Practical ABC intelligence solution for Quadratic Assignment

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Abstract

One of the key challenges in the scope of NP-hard problems is associated with finding a suitable solution algorithm as a result of degrees of complexity integrated in such problems. Among that, Quadratic assignment problems (QAPs) have considerable popularity because of various real-world operation applications such as production line scheduling, facility layout, etc. however as QAP is classified in the group of most difficult combinatorial optimization problems, adopting an appropriate solution approach to that, is a controversial subject among the researchers. In this paper, ABC (Artificial Bee Colony) algorithm as a new and efficient intelligent approach, is proposed and applied due to minimize the total cost of the developed quadratic model including weighted distance and fixed locating cost by adopting optimal assignment decision. Then, two experimental examples are developed and solved through MATLAB 8 software; results are achieved and compared with the outcomes from the mainly used classical algorithm (GA) to prove the efficiency and highlight some managerial issues.

Keywords
Optimization, artificial intelligence, NP-hard, quadratic assignment problem, heuristic

1. Introduction

Quadratic Assignment problem (QAP) is one of the important mathematical modeling types which could be applied in many industrial engineering and operation management problems. For instance, QAP model is fitted in scheduling problems of the parallel production line or in continues facility layout problems. However, finding a proper solution to this problem is a challenging decision in terms of applicability and exactness, as it is categorized in the category of NP-hard problem.

Among the related solutions, Swarm intelligence is more popular in the literature of QAP solutions. Swarm intelligence is a heuristic approach that tries to model the population of entities that are able to self – organize and interact and broadly speaking, refers to the problem-solving method that origins from the interaction of individuals in those population. Then, the computational swarm intelligence could be defined as algorithmic models of intelligence behavior between such individuals (Engelbrecht 2005). Many heuristic solution methods have been developed and applied for solving the mathematical models; for example, classical heuristic solution approaches such as Genetic Algorithm (GA) are applied in the problem by researchers in large extent. In addition, Particle swarm optimization (PSO) and ant colony optimization (ACO) are other two of mainly applied swarm intelligence heuristics which also particularly used in Quadratic Assignment Problems; Furthermore in group bee-inspired meta heuristics: honey bee social foraging algorithm, honey bee mating optimization (HBMO) algorithm, Bee Colony Optimization (BCO) have been used to solve QAP problems; however, here Artificial Bee Colony (ABC) algorithm as a recent bee-inspired meta-heuristic have been proposed to solve developed QAP model. The motivation of applying this algorithm to such problem is as per proved most efficiency to successfully apply to many combinatorial optimization problems (Bolaji et al. 2005). Beside, QAP is classified in NP-Complete difficult combinatorial optimization problems and exact solution methods to that have not been successful when the size of problem increased (Bayat and Sedghi 2009).Therefore, applying an efficient heuristic in QAP is the key challenge here and this work contributes in practically extend ABC algorithm in solving the quadratic assignment problem. Such efficiency is claimed through comparative results analysis with what are achieved with GA heuristic which is mainly used in the literature.

Thus, in this research, one of the important industrial engineering related problems named QAP is chosen to solve. The model is basically trying to locate the facilities at least costs associated with placement, distance and interaction and is considered to solve by an efficient ABC algorithm; The whole paper is classified in the following sections; first the introduction and literature review are provided, next the mathematical QAP model and ABC heuristic algorithm are developed, then the chosen experimental problem is solved through the proposed ABC algorithm and GA and comparative results are achieved. Finally, conclusion and future works are proposed in the last section.

2. Literature Review

Applications of QAP are recognized clearly in many significant areas of operation management and industrial engineering. For example Geoffrion and Graves (1976) fitted QAP to the production line scheduling to minimize change over and production costs. QAP model is applied to facility layout problem as arrangement of facilities within the departments is obtained through quadratic assignments (Forghani et al 2012). Despite, the challenging issue in the literature is in regard with the suitable solution approach to that as it is classified in the category of most difficult problem. Among, the proposed related solutions in the literature, meta-heuristics and particularly swarm intelligence based approaches have been established as one of the most practical approaches and are chosen which are reviewed as follows;

Swarm behavior is one of the considerable features in different colonies of social insects (bees, wasps, ants, termites). For instance Ant Colony Optimization (ACO) introduced by Dorigo et al (2002) mimics ant foraging approach in classical optimization problems including QAP (Dorigo et al. 2002, Dorigo and St¨utzle 2004, and Bidyarthy 2012), particle swarm optimization (PSO) that considers a population of particles moving in search for space by following present particles and modifies the position in order to find optimal solution (Kennedy and Eberhart 1995), and bee-inspired algorithms such as honey bee social foraging algorithm, honey bee mating optimization (HBMO), Bee Colony Optimization (BCO) and Bee Collecting Pollen Algorithm (BCPA) which applied to some areas of facility location and quadratic assignment problems (Lu and Zhou 2008, Fon and Wong 2010, Mirzazadeh et. al 2011 and Chalk 2013).

However, Artificial Bee Colony (ABC) is considered here as new intelligent approach to apply in proposed quadratic assignment. ABC is basically a swarm based meta-heuristic that has been introduced by Karaboga (2005) for applying in numerical optimization problems through imitating the natural approach of honey bee colony recently attracts attention of researchers in area of optimization (Karaboga 2005, Kumar et. al 2010 and Quijano and Passino 2010). Particularly, in order to handle complex integer programming problems according to comparison report by Akay and Karaboga (2009) that compared Artificial Bee Colony (ABC) with particle swarm optimization (PSO), Quantum behaved particle swarm optimization (QBSO) and traditional Branch and Bound Technique, ABC showed the attractive efficiency in truncates the values of variables to the closest integer after producing new solutions.

One of the key differences between the ABC and other swarm intelligence algorithms is about the possible solutions of the algorithm which is considered as food sources and not individuals (honey bees). However, in the algorithms such as PSO, the possible solutions are supposed to be the swarm individuals. Interestingly, in the ABC algorithm the quality of solution (fitness of a food source) is calculated by means of the objective function of the particular problem (Subotic 2012).

In regard with applications of the algorithm, ABC attracts researchers to adopt the use of ABC for many decision making problems such as constrained optimization (Karaboga and Akay 2007) and engineering (Bernardino et. al 2010 and Sonmez 2011). However, need of using practical and simple solution approaches to some particular complex engineering problems, is still felt; in respond to that, here ABC algorithm is applied due to solve a practical quadratic assignment problem.

3. Quadratic Assignment Model Development

The quadratic assignment problem (QAP) was introduced by Koopmans and Beckmann (1957) as a mathematical programming model for optimizing the location of a set of indivisible economical activities. This model usually considers the problem of allocating a set of facilities to a set of sites which the objective function is to minimize the weighted distance based on the related flow between the facilities. Sometime the costs associated with a facility being placed at a certain location as a fixed cost are also considerable. Broadly speaking, the objective is to assign each facility to one location such that the total cost is minimized, however as the mathematical model is classified as constrained NLP and Quadratic programming, the simple and applicable approach for solving the problem is an area of concern. Therefore ABC intelligent algorithm is considered here to develop for quadratic assignment problem which its mathematical model is identified as bellow.

Minimize
$$\sum\sum\sum\sum \varphi_{ij} d_{kl} x_{ik} x_{kl} + \sum_i \sum_j \sigma_{ij} x_{ij} \quad (1)$$

Subject to:
$$\sum_k x_{ik} = 1; \quad for \quad each \quad i \quad (2)$$
$$\sum_i x_{ik} = 1; \quad for \quad each \quad k \quad (3)$$
$$x_{ik} = 1,0 \quad (4)$$

Here, if department i is assigned to site k, x_{ik} would be 1 and get 0 value if such assignment would not applicable. Then, φ_{ij} refers to the volume of flow transported including data, material, etc. between i and j and d_{kl} denotes distance between k and l and finally, σ_{ij} is considered as fixed costs of placement for facility i in potential area of j.

Equation (1) minimizes costs including weighted transportation distance costs and fixed costs of facility establishment. Equations (2) and (3) ensure that each facility gets exactly one site allocated and each site exactly allocate to one facility. Equation (4) identifies model variables as binary.

4. Proposed Meta heuristic solution approach

A colony includes a queen, a few number of drones (males) and thousands of worker bees (infertile females) that are key in ABC model because of their approach in foraging for food (nectar). Foraging behavior contains two main elements: recruitment and abandonment of a nectar source. When food source is discovered by scout bees, they return to the hive and pass the useful information about food source including odor, direction and distance through particular dancing in dance floor to the other bees and enable them to fly directly to food sources.

Based on initial artificial algorithm by Karaboga (2005), ABC model is divided to main phases as initialization, employed bee, onlooker bee and scout bee. In this model food sources are considered as the possible random solutions (population) and nectar amount of each source is fitness value of the solution. For each food source is supposed to have only one employed bee and if the nectar amount decreased or exhausted, foragers abandon the food source and become scouts. There are some control parameters about population size, maximum cycle number and limit are required to be predetermined and all three mentioned bee groups evolve in searching procedure of algorithm and the food sources (population) are subjected to iterative cycles till MCN (Maximum Cycle Number) is achieved. In initialization phase, generally after setting control parameters, the population of food source (x_{mi}) are initialized randomly by scouts with using below equation:

$$x_{mi} = l_i + rand(0,1) * (u_i - l_i) \quad (5)$$

Where x_{mi} is the solution vector, m:1....N is the population size, and i:1....n are variables in each solution vector. Finally l_i and u_i are the lower and upper bounds of the solution respectively. Next, in employed bee phase, employed bees search for new food sources (v_{mi}) with more nectar based on profitability evaluation in neighborhood of previous ones (x_{mi}) by referring to their memory. In other words, they update its solution (position) and the fitness value, in the memory; in the case this fitness value is more, bee memorizes the new position and the old solution is cleared from the memory. v_{mi} is calculated as below:

$$v_{mi} = x_{mi} + rand(-n,n) * (x_{ki} - x_{mi}) \quad (6)$$

Where x_{ki} is a randomly chosen food source, and $rand(-n,n)$ is a random number within this range. Then new food source (v_{mi}) fitness is calculated and a greedy selection is used to select between x_{mi} and v_{mi} . Fitness value for x_{mi} solution ($f_m(x_{mi})$) is calculate as:

$$f_m(x_{mi}) = \begin{cases} \dfrac{1}{1+f_m(x_{mi})} & f_m(x_{mi}) \geq 0 \\ 1+abc(f_m(x_{mi})) & f_m(x_{mi}) < 0 \end{cases} \quad (7)$$

Then, in the onlooker bee phase, a fitness based selection technique such as roulette wheel is applied; the food source searching procedure is based on a probability associated with the nectar fitness obtaining through communication with the employed bees. Here, p_m is the probability value with which x_m is selected as below:

$$p_m = \frac{f_m(x_{mi})}{\sum_m^N f_m(x_{mi})} \quad (8)$$

Flowingly and based on the above probability, one food source is chosen by an onlooker and a neighborhood solution is also produced; final selection is based greedy application similar to which of employed bees.

Finally employed bees whose did not be progressed through pre-set number of trials (limit or abandonment criteria) become scouts and their solutions are left and the new scouts begin searching for new solution and steps repeat again till stop criteria (MCN) happens. The pseudo code of the ABC algorithm can be developed as bellow.

Table 1) pseudo code of the ABC algorithm

```
%% initialization
for m=1:N
   Food source initialization by scout: Equation (5);
   Fit=fitness (foods);
end
for cycle=1:MCN
    % employed bee phase
    for m=1:N
       v=New food sources based on equation (6);
       New Fit Evaluation equation (7);
        if fit v<fit(m)
           foods(m)=v;
         else
          next trial;
      end
   end
   % onlooker bee phase
   Calculate probability of selection equation (8);
   while m<N
       if rand<P(m)
       x=foods(m);
       k=rand(1 N);
       while k==m
       k=rand(1 N);
       end
       v=New food sources in rand (-a, a) based on equation (6);
       New Fit Evaluation equation (7);
       if fit v<fit(m,1)
           foods =v;
         else
          next trial;
       end
       if m==N;
          m=0;
       end
   end
    % scout bee phase
  q= trial>Limit;
  for j=1:q
       New food by Equation (6);
      Evaluate food by Equation (7);
  end
   [fit food]=min (fit);
  if fit<best-fit
      best-fit=fit;
  end
end;
```

5. Experimental Examples and Results

In this section, two cases of quadratic assignment problem are introduced and solved. For the first case, the number of facilities to be assigned is less, however in the second one, this number is doubled to analyze sensitivity of the model to the bigger size problems; both problems are solved through MATLAB software with two heuristic methods including ABC and GA. In all cases, to simplify, is assumed that the potential locations of facility are available and there is no fixed cost associated with such placement.

5.1 Case1

In this case we have 4 facilities and 4 available potential locations for assignment. The matrix of flow and distances are considered in table 2 and 3 respectively.

Table 2) Flow Matrix-Case 1

0	30	70	40
30	0	60	50
70	60	0	20
40	50	20	0

Table 3) Distance Matrix-Case 1

0	10	10	20
10	0	20	10
10	20	0	10
20	10	10	0

After solving the model the below results are achieved as present in table 4 and 5:

Table 4) Optimal Assignment ABC-Case 1

0	1	0	0
1	0	0	0
0	0	0	1
0	0	1	0
Total Cost=7400			

Table 5) Optimal Assignment GA-Case 1

0	0	1	0
0	0	0	1
1	0	0	0
0	1	0	0
Total Cost=7400			

As shown in table 4 and 5, the results are same in this case for both algorithms at MCN=200 and limit of 60 in ABC, however the time is much lesser in GA as 3.4 compared with 0.77 seconds.

5.2 Case2

In this case a bigger size problem is introduced with 8 facilities and 8 available potential locations for assignment with flow and distances matrixes as are identified in table 6 and 7 respectively.

Table 6) Flow Matrix-Case 2

0	50	20	40	10	0	0	60
50	0	30	0	20	20	20	0
20	30	0	0	0	0	0	50
40	0	0	0	50	20	20	100
10	20	0	50	0	100	0	0
0	20	0	20	100	0	50	10
0	20	0	20	0	50	0	100
60	0	50	100	0	10	100	0

Table 7) Distance Matrix-Case 2

0	10	20	30	10	20	30	40
10	0	10	20	20	10	20	30
20	10	0	10	30	20	10	20
30	20	10	0	40	30	20	10
10	20	30	40	0	10	20	30
20	10	20	30	10	0	10	20
30	20	10	20	20	10	0	10
40	30	20	10	30	20	10	0

After solving the model for case 2, the below result are achieved as shown in table 8 and 9 for ABC and GA, respectively.

Table 8) Optimal Assignment ABC-Case 2

0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	1
0	0	0	0	0	0	1	0
0	0	0	0	1	0	0	0
0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0
1	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0
Total Cost=28200							

Table 9) Optimal Assignment GA-Case 2

0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0
0	0	0	0	1	0	0	0
1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	0	0	0	0	1	0	0
0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1
Total Cost=37000							

As shown in table 3 and 4, as size of the problem is increased to double, the results for ABC algorithm are much more better in comparison with that of GA at MCN=200 and limit of 60 in ABC; Although still the time is less in GA as 4.1 compared with 1.8 seconds, but this is much more stable in ABC compared with the previous case. These figures are compared for both algorithm as are presented in figure 1 and 2.

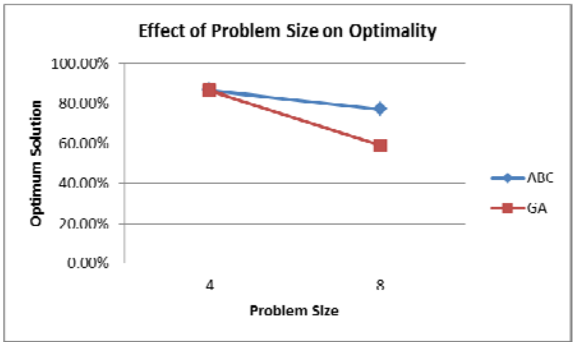

Figure 1) Optimality vs. size in ABC & GA

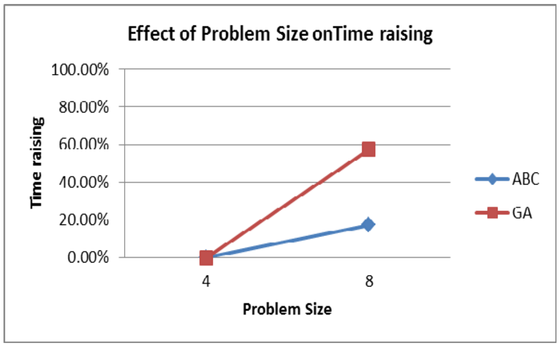

Figure 2) Time raising percentage vs. size in ABC & GA

Graph 1 shows that as size of the problem doubled, efficiency of the solution in heuristic solution falls till almost 80% in ABC and just 60% in GA, however graph 2 presents effects of size incensement on both algorithms which has 17% raise in ABC compared with 57% in GA.

6. Conclusion and future work

This paper discusses solving an applicable problem in operation and industrial engineering named quadratic assignment by means ABC as new intelligence nature-inspired heuristic algorithm. Due to analyze the efficiency of the proposed algorithm in this problem, two experimental examples are solved and the results are compared with that of another heuristic algorithm (GA) which mainly used in literature of such problem. As results compared, some trade-offs consideration can be highlighted, for example ABC algorithm presents better results quality particularly in the case that problem size increases; on the other hand, elapsed time in GA algorithm is much better in comparison to ABC, however, interestingly both efficiency and optimality are more stable in ABC algorithm and these features are not as sensitive as those of GA to the problem size increasing.

To extend this work, some new feature can be added to the model for example developing the model to dynamic quadratic assignment (DQAO) or considering uncertainty. Also, for the solution approach to that ABC algorithm may be hybrid with other heuristics for better results achievement.

Appendix

Graphs have been included here for ABC and GA model iterations as figure 3 and 4 for 200 iterations through algorithms in MATLAB.

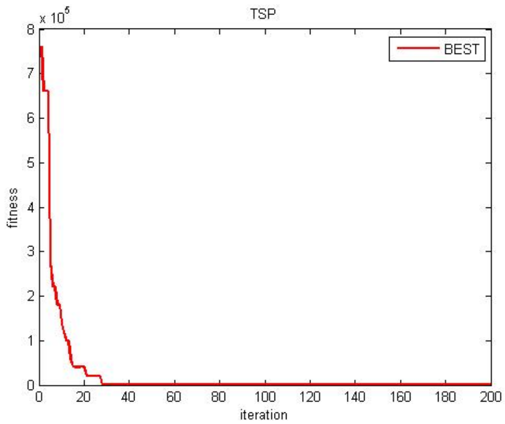

Figure 3) best fitness iterations for GA

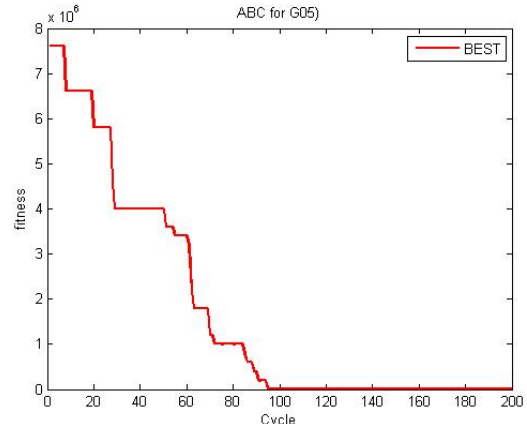

Figure 4) best fitness iterations for ABC

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Biography

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