Reducibility of Some Multi Criteria Scheduling Problems to Bicriteria Scheduling Problems.

Saheed Akande
Department of Mechanical and Mechatronics Engineering,
Afe Babalola University, Ado-Ekiti
Nigeria

Ayodeji.E. Oluleye
Department of Industrial and Production Engineering
University of Ibadan, Ibadan
Nigeria

E.O. Oyetunji
Department of Mechanical Engineering
Lagos State University, Lagos
Nigeria

Abstract

It is well known that optimal or near optimal polynomial time algorithms have been obtained for many single criterion and bicriteria scheduling problems while most multi criteria scheduling problems are NP-Hard. Thus, heuristics like simulated annealing, tabu search and genetic algorithm that produce 'good solutions' are usually employed in solving multi criteria problems. This paper explored the possibility of reducing multi criteria scheduling problems to single criterion or bicriteria scheduling problems to make them more amenable to optimal polynomial time algorithms. A total of four multi criteria scheduling problems were analyzed and reduced to their equivalent single criterion or bicriteria scheduling problems. It is expected that the solution for the reduced single criterion or bicriteria scheduling problems will also solved the parent multi-criteria problem without compromising its linear composite objective function. Validation was carried out by solving some multi criteria problem already solved by heuristics with an existing polynomial time algorithm that solves its equivalent single criteria or bi criteria problem and comparing the linear objective function of the two solution methods.

Keywords
Multi-criteria, reducibility, heuristic, bi-criteria, optimal

1. Introduction

Scheduling objectives are criteria by which the performance of any solution method can be measured (French, 1982). It is not easy to state scheduling objectives, this is because they are numerous, complex and often conflicting (Oyetunji, 2009).

Scheduling problems can be classified in terms of the number of objectives by which the performances of solution methods may be assessed (Oyetunji, 2011). This can be single criterion, bicriteria, or multi criteria scheduling problems. When considering only two criteria, it is called bicriteria scheduling problem (Ehrgott and Grandibleux, 2000).The simplest multi objective problems focus only on two criteria (Abdullah, 2010). Multi criteria scheduling problems deal with three or more objectives.

Since the introduction of scheduling theory in the 1950s, most research has been concentrated on single criterion scheduling problems. This is probably due to the fact that single criterion scheduling problems are simpler. However, in the real-life, multiple and conflicting criteria play a role (Hoogeveen, 2005). The problem of multi criteria have recently been explored by different researchers; For instance, Erne (2007) presented an integer programming model for multi criteria scheduling problem with sequencing dependent setup time to minimize the weighted sum of total completion time, maximum tardiness and maximum earliness. Abdullah (2010) studied the problem of scheduling jobs on a single machine to minimize total tardiness subject to maximum earliness or tardiness for each job. Also, Haidar and Tariq (2011) solved the single machine scheduling problem of minimizing maximum tardiness, maximum earliness and sum of square of completion time using a hierarchical method.

Most multi criteria scheduling problems are NP-Hard in nature (Farhad and Vahid, 2009). Therefore, heuristics methods like simulated annealing, tabu search, genetic algorithms are usually explored in solving multi criteria problems. For instance, Tamer (2007) solved a multi criteria scheduling problem with sequence-dependent setup times on a single machine using tabu search method. Also, Ashwani and Pankaj (2010) framed multi criteria decision making for flow shop scheduling with weighted sum of total tardiness, total earliness and makespan as performance objective using hybrid genetic algorithms. In contrast, some researchers have utilized existing optimal solution methods proposed for some single criterion problems to solve multi criteria scheduling problems (Oyetunji, 2010; Haidar and Tariq, 2011). In view of this, coupled with difficulties in solving multi criteria scheduling problems, it is desired to explore the possibility of reducing some multi criteria problems into either single criterion or bicriteria problems. This will make them amenable to polynomial time solution methods. This is the focus of this paper.

2. Formulation of Multi Criteria Scheduling Problems

Generally, scheduling problem with m criteria can be formulated using three parameters notation given by: $\alpha|\beta|\gamma$ (Lenstra, Rinnooy kan, ; and Garaham et al., 1979)
Where:
α is the number of machines, β is Job characteristics like pre-emption, releases date, due dates, precedence constraints etc. γ is the objective function.

However, multi-objectives scheduling problem is defined as (Branke et al., 2008; Deb, 2001):

$$Min/Max\ \{f_1\ (x), f_2\ (x), f_3\ (x) - - - - - - - - f_m(x)\} \quad (1)$$

Subject to $x \in S$
Where:
S is the set of feasible solutions, ***x is*** the decision vector, ***x*** = (x_1, x_2, x_3, -----------------, x_n,)
f_i is the performance objectives or criteria (*i* = 1,2, …, *m*); *m* is the number of performance objectives/criteria to be minimized or maximized.

There are basically three approaches by which multi criteria scheduling problems may be solved. These are: simultaneous, hierarchical, and pareto-optimal approaches. However, only simultaneous optimization approach will be briefly discussed in this paper because it is the basis of our work.

2.1 Simultaneous Optimization Approach to Multi Criteria Scheduling Problem.

In this method, the objectives are aggregated into a single objective function called the composite objective function (COF). There are two types of composite objective functions; the linear and general composite functions. (Hoogeveen, 2005).

The expression for linear composite objective function with respect to criteria X, Y and Z each carrying relative weights of α, β *and* γ respectively is given as:

$$\mathrm{F(X,Y)} = \alpha \mathrm{X} + \beta \mathrm{Y} + \gamma Z \quad (2)$$

Thus, the linear composite objective function (LCOF) consists of the sum of the relative weights of each objective multiplied by the value of each of the objectives. Mathematically;
Optimize $F(X) = a_1X_1 + a_2X_2 + a_3X_3 + \cdots + a_mX_m = \sum_{k=1}^{m} a_kX_k$
Such that $\sum_{k=1}^{m}(a_k) = 1 \quad 0 < a_k < 1$, $for\ all\ k = 1, 2, \dots m$
Where:
$a_1, a_2, \dots, a_m$ are relative weights of criteria $X_1, X_2, \dots, X_n$ to be optimized respectively.
F(X) is the linear composite function (LCOF) and *m* is number of the criteria to be optimized.

An example of this method is the simultaneous minimization of machining cost (cost associated with completion time), total earliness penalties and makespan in an unrelated parallel machine explored by Farhad and Vahid (2009). The problem was formulated as

$$Min \sum_{i=1}^{n} F_i(x) + C_{\max}(x) + \sum_{i=1}^{n} E_i(x) \qquad (3)$$

Similarly, Tamer (2007) considered a multi criteria scheduling problem with sequence-dependent setup times on a single machine. The objective function of the problem was formulated as simultaneous minimization of the weighted sum of total completion time, maximum tardiness and maximum earliness. This was given by:

$$Min \sum_{i=1}^{n} C_i(x) + T_{\max}(x) + E_{\max}(x). \qquad (4)$$

3. Reducibility of Multi Criteria Scheduling Problems

Parviz (2009) stated that the larger the number of performance objectives in scheduling problems, the higher the degree of computational complexity involved. In this regard, any effort towards reducing a multi-criteria scheduling problem to bicriteria or single criterion scheduling problem without compromising the composite objective function will enhance the chance of proposing an efficient algorithm to solve the problem. Therefore, there is a need to develop procedures for reducing multi criteria scheduling problems to either single criterion or bicriteria problems, so as to make them more amenable to solution methods.

4. Illustrative Examples

Some examples are discussed to illustrate how multi criteria scheduling problems can be reduced to bicriteria or single criterion problems. These examples are grouped into four classes:

a. Three criteria scheduling problem reduced to bicriteria or single criterion problem
b. Four Criteria scheduling problem reduced to bicriteria or single criterion problem
c. Six criteria scheduling problem reduced to bicriteria or single criterion problem
d. Eight Criteria scheduling problem reduced to bicriteria or single criterion problem

The following notations given below are employed in the illustrative examples.

D_j	= Due time of job J
P_J	= Processing time of job J
$\sum_{j=1}^{n} C_j$	= the total completion time
$\sum_{j=1}^{n} E_J$	= the total earliness
$\sum_{j=1}^{n} L_j$	= the total lateness
$\sum_{j=1}^{n} W_j$	= the total waiting time
$\sum_{j=1}^{n} F_J$	= the total flow time
N_E	= the number of early job
N_t	= the number of tardy job
C_{max}	= the makespan
I_M	= the machine idle time
U_M	= the machine utilization time
F_{max}	= the maximum flow time
$\propto, \beta, \gamma, \delta, \mu, \sigma, \tau, \rho,$	= relative weights associated with each of the required performance objectives.

4.1 Reduction of Three Criteria Scheduling Problems

Consider a multi-criteria scheduling problem in which the objective function is minimization of the weighted sum of machine idle time, total flow time and makespan. This problem was explored by Rajendran (1995). The problem was represented as:

$$\text{Min}\left[\propto I_M + \gamma \sum_{j=1}^{n} \mathrm{F_j} + \beta \mathrm{C_{max}}\right] \tag{5}$$

The problem can be reduced to bicriteria or single criterion problem as follows:
The machine idle time is defined as the difference between the makespan ($\mathrm{C_{max}}$) and the total processing time on the machine (French, 1982).

$$I_M = \mathrm{C_{max}} - \sum_{j=1}^{n} \mathrm{P_j} \tag{6}$$

Substitute equation (6) into equation (5), the problem (equation 5) thus becomes;

$$\text{Min}\left[\propto (\mathrm{C_{max}} - \sum_{j=1}^{n} \mathrm{P_j}) + \gamma \sum_{j=1}^{n} \mathrm{F_j} + \beta \mathrm{C_{max}}\right] \tag{7}$$

$$=\text{Min}\left[\propto \mathrm{C_{max}} + \beta \mathrm{C_{max}} + \gamma \sum_{j=1}^{n} \mathrm{F_j} +\right] - \propto \sum_{j=1}^{n} \mathrm{P_j} \tag{8}$$

$$=\text{Min}\left[(\propto + \beta)\mathrm{C_{max}} + \gamma \sum_{j=1}^{n} \mathrm{F_j} +\right] - \propto \sum_{j=1}^{n} \mathrm{P_j} \tag{9}$$

Since ∝, β, γ and$-\sum_{j=1}^{n} \mathrm{P_j}$ are constants, thus (∝+β) and $\propto\sum_{j=1}^{n} \mathrm{P_j}$ are also constants

Assume $\propto + \beta = \rho$ and $\propto \sum_{j=1}^{n} \mathrm{P_j} = K$, Thus, equation (5) becomes

$$\text{Min}\left[\rho \mathrm{C_{max}} + \gamma \sum_{j=1}^{n} \mathrm{F_j} +\right] - K \tag{10}$$

This is a bicriteria scheduling problem of minimizing the weighted sum of total flow time and makespan.

Similarly, the problem (equation 5) can also be reduced to bi-criteria scheduling problem with machine idle time and flow time as the criteria as follows:

From equation (6) , it can be deduced that makespan ($\mathrm{C_{max}}$) is given by the equation:

$$\mathrm{C_{max}} = I_M + \sum_{j=1}^{n} \mathrm{P_j} \tag{11}$$

Substitute equation (11) into equation (5), and obtain:

$$\text{Min}\left[\propto I_M + \gamma \sum_{j=1}^{n} \mathrm{F_J} + \beta(I_M + \sum_{j=1}^{n} \mathrm{P_J})\right] \tag{12}$$

$$= \text{Min}\left[\propto I_M + \beta I_M + \gamma \sum_{j=1}^{n} \mathrm{F_J} + \sum_{j=1}^{n} \mathrm{P_J})\right] \tag{13}$$

$$= \text{Min}\left[I_M(\propto + \beta) + \gamma \sum_{j=1}^{n} \mathrm{F_J})\right] + \beta \sum_{j=1}^{n} \mathrm{P_J} \tag{14}$$

Assume, $\propto + \beta = \rho$, and $\beta \sum_{j=1}^{n} \mathrm{P_J} = K_2$, the equation (14), thus becomes:

$$\text{Min}\left[\rho I_M + \gamma \sum_{j=1}^{n} \mathrm{F_J})\right] + K_2 \tag{15}$$

This is a bicriteria scheduling problem of minimizing the weighted sum of machine idle time and total flow time.

4.2 Reduction of Four Criteria Scheduling Problems

Consider a multi criteria scheduling problem that involves minimization of the following objective functions: total flow time, total earliness, total lateness and total tardiness. This problem was explored by Gur and Sarig (2008). Though, the total lateness cost was represented by due window cost in the work. The problem was represented as:

$$\text{Min}\left[\propto \sum_{j=1}^{n} T_j + \beta \sum_{j=1}^{n} F_J + \gamma \sum_{j=1}^{n} \mathrm{E_j} + \delta \sum_{j=1}^{n} \mathrm{L_j}\right] \quad (16)$$

This problem can be reduced to bicriteria scheduling problem as follows:

Lateness of a job is defined as the difference between the completion time and the due date of the job. Thus, the total lateness (L_{tot}) is the sum of all the lateness of all the jobs.

$$\text{Total lateness } (L_{tot}) = \sum_{j=1}^{n} \mathrm{L_j} = \sum_{j=1}^{n} (\mathrm{C_j} - \mathrm{d_j}) \quad (17)$$

Similarly, Total earliness (E_{tot}) is the difference between the due dates (d_i) and the completion time (C_i) of the job.

$$\text{Therefore, the total earliness } (E_{tot}) = \sum_{j=1}^{n} \mathrm{E_j} = \sum_{j=1}^{n} (\mathrm{d_j} - \mathrm{C_j}) \quad (18)$$

Compare equation (17) and (18), we can deduce that, $E_j = -L_j$

$$\text{Summing over, } n \text{ (number of jobs), thus, } \sum_{j=1}^{n} \mathrm{E_j} = -\sum_{j=1}^{n} \mathrm{L_j} \quad (19)$$

Substitute equation (19) into equation (16), the problem (equation (16)), then,

$$\left[\propto \sum_{j=1}^{n} T_j + \beta \sum_{j=1}^{n} F_J - \gamma \sum_{j=1}^{n} \mathrm{L_j} + \delta \sum_{j=1}^{n} \mathrm{L_j}\right] \quad (20)$$

$$=\left[\propto \sum_{j=1}^{n} T_j + \beta \sum_{j=1}^{n} F_J + (\delta - \gamma) \sum_{j=1}^{n} \mathrm{L_j}\right] \quad (21)$$

$$\text{Similarly, from equation (17), total lateness, } \sum_{j=1}^{n} \mathrm{L_j} = \sum_{j=1}^{n} \mathrm{C_j} - \sum_{j=1}^{n} \mathrm{d_j} \quad (22)$$

$$\text{Therefore, } \sum_{j=1}^{n} \mathrm{C_j} = \sum_{j=1}^{n} \mathrm{L_j} + \sum_{j=1}^{n} \mathrm{d_j} \quad (23)$$

Furthermore, the flow time is the time interval between the time a job is released to the shop and the time the processing of the job is completed. It is given as the difference between the completion time and the release date of job. The total flow time (F_{tot}) is the sum of all the flow times of all the jobs.

$$\text{Total Flow time } (\mathrm{F}_{tot}) = \sum_{j=1}^{n} F_j = \sum_{j=1}^{n} (\mathrm{C_j} - \mathrm{r_j}) = \sum_{j=1}^{n} \mathrm{C_j} - \sum_{j=1}^{n} \mathrm{r_j} \quad (24)$$

$$\sum_{j=1}^{n} \mathrm{C_j} = \sum_{j=1}^{n} \mathrm{F_j} + \sum_{j=1}^{n} \mathrm{r_j} \quad (25)$$

Compare equation (23) and (25), this gives

$$\sum_{j=1}^{n} \mathrm{L_j} + \sum_{j=1}^{n} \mathrm{d_j} = \sum_{j=1}^{n} \mathrm{F_j} + \sum_{j=1}^{n} \mathrm{r_j} \quad (26)$$

Therefore

$$\sum_{j=1}^{n} \mathrm{L_j} = \sum_{j=1}^{n} \mathrm{F_j} + \sum_{j=1}^{n} \mathrm{r_j} - \sum_{j=1}^{n} \mathrm{d_j} \tag{27}$$

$$\sum_{j=1}^{n} \mathrm{F_j} = \sum_{j=1}^{n} \mathrm{L_j} + \sum_{j=1}^{n} \mathrm{d_j} - \sum_{j=1}^{n} \mathrm{r_j} \tag{28}$$

Substitute equation (22) into equation (16), this gives

$$\left[\propto \textstyle\sum_{j=1}^{n} T_j + \beta \sum_{j=1}^{n} F_J + (\delta - \gamma)(\sum_{j=1}^{n} \mathrm{F_j} + \sum_{j=1}^{n} \mathrm{r_j} - \sum_{j=1}^{n} \mathrm{d_j})\right] \tag{29}$$

$$= \left[\propto \textstyle\sum_{j=1}^{n} T_j + \beta \sum_{j=1}^{n} F_J + (\delta - \gamma)(\sum_{j=1}^{n} \mathrm{F_j})\right] + \left[(\delta - \gamma)(\sum_{j=1}^{n} \mathrm{r_j} - \sum_{j=1}^{n} \mathrm{d_j})\right] \tag{30}$$

$$= \left[\propto \textstyle\sum_{j=1}^{n} T_j + (\beta + \delta - \gamma)(\sum_{j=1}^{n} \mathrm{F_j})\right] + \left[(\delta - \gamma)(\sum_{j=1}^{n} \mathrm{r_j} - \sum_{j=1}^{n} \mathrm{d_j})\right] \tag{31}$$

Since $\propto$, $\beta, \sum_{j=1}^{n} \mathrm{r_j}$, $and \sum_{j=1}^{n} \mathrm{d_j}$ are constants, it follows that $(\delta - \gamma)$, $(\beta + \delta - \gamma)$, $(\sum_{j=1}^{n} \mathrm{r_j} - \sum_{j=1}^{n} \mathrm{d_j})$ and $(\delta - \gamma)(\sum_{j=1}^{n} \mathrm{r_j} - \sum_{j=1}^{n} \mathrm{d_j})$ are also constants.

Thus, assuming, $\delta - \gamma = K_1$, $\beta + \delta - \gamma = K_2$, and $(\delta - \gamma)(\sum_{j=1}^{n} \mathrm{r_j} - \sum_{j=1}^{n} \mathrm{d_j}) = \mathrm{K_3}$
the equation (31) becomes:

$$\left[\propto \textstyle\sum_{j=1}^{n} T_j + (K_2)(\sum_{j=1}^{n} \mathrm{F_j} +\right] + K_3 \tag{32}$$

This is a bicriteria scheduling problem of minimizing weighted sum of the total tardiness and total flow time.

Similarly, if equation (28) is substituted into equation (21), the problem thus becomes:

$$\left[\propto \textstyle\sum_{j=1}^{n} T_j + \beta(\sum_{j=1}^{n} \mathrm{L_j} + \sum_{j=1}^{n} \mathrm{d_j} - \sum_{j=1}^{n} \mathrm{r_j}) + (\delta - \gamma) \sum_{j=1}^{n} \mathrm{L_j}\right] \tag{32}$$

$$= \left[\propto \textstyle\sum_{j=1}^{n} T_j + \beta \sum_{j=1}^{n} \mathrm{L_j} + (\delta - \gamma) \sum_{j=1}^{n} \mathrm{L_j} + \beta \sum_{j=1}^{n} \mathrm{d_j} - \beta \sum_{j=1}^{n} \mathrm{r_j}\right] \tag{33}$$

$$= \left[\propto \textstyle\sum_{j=1}^{n} T_j + (\beta + \delta - \gamma) \sum_{j=1}^{n} \mathrm{L_j}\right] + \beta \sum_{j=1}^{n} \mathrm{d_j} - \beta \sum_{j=1}^{n} \mathrm{r_j} \tag{34}$$

Assume $\beta + \delta - \gamma = K_4$ and $\beta \sum_{j=1}^{n} \mathrm{d_j} - \beta \sum_{j=1}^{n} \mathrm{r_j} = \mathrm{K_5}$, equation (34) becomes

$$= \left[\propto \textstyle\sum_{j=1}^{n} T_j + (K_4) \sum_{j=1}^{n} \mathrm{L_j}\right] + \mathrm{K_5} \tag{35}$$

This is a bicriteria scheduling problem of minimizing the weighted sum of the total tardiness and total lateness

4.3 Reduction of Six Criteria Scheduling Problems

Consider a flexible manufacturing system in which all parts to be assembled are available at the same time and required to schedule the jobs in order to optimize the following performance objectives: number of early jobs, number of tardy jobs, and maximum flow time, makespan, machine idle time and machine utilization. The problem is represented as:

$$\text{Min } [\propto N_E \quad + \beta N_T + \gamma F_{max} + \ \delta C_{max} + \mu I_m + \rho U_m] \qquad (36)$$

The problem can be reduced to bi criteria problem as follows:
A job is said to be early if it completes before its due date. Scheduling criteria based on number of early jobs are listed below.

$$\text{Let } U_i = \begin{cases} 1 \ ; if \ C_i > d_i \\ 0 \ otherwise \end{cases}$$

$$\text{Number of early jobs (NE)} = \sum_{i=1}^{n}(1 - U_i) \qquad (37)$$

Similarly, a job is said to be late or tardy if it completes after its due date. Scheduling criteria based on number of tardy jobs are listed below.

$$\text{Let } U_i = \begin{cases} 1 \ ; if \ C_i > d_i \\ 0 \ otherwise \end{cases}$$

$$\text{Number of tardy jobs } (\mathrm{N_T}) = \sum_{i=1}^{n}(U_i) \qquad (38)$$

The total number of jobs in any given problem equals number of early jobs plus number of tardy jobs

$$N_E + \ \ N_T \ = \mathrm{N} \qquad (39)$$

$$\text{This gives; } N_T = N - \ \ N_E \qquad (40)$$

Similarly, from equation (6),

$$I_M = \mathrm{C_{max}} - \sum_{j=1}^{n} \mathrm{P_j} \qquad (6)$$

Also, according to Charles-Owaba (2002), $U_m \ + I_m \ = 1$ (41)

$$U_m \ = \ \ 1 - I_m \qquad (42)$$

Substituting equation (6) into equation (42), gives;

$$\mathrm{U_m} = 1 - \mathrm{C_{max}} + \sum_{j=1}^{n} \mathrm{P_j} \qquad (43)$$

In a system where the ready time of all jobs are equal, $\mathrm{F_{max}} = \ \mathrm{C_{max}}$ (44)

Substituting equations (6), (40), (43) and (44) into equation (36), gives

$$\text{Min } \left[\propto N_E + \beta(N - \ \ N_E) \ + \ \gamma C_{max} + \ \delta C_{max} + \mu(\mathrm{C_{max}} - \sum_{j=1}^{n} \mathrm{P_j}) + \rho(1 - \mathrm{C_{max}} + \sum_{j=1}^{n} \mathrm{P_j})\right] \qquad (45)$$

$$\text{Min } \left[(\propto -\beta)N_E + \beta N + (\gamma + \delta)C_{max} + \mu \mathrm{C_{max}} - \mu \sum_{j=1}^{n} \mathrm{P_j} + \ \rho - \rho \mathrm{C_{max}} + \rho \sum_{j=1}^{n} \mathrm{P_j}\right] \qquad (46)$$

$$\text{Min } \left[(\propto -\beta)N_E + (\gamma + \delta + \ \mu - \rho)C_{max} + \ \rho + \beta N + (\rho - \mu) \sum_{j=1}^{n} \mathrm{P_j}\right] \qquad (47)$$

$$\text{Min } \left[(\propto -\beta)N_E + (\gamma + \delta + \ \mu - \rho) \ C_{max} + \ \rho + \beta N + (\rho - \mu) \sum_{j=1}^{n} \mathrm{P_j}\right] \qquad (48)$$

Assuming $K_1 = \propto -\beta$; $K_2 = \gamma + \delta + \ \mu - \rho$; $K_3 = \rho + \beta N + (\rho - \mu) \sum_{j=1}^{n} \mathrm{P_j}$, This gives;

$$\text{Min } (K_1 N_E + K_2 C_{max}) + K_3 \qquad (49)$$

This is a bicriteria scheduling problem of minimizing the weighted sum of the makespan and number of early job

4.4 Reduction of Eight Criteria Scheduling Problems

Consider a flexible manufacturing system in which all parts to be assembled are available at the same time and is required to schedule jobs or parts in order to optimize multi criteria objective functions based inventory and resources utilization. The following objective functions are considered under each category

i. Inventory : Number of jobs actually being processed at time t, N_p, Number of job waiting for processing at time t, N_w, Number of jobs completed at time t N_c

ii. Resources utilization: Makespan, maximum flow time, machine idle time, machine utilization

The problem can be represented as:

$$\text{Min}\left[\alpha N_p + \beta N_w + \gamma N_c + \mu F_{max} + \rho C_{max} + \sigma I_m + \tau U_m\right] \tag{50}$$

The problem can be reduced to bi criteria as follows:

$$\text{It is known that (French, 1982); } N_p + N_w + N_c = N \tag{51}$$

Where N is the total number of jobs.

$$\text{Therefore, } \alpha N_p + \beta N_w + \gamma N_c = KN \tag{52}$$

If we assume that the weights of $N_p and\ N_w$ are equal. This is realistic as both jobs are expected at either in storage or around the workshop. $\alpha = \beta = K_1$. Thus, equation (52) becomes;

$$\text{Thus, } \alpha N_p + \beta N_w = K_1(N_p + N_w) \tag{53}$$

$$\alpha N_p + \beta N_w + \gamma N_c = KN \tag{54}$$

$$K_1(N_p + N_w) + \gamma N_c = KN \tag{55}$$

$$K_1\left(N_p + N_w\right) = KN - \gamma N_c \tag{56}$$

$$\text{From equation (44), } C_{max} = F_{max}\ (since\ r = 0)$$

$$\text{Also, from equation (6) } I_M = \mathrm{C_{max}} - \textstyle\sum_{j=1}^{n} \mathrm{P_j}$$

Substitute equation (44) into equation (6), this gives

$$I_M = \mathrm{F_{max}} - \textstyle\sum_{j=1}^{n} \mathrm{P_j} \tag{57}$$

$$\text{Also from equation (43), } U_m = 1 - \mathrm{C_{max}} - \textstyle\sum_{j=1}^{n} \mathrm{P_j}$$

Substitute equation (44) into equation (43), this gives

$$U_m = 1 - \mathrm{F_{max}} - \textstyle\sum_{j=1}^{n} \mathrm{P_j} \tag{58}$$

Substitute equations (56), (57), and (58) into equation (50), this gives

$$\text{Min}\left[KN - \gamma N_c + \mu F_{max} + \rho F_{max} + \sigma(\mathrm{F_{max}} - \textstyle\sum_{j=1}^{n} \mathrm{P}_j) + \tau(1 - \mathrm{F_{max}} - \sum_{j=1}^{n} P_{\mathrm{j}})\right] \tag{59}$$

$$\text{Min}\left[-\gamma N_c + \mu F_{max} + \rho F_{max} + \sigma \mathrm{F_{max}}) + \tau \mathrm{F_{max}})\right] + KN - \sigma \textstyle\sum_{j=1}^{n} P_{\mathrm{j}} + \tau - \tau \sum_{j=1}^{n} P_{\mathrm{j}} \tag{60}$$

$$\text{Min } [-\gamma N_c + (\mu+\rho+\sigma+\tau)\,\mathrm{F}_{\max})] + K_N - \sigma\sum_{j=1}^{n} P_{\mathrm{j}} + \tau - \tau\sum_{j=1}^{n} P_{\mathrm{j}} \qquad (61)$$

$$\mu+\rho+\sigma+\tau = \mathrm{K}_2,\ \sigma\sum_{j=1}^{n} P_{\mathrm{j}} + \tau - \tau\sum_{j=1}^{n} P_{\mathrm{j}} = K_3$$

The problem thus becomes

$$\text{Min } [-\gamma N_c + (\mathrm{K}_2\mathrm{F}_{\max})] + KN - K_3 \qquad (62)$$

This is a bicriteria criteria scheduling problem of minimizing the number of completed job at time t and the maximum flow time with all the jobs released at time zero.

5.0 Conclusion

It is a known fact that the complexities of scheduling problems grows as the number of criteria to be optimized increase. Also,it is known that optimal or near-optimal solution methods exist for a number of single criterion or bicriteria scheduling problems. In this paper we have explored the possibility of reducing multi criteria scheduling problems to their equivalent bicriteria scheduling problems. To illustrate this, four multi criteria problems have been reduced to equivalent bicriteria problems.

One of the consequences of this work is that if solution methods can be found to the reduced problem, then these can be applied to the original multi criteria problem. Work is in progress to explore this thrust.

Acknowledgements

The authors are grateful to all that support this research work. Special thanks to Afe Babalola University, Ado-Ekiti, Ekiti state for their support, the University of Ibadan Post graduate School and the Department of Industrial and Production Engineering, University of Ibadan. Also, we thank the Lagos state university, faculty of Engineering for their immense support and contribution.

REFRENCES

Abdullah, H.F., Multi criteria Scheduling Problems to Minimize Total Tardiness Subject to Maximum Earliness or Tardiness, Ibn Al- Haitham Journal For Pure and Applied Science Vol. 23, No. 1, pp. 311-320 , 2010.

Ashwani, D., and Pankaj, C., Hybrid Genetic Algorithm for Multi criteria Scheduling with Sequence Dependent Set up Time, International Journal of Engineering,Vol. 3, no. 5, pp. 510-520. 2009.

Bhaba, R., Sarker, Junfang, Y., Deniz, M., Mohammad A., Rahman, and Sultana P., Pareto-optimal solution of a scheduling problem on a single machine with periodic maintenance and non-pre-emptive jobs. Proceedings of the International Conference on Mechanical Engineering. Bangladesh. pp. 29-31, 2007.

Branke, J., Belton, V., Eskelinen, P., Greco, S., Molina, J., Ruiz, F. and Slowinski, R., Multiobjective optimization: interactive and evolutionary approaches, pp. 405-434. 2008.

Charles- Owaba, O. E., Organisation Design: A quantitative Approach. Oputuru Books, Nigeria, 2012.

Deb, K., Multi-Objective Optimization Using Evolutionary Algorithms. Wiley, 2001.

Eren, T., A Multi criteria Scheduling with Sequence-Dependent Setup Times. Journal of Applied Mathematical Sciences, vol. 1, no. 58, pp. 2883 – 2894, 2007.

French, S., Sequencing and Scheduling. Ellis Horwood Limited, 1982.

Farhad, K., and Vahid K., A Heuristic Algorithm Approach for Scheduling of Multi- criteria Unrelated Parallel Machines. World Academy of Science, Engineering and Technology. vol. 59, pp. 253, 2009.

Graham, R.L., Lawler, E.L., Lenstra, J.K., Rinnooy Kan, A.H.G., Optimization and approximation in deterministic sequencing and scheduling: a survey. Ann. Discrete Math. vol. 5, pp. 287-326, 1979.

Haidar, Y.K., and Tariq, S.A., Single machine scheduling to minimize three hierarchically Criteria.Journal of Basrah Researches Sciences Vol. 37. No 4.pp263-271 2011.

Hoogeveen, J.A., Multi criteria Scheduling. European Journal of Operational research, Vol.167, no. 3, pp. 592-623, 2005.

Oladokun, V., and Owaba O.E., The Sequence Dependent Machine Set-Up Problem: A Brief Overview. The Pacific Journal of Science and Technology. Vol. 9 no 1, pp 36-44. 2008.

Oyetunji, E.O., Some Common Performance Measures in Scheduling Problems. Research Journal of Applied Sciences, Engineering and Technology Vol.1, no. 2, pp. 6-9, 2009.

Oyetunji, E.O. Truncation and Composition of Schedules: A Good Strategy for Solving Bicriteria

Scheduling Problems, American Journal of Scientific and Industrial Research, Vol. 1, no. 3, pp. 427-434, 2010.

Oyetunji, E.O., and Oluleye, A.E., Heuristic for Minimizing Total Completion Time on Single Machine with Job Release Date. Proceeding of NIIE Conference. 2009.

Oyetunji, E.O., Assessing solution methods to mixed multiobjectives scheduling problems International. Journal of. Industrial and Systems Engineering, Vol. 9, no.2, 2011.

Parviz, F., A Hybrid Multi Objective Algorithm for Flexible Job Shop Scheduling. World Academy of Science, Engineering and Technology, Vol. 50, pp 551-556, 2009.

Pinedo, M., Scheduling: Theory, Algorithms and Systems. Prentice Hall, New Jersey, second edition 2002.

Rajendran, C., Theory and methodology heuristics for scheduling in flow shop with multiple objectives. European Journal of Operation Research, Vol. 82, pp. 540 −555, 1995.

BIOGRAPHY

Akande Saheed is a Lecturer (II) in Afe Babalola University, Ado-Ekiti, Department of Mechanical Engineering. He earned B.Sc. in Metallurgical and Materials Engineering from Obafemi Awolowo University, Ile-ife, Nigeria. Masters in Industrial and Production Engineering, University of Ibadan. Nigeria. He is currently a Ph.D candidate in the Department of Industrial Engineering from University of Ibadan, Nigeria. He has published some journal and conference papers. His research interests include scheduling, manufacturing, simulation, optimization.

Aydeji E. Oluleye is a Professor in the University of Ibadan's Department of Industrial and Production Engineering. He earned B.Sc (Hons.) in Agricultural Engineering from University of Ibadan, Nigeria. Master in Agricultural Machinery Engineering, from Cranfield Institute of Technology, England and Ph.D in Industrial Engineering, University of Ibadan. His research interest includes production scheduling/operations management (optimization), and algorithm design, engineering economics amongst, others. He has published extensively in prestigious journals. He is a Registered Engineer (COREN) and a Member of the Nigeria Society of Engineers (NSE), and a Fellow of the Nigerian Institute of Industrial Engineering (NIIE).

E.O. Oyetunji is currently a Professor of Industrial and Production Engineering in the Department of Mechanical Engineering, Faculty of Engineering, Lagos State University, Epe Campus, Lagos, Nigeria. He graduated with a BSc (Hons) in Electrical Engineering from the University of Ilorin, Ilorin, Nigeria. He has an MSc and PhD in Industrial Engineering from the University of Ibadan, Ibadan, Nigeria. He has attended numerous national and international conferences. His research interest includes production scheduling/operations management (optimization), and algorithm design amongst others. He has published extensively in prestigious local and international journals. He is a Registered Engineer (COREN) and a Member of the following professional bodies: Nigeria Society of Engineers (NSE), Nigeria Institute of Industrial Engineering (NIIE),