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Multi-Objective Path Search Problem Based on an Extended Network Model

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Abstract

Motor vehicle drivers select paths to their destinations that meet their wishes as much as possible. We study a multi-objective path search problem that considers a driver's various and vague requests, and propose a method for solving it based on an extended network model. This method minimizes the total route distance and the total angular degrees to which the driver turns to the right or left. Using numerical computation, our method selects links in such a way as to minimize total route distance and angular turning, and selects as far as possible a route composed of arterial roads. In addition when the weight or cost of route distance and angular turning change slightly, the numerical computation shows that the selected route changes dynamically. In conclusion, it is clear that the model is useful for reducing driver stress and environmental load, and promoting of safe driving.

Keywords
Path search problem, multipurpose optimization, stochastic processes, network extended model

1. Introduction
In motor-vehicle traffic, most drivers aim to run the shortest route in time or distance. It is called the shortest path problem to get the route. In the problem we are given an undirected and connected network with two special nodes called the origin and the destination. Associated with each of the links (undirected arc) is a non-negative distance. The objective is to find the shortest path (the path with the minimum total distance) from the origin to the destination. From the standpoint of an optimization problem, shortest path problems are solved by minimizing or maximizing only one objective function, which implies optimizing with respect to only one parameter. However, in reality, there are several parameters and various and vague requests.

Although there are numerous studies on the routing problem and its algorithms, we are not aware of any previous study relevant to our research. We studied the shortest path algorithms for finding a path between two nodes in a graph such that the sum of the weights of its edges is minimized. Generally the weight is one type of cost (for instance time or distance), but in this paper, we assign distance to one type of link and angular turning to the left or right to other links as the weights in a road network. The multi-objective shortest path problem (MSPP) and bi-criterion shortest path problem (BSP) are most relevant to our study. Current and Marsh(1993) reviewed earlier papers on multi-objective design of transportation networks, examining forty-one articles, and showed that the diversity of objectives demonstrates the multi-objective nature of the network design and routing problem. Moreover they showed the difficulty of measuring these objectives and the complexity of transportation network analysis.

Hansen (1980) defined reliability as the objective in his formulation and studied 10 different BSP problems and their solution procedures. His algorithm was used for comparison with later algorithms in Mote et al. (1991) and Huarng et al. Martins generalized Hansen's algorithm (1984), which can be seen as a generalization of Dijkstra's shortest path algorithm to multiple criteria. It assumes that all link-coefficients are non-negative. Most drivers want to arrive at their destination in minimum time and do not want to reduce their speed as a result of turning left or right at intersections. From this viewpoint, Osuna et al.(2012) proposed an extended network model to optimize with respect to two parameters, one is the route distance and other is the total amount of angular turning to the left or right. They examined an angular function. Osuna et al.(2013) studied a sensitivity analysis of the angular function.

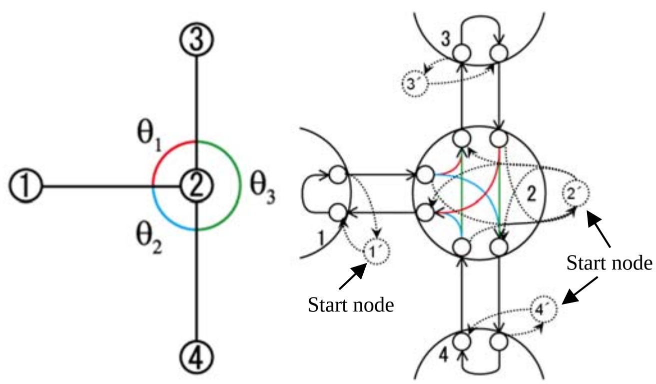
In this paper, we explain in detail a multi-objective path search problem based on an extended network model. We show that our model makes it possible to solve these multi-objective shortest path problems using the road network obtained from extended GIS (Geographic Information Systems) data.

2. Framework for the model

2.1 Network extended model

In this section, we explain our extended network model. A road network can be considered a graph with a positive weight. The nodes represent road junctions and curve points. Each link in the graph is associated with a road segment between two junctions. The weight of a link may correspond to the length of the associated road segment, the time needed to traverse the segment or the cost of traversing the segment. In this paper we use the directed graph because we can model a one-way street. In this graph we can treat some highway links as more important than others for long distance travel.

In our extended model, the link corresponding to a road segment is set to the length of the associated road segment, and the link corresponding to a turn point is set to the cost as a function of the angular turning. Figure 1 shows the original network and our extended model. We assume an original network which is constructed using a road center line from the Geographic Information Systems (GIS). The road center line from the digital map data is a non-directed graph as shown on the left in Figure 1. We change the network to a directed graph and create the links to every connected direction inside the intersection node, which is shown as node 2 in the figure. This process enables the start node to split up into several nodes. Thus we create new nodes for the start and destination nodes. Vehicles enter the network from the start node and follow the directed links to the destination node.



(the original network) (the extended network)

Figure 1: an extended networks

When a node is used just for a curve, our model creates an angular link inside the node.
Consequently, after the calculation, we obtain two types of links, road segment links and angular turning links.

2.2 Mathematical formulation

A natural linear programming formulation is defined for our shortest path problem in this section.

Given a directed graph (V, E) with source node s, target node t and cost function $w(i, j)$ for each link (i, j) in A, consider the program with variables $x_{ij} \in \{0 \text{ and } 1\}$:

$$\min \quad z = \sum_{i, j \in A} w(i, j) x_{ij} \,, \tag{1}$$

subject to

$$\sum_j x_{ij} - \sum_j x_{ji} = \begin{cases} 1 & \text{if} \quad i = s \\ \text{-1} & \text{if} \quad i \neq t \\ 0 & \text{othewise} \end{cases} \,, \tag{2}$$

where $w(i, j)$ is defined as:

$$w(i, j) = \begin{cases} r(i, j) & \text{if } (i, j) \text{is road segment link} \\ \text{c}(\theta) & \text{if } (i, j) \text{is angular link} \end{cases} \,, \tag{3}$$

where r is the length of the road segment link (i, j). In this paper, we use the function below:

$$c(\theta) = t \times \theta \,, \tag{4}$$

where t is a parameter. We minimize the route distance and total amount of angular turning using the shortest path finding algorithm such as the Dijikstra's method.

3. Numerical Computation

In this section, we present two case studies for the proposed model. Case 1 is a calculation on a small network. We will show the change of route caused by changing the parameter for the angular function. In Case 2, we examine our model using a large network, which is part of the Tokyo metropolitan area. In this study, we used DRM (Digital Road Map) data. The links for this digital map contain data on many attributes which include the traffic volume of the road traffic census survey held by Ministry of Land, Infrastructure, Transport and Tourism of Japan in 2005. We demonstrate that the links selected by our model have comparatively improved flow and our model tends to find arterial roads for the path.

3.1 Case study: A small network at a city ward level

In this study the network extracted from GIS and shown in Figure 2 is used for the numerical experiment. This region is the Tama Ward in the City of Kawasaki, Kanagawa Prefecture. The original network is a non-directed graph, which has 6,412 links and 2,973 nodes. The thicker lines are arterial roads. Using the process to change networks as presented in Section 2.1, we obtain the extended network. Table 1 shows the number of links and nodes for both networks. As can be seen, the number of links is increased 3.25 times and the number of nodes 5.31 times.

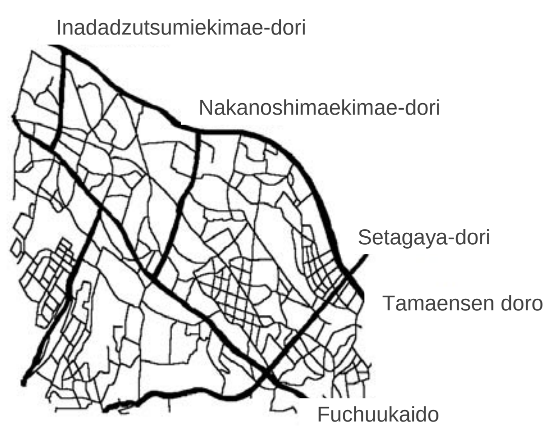


Figure 2: Map of the region.

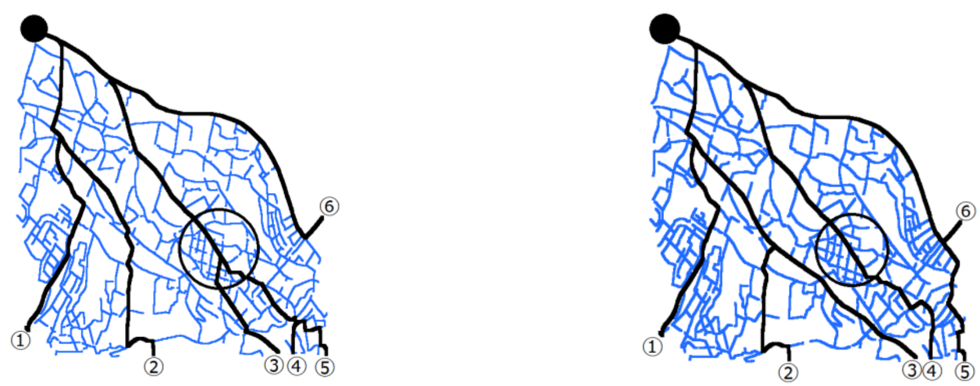
The results of numerical calculation are shown in Figures 3 and 4. The filled black circles are the start points and points 1 to 6 are destination points. The thicker line is the selected route. The figure on the left is calculated with the parameter *t=0*, implying the route is the shortest path as determined using only the segment length. The figure on the right is calculated with the parameter *t=100*, and shows that a part of calculated route is different from the shortest path.

In Figure 3, the routes to point 3 and point 5 represent a substantial change. Both tend to take arterial road links. Observing the inside of the circle in the center of the map, we note this is an area with a high concentration of houses and intersections. The route $t = 100$ does not run through this area because our model tends to minimize the total number of intersections.

Table 1: Link and node numbers in both networks.

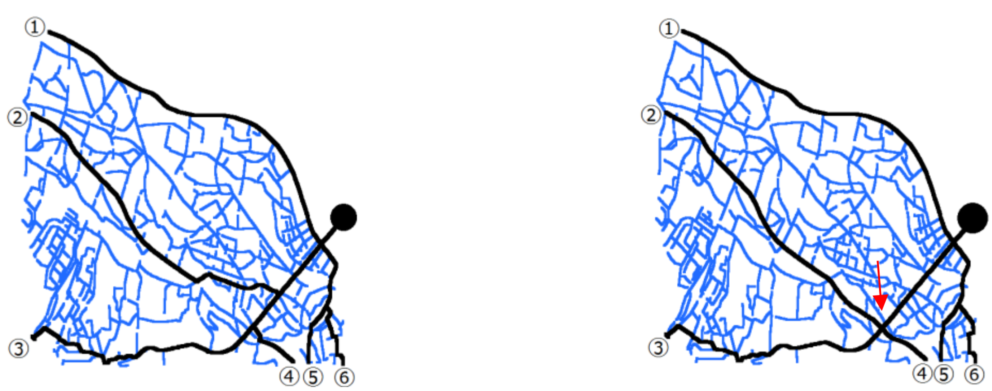
	Original network	Extended network	rate of increase
	(a)	(b)	(b)/(a)
Number of links	6,412	20,864	3.25
Number of nodes	2,973	15,787	5.31

In the next numerical experiment, we changed the start point. In Figure 4, we observe hardly any change in the routes. Although in the case for the parameter $t = 0$, the route to point 2 runs through a non-arterial road, for the parameter $t = 100$, the route takes only a single right turn (shown by the arrowhead).



$t = 0$ $t = 100$

Figure 3: Results for Case 1



$t = 0$ $t = 100$

Figure 4: Results for Case 2

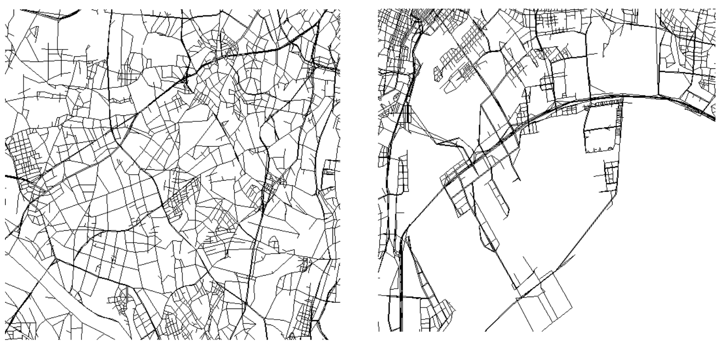
We compare the route distance between the shortest path and our model in Table 2. The route distance using our model is rarely different from the shortest path and is about 1.05 times in the maximum case.

Table 2: Comparison of route distance [m]

		Shortest path	Our model	Difference	Rate of increase
		(a)	(b)	(b)-(a)	(b)/(a)
Figure 2	②	3,699.4	3,761.5	62.1	1.01
	③	4,158.9	4,294.7	135.7	1.03
	④	4,342.9	4,387.8	45.0	1.01
	⑤	4,763.9	5,023.7	259.8	1.05
Figure3	②	4,290.2	4,440.2	150.1	1.03
	④	1,877.1	1,922.7	45.6	1.02
	⑥	1,498.6	1,520.0	21.4	1.01

3.2 Case study of the large network at the Tokyo'23 ward level

In this case, we calculate the number of links that run through the shortest path between every node, using Dijkstra's method on the extended network, to study the relationship between the 12 hours traffic volume which the links of the DRM contain on attributes and the number of links. We used the large network of the Tokyo'23 wards. The target areas are the southwest and south parts of Tokyo. Figure 5 shows a map of the area. In Table 3, the outline of the target area's network is summarized.



(Southwest part of Tokyo) (South part of Tokyo)

Figure 5: Results of numerical experiments 1

Table 3: Number of Links and nodes number in both networks.

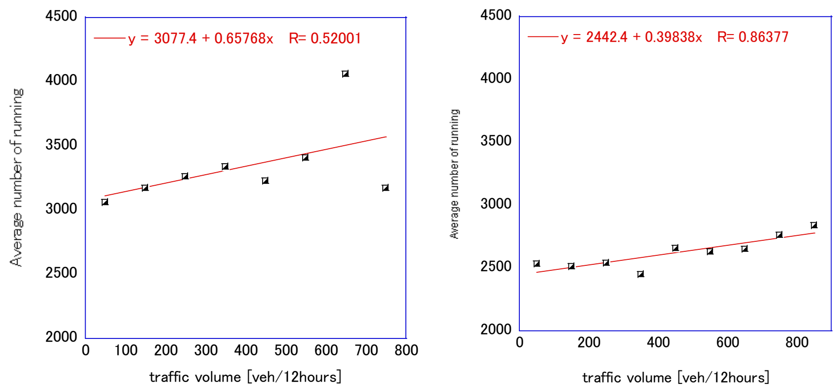
area		Original network	The extended network	Rate of increase
		(a)	(b)	(b)/(a)
Southwest part of Tokyo	Number of links	11,918	62,547	5.24
	Number of nodes	4,115	27,951	6.79
South part of Tokyo	Number of links	9,110	48,454	6.90
	Number of nodes	3,084	21,304	5.31

When we compare the southwest part of Tokyo in Table 3 with Table 1, the number of links increase 1.8 times and nodes 1.3 times. This is because the DRM data were originally in the form of a directed graph. It is clear that the network became more accurate. The rate of increase in nodes was about 6.0 as in the previous case. However, the rate of increase was different for the number of links. The results for Case 2 are twice those for Case 1.

Table 4: Outline of traffic volume and number of times used

area	Southwest part of Tokyo	South part of Tokyo
Average traffic volume [100veh./12 h]	287.9	290.4
Standard deviation of traffic volume	134.3	160.8
Average number of times links used	3251.3	2550.6
Standard deviation of number of times links used	990.4	702.3

We establish the average number of links that run through the shortest path for 50 vehicles. Figure 6 plots the relationship between the average number of links and traffic volume. It shows that as the average vehicle volume increased, the average number of links used increased.



(Southwest part of Tokyo) (South part of Tokyo)

Figure 6: Average use count and traffic volume at links

4. Conclusions

In this study, we developed the network extended model for the route finding problem considering the total angular turning to the left or right. We extracted a road network using Geographic Information Systems data and changed the network to the extended network. The parameters of segment length and angular turning are set for each link. In a case study of a small network at a city ward level, we confirmed that the parameters of the angular t influence the route search. From the results of the route map, our model tends to select arterial links depending on the setting of parameter t.

In the case study of a large network at Tokyo'23 ward level, we studied the relationship between the 24 hour link traffic volume from traffic census data and the number of links that the shortest path ran through. Our results show the selected path using our model tends to have a large volume of the use of traffic links. This implies that our model has the feature that it can find the arterial link if it does not use the link attributes.

Our study assists drivers in finding routes which are easy and safe. Thus it finds the arterial links in cases where the digital road map has no attributes at the links. This approach is useful for finding a route on the map of a developing country. Shimakawa (2013) has plans to use this model to make an analysis of the traffic flow in the Bangkok metropolitan area.

For future research, a multi-objective shortest path algorithm will be developed. The network extended model has the problem that becomes too large, despite its original network not being so. Using the algorithm, we can implement our approach on an original large scale network, which will reduce the computation load.

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Biography

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