimal solutions. Computations are performed using CPLEX 12.1 on a Dell Studio 1558 laptop, CPU 2.8 GHz, with 4GB of RAM, running the Windows operating system.

Table 1. CPU Times for Different Problem Sizes

Number of regions	CPU times (secs)
20	0.84
40	1.3
70	2.6
80	2.5
90	4.5
100	5.28
120	8
150	11.2

The computational results show that this algorithm is able to solve the problem instances containing up to 150 regions and find 5 Pareto optimal solutions within a period of less than 12 seconds which is a reasonable time. Problem instances which contain 150 regions are large enough to model real problems in metropolises and shows that this model and the algorithm can be applied to real world problems. Furthermore, this algorithm provides a set of Pareto optimal solutions for the decision makers which give them the opportunity to make their preferences, consider the situations and make the best decision from among the available options.

6. Conclusion

In this study, a bi-objective model within the integer linear programming framework is presented to determine the location of emergency stations, the regions covered by each of these stations and the allocation of ambulances to these stations. Objectives of this model are minimizing the cost of locating the stations and ambulances as well as the maximum time it takes to serve the demands. In this model, service time is the interval between dispatching the ambulance and transporting the patient to the closest hospital. Thus, the locations of existing hospitals are taken into account in making decisions while this parameter has been ignored in most of the models available in the literature review.

Solving this model by using the $\varepsilon-$constraint method, provides a set of Pareto optimal solutions for the decision makers which give them the opportunity to make their preferences, consider the situations and make the best decision from among the available options. The presented algorithm is able to solve problem instances containing up to 150 regions within a period of less than 12 seconds which is a reasonable time. For further study, modern heuristics such as genetic or tabu search type algorithms and simulated annealing can be developed for larger problems to reach fairly “good” solutions in short time. Also a decision support system based on our model can be developed to assist optimally decide the locations and capacities of emergency stations.

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