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Application of ELECTRE Method for Sub-Contractor Selection using Interval-Valued Fuzzy Sets - Case Study

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Abstract

Sub-contractors selection is a multi-criteria decision making (MCDM) problem, in which some conflicted quantitative and qualitative factors are influencing for making decisions. While selecting sub-contractors is one of the most important issues in general contractor organizations, selecting the right sub-contractors becomes more difficult if accurate input information is not achievable. According to the above mentioned, defining appropriate criteria and applying a practical method would be a vital stage in project management. Therefore, in this paper, the ELECTRE method, developed based on the interval – valued fuzzy sets, has been applied for selecting sub-contractors in general contractor organizations. An experimental problem, based using six pre-defined criteria of decision making and three alternatives, has been defined whereas three alternatives are considered to satisfy the project managers. Eventually results have been evaluated comparing to local experts' points of view.

Keywords: Project Management, Sub-contractor Selection, Multi-Criteria Decision Making, Interval-valued fuzzy Set, ELECTRE.

1. Introduction

Project selection is a process of selecting one or more promising projects considering real limitations in order to satisfy corporate goals or objectives. In general, project selection is limited by restricted resources such as capital research talent, laboratory space, equipments, etc. In governmental dimensions, decision makers are often facing with multiphase projects in which each phase requires especial knowledge and abilities. Project management consists of general and technical skills corresponding to the role of management (Sebastian, 2007). In the real world, the general contractor of Engineering Procurement Construction (EPC) projects hires specialized sub-contractors to perform all or portions of the construction work so selecting an appropriate sub-contractor would be a vital concern. In the process of selecting sub-contractors, most criteria are needed to be defined as well as the best method of analyzing is required to be applied (Hamilton, 2004). In construction industries, the performances of sub-contractors are crucial factors in their awards of a new job by general contractor. Chien et al. (Chien et al., 2007) proposed a Sub-contractor Performance Evaluation Model (SPEM) using Evolutionary Fuzzy Neural Inference Model. The effectiveness of their proposed model has been validated by performing in a case study and results show that the proposed method accurately measures sub-contractor's performance. A web-based sub-contractor evaluation system called WEBSES has also been proposed to evaluate Sub-Contractors (SCs) based on a combined criterion such as quality of production, efficiency, employment of qualified members, reputation of the company, accessibility to the company, completion of the work on time (Gokhan et al., 2008). The proposed system enables general contractor to select the most appropriate SCs for their relevant tasks, speed up the selection process and gain time and cost savings during the bidding process.

A wide range of methodologies are being used for analytical process of criteria defining numerical measures. But in the practice, there are some criteria, which cannot be defined by numeric aspects, so using linguistic variables would be implemented by fuzzy set approaches (Vahdani, et al, 2009, 2010). The well-known approach of Multi Criteria Decision Making (MCDM) provides an effective framework based on the evaluation of multiple conflict criteria for comparison while many concrete problems can be represented by several (conflicting) criteria (Triantaphyllou, 2000). Due to lack of information regarding to attributes, the classical MCDM methods cannot effectively be applied in the problem solving. Hence in the both deterministic and random process, it is a wise decision to tend to be less effective in conveying the imprecision and fuzziness characteristics. The concept of defining linguistic variables was firstly proposed as fuzzy set theory by Zadeh (Zadeh, 1965, 1975) and applied later as a powerful tool to handle imprecise data and fuzzy expressions.

At the first time, interval valued fuzzy sets were suggested by (Gorzlczany, 1987) and (Turksen, 1996). They were later defined as interval valued fuzzy numbers (IVFN) and used in the relevant extended operations (Wnag and Li, 1998). More descriptions about that the presentation of the linguistic expression in the form of ordinary fuzzy sets is not clear enough have been later discussed by (Karnik, 2001) and (Cornelis et al., 2006). Interval-valued fuzzy sets have been widely used in real-world applications for instance, in a CLINAID system (Kohout and Bandler, 1998), thyrodian pathology (Sambuc, et al., 1975), approximate reasoning (Gorzlczany, 1975), (Bustine, 1994), and eventually in interval valued logic and preference modeling (Turksen, 1986). Based on definition of interval-valued fuzzy set in (Gorzlczany, 1987), an interval-valued fuzzy set is given by:

$$
\begin{aligned}
&A = \left\{\left(x, \left[\mu_A^L(x), \mu_A^U(x)\right]\right)\right\} \\
&\mu_A^L, \mu_A^U : X \to [0,1] \qquad \forall x \in X, \qquad \mu_A^L \le \mu_A^U \\
&\overline{\mu}_A(x) = [\mu_A^L(x), \mu_A^U(x)] \\
&A = \left\{\left(x, \overline{\mu}_A(x)\right)\right\}, x \in (-\infty, +\infty)
\end{aligned}
\tag{1}
$$

Where, $\mu_A^L(x)$ and $\mu_A^U(x)$ are respectively the lower and upper limits of degree of membership.

In fuzzy sets theory it is often difficult for an expert to quantify his or her opinion as a number in interval [0, 1]. Therefore, it is more suitable to represent this degree of certainty by another method of analyzing. Because the presentation of a linguistic expression in the form of fuzzy sets is not always enough (Sambuc, et al., 1975 & Grattan, 1975), the main contribution in this paper lies on the application of interval-valued fuzzy (Vahdani, et al, 2009, 2010) in order to select sub-contractor. After this section, a brief discussion is proposed regarding to the concept behind interval-valued fuzzy, followed by presenting characteristics of case study and applying the methodology. Results are being discussed in the last section where further researches are conducted to do future studies.

2. The Interval-Valued Fuzzy Set and ELECTRE

The concept of linguistic variable is very useful in dealing with situations that are too complex or ill-defined to be reasonably described in conventional quantitative expressions (Zadeh, 1975). These linguistic variables can be converted to triangular interval-valued fuzzy numbers for ratings and the importance of each criterion as depicted in Table 1 (Vahdani, et al, 2009, 2010).

Table 1: Definitions of linguistic variables

For the ratings		For the importance of each criterion	
Very Poor (VP)	[(0,0);0;(1,1.5)]	Very Low (VL)	[(0,0);0;(0.1,0.15)]
Poor (P)	[(0,0.5);1;(2.5,3.5)]	Low (L)	[(0,0.05);0.1;(0.25,0.35)]
Moderately Poor (MP)	[(0,1.5);3;(4.5,5.5)]	Medium Low (ML)	[(0,0.15);0.3;(0.45,0.55)]
Fair (F)	[(2.5,3.5);5;(6.5,7.5)]	Medium (M)	[(0.25,0.35);0.5;(0.65,0.75)]
Moderately Good (MG)	[(4.5,5.5);7;(8,9.5)]	Medium High (MH)	[(0.45,0.55);0.7;(0.8,0.95)]
Good (G)	[(5.5,7.5);9;(9.5,10)]	High (H)	[(0.55,0.75);0.9;(0.95,1)]
Very Good (VG)	[(8.5,9.5);10;(10,10)]	Very High (VH)	[(0.85,0.95);1;(1,1)]

Let $\widetilde{X} = [\widetilde{x}_{ij}]_{n\times m}$ be a fuzzy decision matrix for a multi criteria decision making problem in which $A_1, A_2, ..., A_m$ are n possible alternatives and $C_1, C_2, ..., C_m$ are m criteria. So the performance of alternative A_j with respect to criterion C_i is denoted as $\widetilde{x}_{ij}$. As illustrated in Fig. 1, $\widetilde{x}_{ij}$ and $\widetilde{w}_i$ are expressed by triangular interval valued fuzzy numbers (Vahdani, et al, 2009, 2010).

$$\widetilde{x} = \begin{cases} (x_1, x_2, x_3) \\ (x_1', x_2, x_3') \end{cases}$$

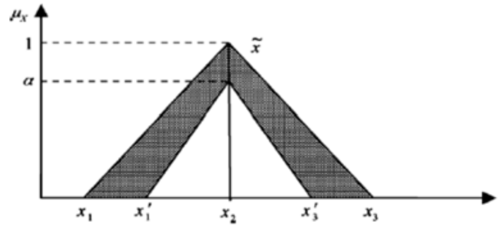

Figure 1: Interval valued triangular fuzzy number.

Where, $\widetilde{x}$ is demonstrated as $\widetilde{x} = [(x_1, x_1'); x_2; (x_3', x_3)]$. In a group decision environment with K experts, the importance of the criteria and the rating of alternatives with respect to each criterion can be calculated as (Vahdani, et al, 2009, 2010):

$$\widetilde{x}_{ij} = \frac{1}{K}[\widetilde{x}_{ij}^1 + \widetilde{x}_{ij}^2 + ... + \widetilde{x}_{ij}^k] \quad (2)$$

$$\widetilde{w}_{ij} = \frac{1}{K}[\widetilde{w}_{ij}^1 + \widetilde{w}_{ij}^2 + ... + \widetilde{w}_{ij}^k] \quad (3)$$

Equations (2) and (3) represent the average values of $\widetilde{x}_{ij}$ and $\widetilde{w}_{ij}$ denoted by experts. Output variable is also an interval value fuzzy number. In this case, the proposed approach to develop the ELECTRE for interval valued fuzzy data (IVF- ELECTRE) can be defined as follows (Vahdani, et al., 2010).:

Step 1: Given $\widetilde{x}_{ij} = [(a_{ij}, a_{ij}'); b_{ij}; (c_{ij}', c_{ij})]$, the normalized performance Rating can be calculated as:

$$\widetilde{n}_{ij} = \left[\left(\frac{a_{ij}}{c_j^+}, \frac{a_{ij}'}{c_j^+}\right); \frac{b_{ij}}{c_j^+}; \left(\frac{c_{ij}'}{c_j^+}, \frac{c_{ij}}{c_j^+}\right)\right], \quad i = 1, ..., n, \quad j \in \Omega_b \quad (4)$$

$$\widetilde{n}_{ij} = \left[\left(\frac{a_j^-}{c_{ij}'}, \frac{a_j^-}{c_{ij}}\right); \frac{a_j^-}{b_{ij}}; \left(\frac{a_j^-}{a_{ij}'}, \frac{a_j^-}{a_{ij}}\right)\right], \quad i = 1, ..., n, \quad j \in \Omega_c \quad (5)$$

Where, $c_j^+ = \underset{i}{\mathrm{Max}} c_{ij}, \quad j \in \Omega_b \; a_j^- = \underset{i}{\mathrm{Min}} a_{ij}, \quad j \in \Omega_c$.

Hence, the normalized matrix $\widetilde{N} = [\widetilde{n}_{ij}]_{n\times m}$ can be obtained. The above mentioned normalization method is to preserve the property that the ranges of normalized interval numbers fall within the closed interval [0, 1].

Step 2: Determine the weighted normalized matrix

By considering the different importance of each criterion, the weighted normalized fuzzy decision matrix is constructed using equations (6) and (7) and according to definition 1, the multiply operator can be applied as (8):

$$\widetilde{V} = [\widetilde{v}_{ij}]_{n\times m} \quad (6)$$

$$\widetilde{v}_{ij} = \widetilde{w}_j \times \widetilde{n}_{ij}. \quad (7)$$

$$\tilde{v}_{ij} = \left[\left(\tilde{n}_{1ij} \times \tilde{w}_{1j}, \tilde{n}'_{1ij} \times \tilde{w}'_{1j}\right); \tilde{n}_{2ij} \times \tilde{w}_{2j}; \left[\tilde{n}'_{3ij} \times \tilde{w}'_{3j}; \tilde{n}_{3ij} \times \tilde{w}_{3j}\right]\right] \tag{8}$$

Step 3: Specify concordance and discordance interval valued fuzzy sets

Specify concordance and discordance interval valued fuzzy sets for each interval valued fuzzy pairs of *k* and *l* alternatives $k,l = 1,2,...,m; l \neq k$. The set of available interval valued fuzzy indicators ($J = \{ j \mid j = 1,2,...,n\}$) is divided into two different sets of concordance interval valued fuzzy set $\left(\tilde{S}_{kl}\right)$ and discordance interval valued fuzzy set $\left(\tilde{D}_{kl}\right)$. Concordance set $\left(\tilde{S}_{kl}\right)$ of alternatives A_k and A_l, consists of all indicators which for each of them A_k is superior to A_l. Namely:

$$\tilde{S}_{kl} = \{ j \mid \left[\left(\tilde{v}_{1kj}, \tilde{v}'_{1kj}\right); \tilde{v}_{2kj}; \left(\tilde{v}'_{3kj}, \tilde{v}_{3kj}\right)\right] \geq \left[\left(\tilde{v}_{1lj}, \tilde{v}'_{1lj}\right); \tilde{v}_{2lj}; \left(\tilde{v}'_{3lj}, \tilde{v}_{3lj}\right)\right]\} \tag{9}$$

Vice versa the complementary subset named interval valued fuzzy discordance set is a set of indicators that for each of them we have:

$$\tilde{D}_{kl} = \{ j \mid \left[\left(\tilde{v}_{1kj}, \tilde{v}'_{1kj}\right); \tilde{v}_{2kj}; \left(\tilde{v}'_{3kj}, \tilde{v}_{3kj}\right)\right] < \left[\left(\tilde{v}_{1lj}, \tilde{v}'_{1lj}\right); \tilde{v}_{2lj}; \left(\tilde{v}'_{3lj}, \tilde{v}_{3lj}\right)\right]\} = J - \tilde{S}_{kl} \tag{10}$$

(The interval $\left[\left(\tilde{v}_{1ij}, \tilde{v}'_{1ij}\right); \tilde{v}_{2ij}; \left(\tilde{v}'_{3ij}, \tilde{v}_{3ij}\right)\right]$ is given with positive utility).

Step 4: Calculate the concordance interval valued fuzzy matrix

Possible values of concordance interval valued fuzzy set are measured by means of available interval valued fuzzy weights of concordance indexes which exist in that set. In other words, concordance interval valued fuzzy index is equivalent to the sum of interval valued fuzzy weights $\left[\left(\tilde{w}_{1j}, \tilde{w}'_{1j}\right); \tilde{w}_{2j}; \left(\tilde{w}'_{3j}, \tilde{w}_{3j}\right)\right]$ for those indexes which form the set $\left(\tilde{S}_{kl}\right)$. Thus concordance interval valued fuzzy index $\left[\left(\tilde{I}_{1k,l}, \tilde{I}'_{1k,l}\right); \tilde{I}_{2k,l}; \left(\tilde{I}'_{3k,l}, \tilde{I}_{3k,l}\right)\right]$ between A_l, A_k is as follow:

$$\tilde{I}_{k,l} = \left[\left(\tilde{I}_{1k,l}, \tilde{I}'_{1k,l}\right); \tilde{I}_{2k,l}; \left(\tilde{I}'_{3k,l}, \tilde{I}_{3k,l}\right)\right] = \sum_{j \in S_{k,l}} \left[\left(\tilde{w}_{1j}, \tilde{w}'_{1j}\right); \tilde{w}_{2j}; \left(\tilde{w}'_{3j}, \tilde{w}_{3j}\right)\right] \tag{11}$$

Concordance interval valued fuzzy index $\left[\left(\tilde{I}_{1k,l}, \tilde{I}'_{1k,l}\right); \tilde{I}_{2k,l}; \left(\tilde{I}'_{3k,l}, \tilde{I}_{3k,l}\right)\right]$ is reflective of relative significance of A_k with respect to A_l. The high value of $\left[\left(\tilde{I}_{1k,l}, \tilde{I}'_{1k,l}\right); \tilde{I}_{2k,l}; \left(\tilde{I}'_{3k,l}, \tilde{I}_{3k,l}\right)\right]$ shows both the superiority and concordance of A_k with regard to A_l. Therefore, obtained value of indexes $\left[\left(\tilde{I}_{1k,l}, \tilde{I}'_{1k,l}\right); \tilde{I}_{2k,l}; \left(\tilde{I}'_{3k,l}, \tilde{I}_{3k,l}\right)\right]$ ($k,l = 1,2,...,m; l \neq k$), constructs the asymmetrical concordance interval valued fuzzy matrix $\left(\tilde{I}\right)$ as follow:

$$\tilde{I} = \begin{vmatrix} - & \left[\left(\tilde{I}_{1(1,2)}, \tilde{I}'_{1(1,2)}\right); \tilde{I}_{2(1,2)}; \left(\tilde{I}'_{3(1,2)}, \tilde{I}_{3(1,2)}\right)\right] & \cdots & \left[\left(\tilde{I}_{1(1,m)}, \tilde{I}'_{1(1,m)}\right); \tilde{I}_{2(1,m)}; \left(\tilde{I}'_{3(1,m)}, \tilde{I}_{3(1,m)}\right)\right] \\ \vdots & - & \cdots & \vdots \\ \vdots & \vdots & - & \vdots \\ \left[\left(\tilde{I}_{1(m,1)}, \tilde{I}'_{1(m,1)}\right); \tilde{I}_{2(m,1)}; \left(\tilde{I}'_{3(m,1)}, \tilde{I}_{3(m,1)}\right)\right] & \cdots & \left[\left(\tilde{I}_{1(m,m-1)}, \tilde{I}'_{1(m,m-1)}\right); \tilde{I}_{2(m,m-1)}; \left(\tilde{I}'_{3(m,m-1)}, \tilde{I}_{3(m,m-1)}\right)\right] & - \end{vmatrix}$$

Step 5: Calculate the discordance interval valued fuzzy matrix

Discordance interval valued fuzzy index (matched with set $\tilde{D}_{k,l}$) contrary to the index $\left[\left(\tilde{I}_{1k,l}, \tilde{I}'_{1k,l}\right); \tilde{I}_{2k,l}; \left(\tilde{I}'_{3k,l}, \tilde{I}_{3k,l}\right)\right]$ indicates that A_k is strongly superior with respect to A_l. The index $\left[\left(N\tilde{I}_{1k,l}, N\tilde{I}'_{1k,l}\right); N\tilde{I}_{2k,l}; \left(N\tilde{I}'_{3k,l}, N\tilde{I}_{3k,l}\right)\right]$ is calculated using the members of matrix *V* (weighted scores) for each element of discordance interval valued fuzzy set $\left(\tilde{D}_{kl}\right)$ as follow:

$$\widetilde{NI}_{k,l}\left[\left(\widetilde{NI}_{1k,l},\widetilde{NI}'_{1k,l}\right);\widetilde{NI}_{2k,l};\left(\widetilde{NI}'_{3k,l},\widetilde{NI}_{3k,l}\right)\right]= \frac{\underset{j\in D_{k,l}}{\operatorname{Max}}\left|\left[\left(\tilde{v}_{1kj},\tilde{v}'_{1kj}\right);\tilde{v}_{2kj};\left(\tilde{v}'_{3kj},\tilde{v}_{3kj}\right)\right]-\left[\left(\tilde{v}_{1lj},\tilde{v}'_{1lj}\right);\tilde{v}_{2lj};\left(\tilde{v}'_{3lj},\tilde{v}_{3lj}\right)\right]\right|}{\underset{j\in J}{\operatorname{Max}}\left|\left[\left(\tilde{v}_{1kj},\tilde{v}'_{1kj}\right);\tilde{v}_{2kj};\left(\tilde{v}'_{3kj},\tilde{v}_{3kj}\right)\right]-\left[\left(\tilde{v}_{1lj},\tilde{v}'_{1lj}\right);\tilde{v}_{2lj};\left(\tilde{v}'_{3lj},\tilde{v}_{3lj}\right)\right]\right|} \tag{12}$$

Where, discordance interval valued fuzzy matrix for all pairwise comparisons of alternatives converts into a matrix with absolute numbers which is defined by equation (13). It deserves to note that the available values of $\tilde{I}$ and *NI* differs noticeably with each other, yet they have complementary relationship such that interval valued fuzzy matrix $\tilde{I}$ is illustrative of the weights $\left[\left(\tilde{w}_{1j},\tilde{w}'_{1j}\right);\tilde{w}_{2j};\left(\tilde{w}'_{3j},\tilde{w}_{3j}\right)\right]$ resulted from concordance indexes, and asymmetrical matrix NI reflects the high relative difference of $\tilde{v}_{ij}=\left[\left(\tilde{n}_{1ij}\times\tilde{w}_{1j},\tilde{n}'_{1ij}\times\tilde{w}'_{1j}\right);\tilde{n}_{2ij}\times\tilde{w}_{2j};\left[\tilde{n}'_{3ij}\times\tilde{w}'_{3j};\tilde{n}_{3ij}\times\tilde{w}_{3j}\right]\right]$ for each discordance indexes.

$$NI=\begin{vmatrix} - & NI_{1,2} & NI_{1,3} & \cdots & NI_{1,m} \\ NI_{2,1} & - & NI_{2,3} & \cdots & NI_{2,m} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ NI_{m,1} & NI_{m,2} & \cdots & NI_{m(m-1)} & - \end{vmatrix} \tag{13}$$

Step 6: Specify the effective concordance interval valued fuzzy matrix

The values of indexes $\left[\left(\tilde{I}_{1k,l},\tilde{I}'_{1k,l}\right);\tilde{I}_{2k,l};\left(\tilde{I}'_{3k,l},\tilde{I}_{3k,l}\right)\right]$ of concordance interval valued fuzzy matrix have to be compared against a threshold value so that the superiority chance of A_k with respect to A_l are better judged. In the case when $\left[\left(\tilde{I}_{1k,l},\tilde{I}'_{1k,l}\right);\tilde{I}_{2k,l};\left(\tilde{I}'_{3k,l},\tilde{I}_{3k,l}\right)\right]$ exceeds from a minimum threshold $\bar{I}=\left[\left(\tilde{\bar{I}}_{1k,l},\tilde{\bar{I}}'_{1k,l}\right);\tilde{\bar{I}}_{2k,l};\left(\tilde{\bar{I}}'_{3k,l},\tilde{\bar{I}}_{3k,l}\right)\right]$, this chance increases. Besides, we can calculate the average of each arbitrary interval valued fuzzy index $\left[\bar{I}^{l},\bar{I}^{u}\right]$ from concordance interval valued fuzzy indexes in the following manner:

$$\bar{I}=\left[\left(\tilde{\bar{I}}_{1k,l},\tilde{\bar{I}}'_{1k,l}\right);\tilde{\bar{I}}_{2k,l};\left(\tilde{\bar{I}}'_{3k,l},\tilde{\bar{I}}_{3k,l}\right)\right]=\sum_{k=1}^{m}\sum_{l=1}^{m}\left[\left(\tilde{I}_{1k,l},\tilde{I}'_{1k,l}\right);\tilde{I}_{2k,l};\left(\tilde{I}'_{3k,l},\tilde{I}_{3k,l}\right)\right]/\,m(m-1) \tag{14}$$

Following the above, based on minimum threshold $\bar{I}=\left[\left(\tilde{\bar{I}}_{1k,l},\tilde{\bar{I}}'_{1k,l}\right);\tilde{\bar{I}}_{2k,l};\left(\tilde{\bar{I}}'_{3k,l},\tilde{\bar{I}}_{3k,l}\right)\right]$, we construct a Boolean matrix F which has elements 0 and 1 as:

$$\begin{cases} f_{kl}=1 & \text{if} \quad \left[\left(\tilde{I}_{1k,l},\tilde{I}'_{1k,l}\right);\tilde{I}_{2k,l};\left(\tilde{I}'_{3k,l},\tilde{I}_{3k,l}\right)\right]\geq\left[\left(\tilde{\bar{I}}_{1k,l},\tilde{\bar{I}}'_{1k,l}\right);\tilde{\bar{I}}_{2k,l};\left(\tilde{\bar{I}}'_{3k,l},\tilde{\bar{I}}_{3k,l}\right)\right] \\ f_{kl}=0 & \text{if} \quad \left[\left(\tilde{I}_{1k,l},\tilde{I}'_{1k,l}\right);\tilde{I}_{2k,l};\left(\tilde{I}'_{3k,l},\tilde{I}_{3k,l}\right)\right]<\left[\left(\tilde{\bar{I}}_{1k,l},\tilde{\bar{I}}'_{1k,l}\right);\tilde{\bar{I}}_{2k,l};\left(\tilde{\bar{I}}'_{3k,l},\tilde{\bar{I}}_{3k,l}\right)\right] \end{cases} \tag{15}$$

Element identified by 1 in matrix F (effective concordance interval valued fuzzy matrix) indicates that it is an effective and dominant alternative against the other alternatives.

Step 7: Specify the effective discordance matrix

Elements $NI_{k,l}$ from discordance matrix as done in step 6 should be evaluated with respect to a threshold value. This threshold value ($\overline{NI}$) is calculated with the following formula.

$$\overline{NI}=\sum_{k=1}^{m}\sum_{l=1}^{m}N_{k,l}\,/\,m(m-1) \tag{16}$$

Boolean matrix G is constructed by (17) in which unit elements indicate dominance relations among alternatives.

$$\begin{cases} g_{kl} = 1 & \text{if} \quad NI_{k,l} \leq \overline{NI} \\ g_{kl} = 0 & \text{if} \quad NI_{k,l} > \overline{NI} \end{cases} \tag{17}$$

Step 8: Specify effective and outranking matrix

Common elements $(h_{k,l})$ construct outranking matrix (H) for making decision from matrix F and matrix G with the following formula.

$$h_{k,l} = f_{k,l} . g_{k,l} \tag{18}$$

Step 9: Eliminate the less attractive alternatives

Outranking matrix (H) indicates the order of relative superiority of alternatives. This means that if, $h_{k,l} = 1$, A_k is superior to A_l in terms of both concordance and discordance indexes. However, A_k might be still dominated by other alternatives. Therefore, the condition which makes A_k an effective alternative is defined as follows:

$$\begin{cases} h_{k,l} = 1 & \text{for at least one unit element} & \text{for} & l = 1,2,...,m; k \neq l \\ h_{k,l} = 0 & \text{for all } i & \text{for} & l = 1,2,...,m; i \neq k; i \neq l \end{cases} \tag{19}$$

In the situations where two conditions are not simultaneously fulfilled, we can identify the effective alternatives from matrix (H). For this purpose, we can eliminate those columns of (H) which at least possess a unit element (1) from matrix (H) because those columns are dominated by other row or rows.

3. Application of Method in a Real Problem

In this research work, a general contractor organization has been selected as case study which in managers are tending to select the best sub-contractor among three candidates for out-sourcing of the pre-selected parts of project. Three candidates are considered to be evaluated based on six experimental criteria. Four experts, who are able to judge on the criteria corresponding to each candidate, are nominated to fill out the questionnaires. Criteria are exclusively considered in the project are as follows:

- Related Execution History (C_1)
- Available Facilities (C_2)
- Adequacy of Technical Team (C_3)
- Financial Ability (C_4)
- Company's Antiquity (C_5)
- Previous Employers Satisfaction (C_6)

Considering to the method of ranking candidates, different weights are engaged for each criteria number of 1 to 6 by 0.2, 0.3, 0.1, 0.1, 0.2, and 0.1 respectively. Three candidates Company 1 (A_1), company 2 (A_2) and company 3 (A_3) are selected to be ranked. The selected method is currently applied to solve this problem; the computational procedure is summarized as follows:

Step 1: Four committee members labeled as DM1, DM2, DM3 and DM4. Each decision maker has presented his assessment based on linguistic variable for importance of each criterion and rating performance by a linguistic variable as depicted in tables 2 and 3, respectively. The process of problem solving will be done using interval-valued fuzzy set. Table 4 shows the final judgment of the decision makers through applying Eqs. (2) and (3).

Step 2: Construct the interval valued fuzzy normalized decision matrix by making use of Eqs. (4) and (5) and results are tabulated in Table 5.

Step 3: Construct the interval valued fuzzy weighted normalized decision matrix by making use of Eqs. (7) and (8). The obtained results are reported in Table 6.

Step 4: Specify the concordance and discordance interval valued fuzzy set by making use of Eqs. (9) and (10). The concordance and discordance interval valued fuzzy set is as follows:

$\tilde{S}_{1,2} = \{1,2,6\}$ $\quad$ $\tilde{D}_{1,2} = \{3,4,5\}$

$\tilde{S}_{1,3} = \{1,2,6\}$ $\quad$ $\tilde{D}_{1,3} = \{3,4,5\}$

$\tilde{S}_{2,1} = \{3,4,5\}$ $\quad$ $\tilde{D}_{2,1} = \{1,2,6\}$

$\tilde{S}_{2,3} = \{3,4,5\}$ $\quad$ $\tilde{D}_{2,3} = \{1,2,6\}$

$\tilde{S}_{3,1} = \{3,4,5\}$ $\quad$ $\tilde{D}_{3,1} = \{1,2,6\}$

$\tilde{S}_{3,2} = \{1,2,6\}$ $\quad$ $\tilde{D}_{3,2} = \{3,4,5\}$

Step 5: Calculate the concordance interval valued fuzzy matrix by making use of Eq. (11).

$$\tilde{I} = \begin{vmatrix} - & [(0.9,1.3);1.725;(2.0625,2.3375)] & [(0.9,1.3);1.725;(2.0625,2.3375)] \\ [(1.8,2.25);2.625;(2.775,2.9625)] & - & [(1.8,2.25);2.625;(2.775,2.9625)] \\ [(1.8,2.25);2.625;(2.775,2.9625)] & [(0.9,1.3);1.725;(2.0625,2.3375)] & - \end{vmatrix}$$

Step 6: Calculate the discordance interval valued fuzzy matrix by making use of Eq. (12).

$$NI = \begin{vmatrix} - & 0.9652 & 0.9342 \\ 1 & - & 0.7787 \\ 1 & 1 & - \end{vmatrix}$$

Step 7: Specify effective concordance matrix by making use of Eqs. (14) and (15) and for matrix F we have:

$$\bar{I} = [(1.35,1.775);2.175;(2.41875,2.65)]$$

$$F = \begin{vmatrix} - & 0 & 0 \\ 1 & - & 1 \\ 1 & 0 & - \end{vmatrix}$$

Step 8: Specify effective discordance matrix by making use of Eqs. (16) and (17). $\overline{NI} = 0.946346$, Matrix G is obtained as follow:

$$G = \begin{vmatrix} - & 0 & 1 \\ 0 & - & 1 \\ 0 & 0 & - \end{vmatrix}$$

Step 10: Determine the outranking matrix H by making use of Eqs. (18) and (19).

$$H = \begin{vmatrix} - & 0 & 0 \\ 0 & - & 1 \\ 0 & 0 & - \end{vmatrix}$$

Step 11: Eliminate the less attractiveness alternatives.

As can be seen from matrix H, the third column has at least one unit element and therefore we can eliminate it. So alternative A_2 is placed in the first rank and alternatives A_3 and A_1 are placed in the second and third ranks, respectively.

Table 2: The importance of criterion

Criterion	DM1	DM2	DM3	DM4

C_1	MH	MH	H	VH
C_2	M	M	ML	MH
C_3	MH	MH	H	MH
C_4	H	MH	H	VH
C_5	VH	VH	H	MH
C_6	ML	ML	M	ML

Table 3: Decision makers' assessments for each criterion

Criterion	Alternatives	Decision makers			
		DM1	DM2	DM3	DM4
C_1	(A_1)	MP	MP	M	M
	(A_2)	G	G	MG	G
	(A_3)	G	F	F	F
C_2	(A_1)	MG	G	F	MG
	(A_2)	F	F	F	MG
	(A_3)	MG	MG	F	F
C_3	(A_1)	F	F	F	MG
	(A_2)	F	MG	MG	G
	(A_3)	MG	MG	MG	G
C_4	(A_1)	F	F	F	G
	(A_2)	MG	MG	G	G
	(A_3)	F	F	F	F
C_5	(A_1)	MG	F	F	MP
	(A_2)	G	G	G	MG
	(A_3)	MG	MG	MG	MG
C_6	(A_1)	F	F	MP	F
	(A_2)	MG	G	G	G
	(A_3)	F	F	MP	F

Table 4: The interval-valued fuzzy decision matrix and weights

	C_1	C_2	C_3
(A_1)	[(1.25,2.5);4;(5.5,6.5)]	[(4.25,5.5);7;(8,9.125)]	[(3,4);5.5;(6.875,8)]
(A_2)	[(5.25,7);8.5;(9.125,9.875)]	[(3,4);5.5;(6.875,8)]	[(4.75,5.5);7;(8,9.125)]
(A_3)	[(3.25,4.5);6;(7.25,8.125)]	[(3.5,4.5);6;(7.25,8.5)]	[(4.75,6);7.5;(8.375,9.625)]
Weight	[(0.575,0.7);0.825;(0.8875,0.95)]	[(0.2375,0.35);0.5;(0.6375,0.75)]	[(0.475,0.6);0.75;(0.8375,0.9625)]

Table 4: The interval-valued fuzzy decision matrix and weights (Continued)

	C_4	C_5	C_6
(A_1)	[(3.25,4.5);6;(7.25,8.125)]	[(2.375,3.5);5;(6.375,7.5)]	[(1.875,3);4.5;(6,7)]
(A_2)	[(5,6.5);8;(8.75,9.75)]	[(5.25,7);8.5;(9.125,9.875)]	[(5,6.5);8;(8.75,9.75)]
(A_3)	[(2.5,3.5);5;(6. 5,7.5)]	[(4.5,5.5);7;(8,9.5)]	[(1.875,3);4.5;(6,7)]
Weight	[(0.6,0.75);0.875;(0.925,0.9875)]	[(0.675,0.8);0.9;(0.9375,0.9875)]	[(0.0625,0.2);0.35;(0.5,0.6)]

Table 5: Normalized decision matrix

	C_1	C_2	C_3	C_4	C_5	C_6
(A_1)	[(0.19,0.23);0.31;(0.5,1)]	[(0.49,0.62);0.78;(0.87,1)]	[(0.31,0.42);0.57;(0.71,0.83)]	[(0.19,0.31);0.46;(0.61,0.72)]	[(0.19,0.3);0.46;(0.61,0.71)]	[(0.19,0.23);0.31;(0.5,1)]
(A_2)	[(0.13,0.14);0.15;(0.18,0.24)]	[(0.36,0.47);0.62;(0.75,0.88)]	[(0.49,0.62);0.78;(0.87,1)]	[(0.51,0.67);0.82;(0.9,1)]	[(0.53,0.71);0.86;(0.92,1)]	[(0.13,0.15);0.17;(0.21,0.26)]
(A_3)	[(0.17,0.19);0.25;(0.36,0.5)]	[(0.47,0.57);0.73;(0.83,0.99)]	[(0.47,0.57);0.73;(0.83,0.99)]	[(0.26,0.36);0.51;(0.67,0.77)]	[(0.46,0.56);0.71;(0.81,0.96)]	[(0.18,0.21);0.28;(0.42,0.67)]

Table 6: Weighted normalized decision matrix

	C_1	C_2	C_3	C_4	C_5	C_6
(A_1)	[(0.11,0.14);0.27;(0.92,0.99)]	[(0.12,0.17);0.39;(0.64,0.75)]	[(0.15,0.19);0.43;(0.7,0.8)]	[(0.12,0.15);0.43;(0.69,0.72)]	[(0.13,0.16);0.43;(0.69,0.71)]	[(0.01,0.04);0.11;(0.5,0.6)]
(A_2)	[(0.08,0.09);0.13;(0.22,0.23)]	[(0.09,0.13);0.31;(0.56,0.66)]	[(0.23,0.3);0.58;(0.84,0.96)]	[(0.32,0.41);0.76;(0.96,1)]	[(0.37,0.45);0.82;(0.975,1)]	[(0.01,0.03);0.06;(0.13,0.16)]
(A_3)	[(0.1,0.125);0.22;(0.46,0.49)]	[(0.11,0.16);0.36;(0.63,0.74)]	[(0.22,0.28);0.54;(0.83,0.95)]	[(0.16,0.2);0.47;(0.74,0.77)]	[(0.32,0.39);0.67;(0.94,0.96)]	[(0.01,0.04);0.1;(0.33,0.4)]

4. Summary and Conclusion

Multi-Criteria Decision Making is an interesting technique applied in the real world especially when dealing with MCDM problem manager of organization would like to consider all aspects. Inter-dependence property among criteria is very important for decision makers. Group decision making is more helpful to determine such an inter-dependent property than to decide by only one or two decision makers. Decision makers should consider very carefully about some quantitative and qualitative factors such as cost for project, time for project, probability of success, suitability and so on of the projects. Quantitative and qualitative factors can be obtained by gathering information from experts.

In this research work, the ELECTRE method, developed based on interval – valued fuzzy sets, has been applied to select sub-contractors using experts' points of view. Six criteria including related execution history, available Facilities, adequacy of technical team, financial ability, company’s antiquity, and previous employers’ satisfaction are nominated exclusively to apply the proposed method. Because of applications of the problem in the real world and different opinion of managers in each level of decision, three alternatives have been considered for ranking alternatives and satisfy decision makers. Results have been evaluated and checked by some local experts' points of view and they accepted them. It means that this method is cable to select sub-contractors in engineering procurement construction projects by general contractors who are keen to out-source some parts of their projects. For researchers interested in this field it is recommended to focus on develop methodology of application fuzzy sets considering the real area of uncertainty in high level of educational process and applying this method in the other problem mainly in personal selection.

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