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A Single Manufacturer – A Single Buyer Partnership with Vendor Managed Inventory and Consignment

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Abstract

Due to global competitiveness, a single manufacturer and a single buyer agree to work together to jointly optimize their performance. The aim of this paper is to propose two integration inventory approaches in order to reduce the total inventory costs: Vendor Managed Inventory (VMI) and Consignment. VMI is an integration tool where manufacturer is authorized to manage the buyer's inventory level. In consignment, manufacturer receives the payment only after the buyer discharges the product from the storage to be used in the production. In this case, the total inventory cost involves transportation cost that charged to the manufacturer. Algorithms are developed to obtain the optimal strategy. Analysis solution to the ratio of the manufacturer's transportation cost to the buyer's ordering cost indicates that the conformance partnership with VMI and consignment is determined by the value of the parameters used. If the manufacturer's transportation cost lower than the buyer's ordering cost, then use VMI, since it is not feasible to continue the partnership with the consignment. Conversely, if the manufacturer's transportation cost is much higher than the buyer's ordering cost, then the partnership is appropriate to proceed to the consignment. Numerical examples are given to illustrate the results.

Keywords
Total inventory cost, partnership, vendor managed inventory, consignment.

1. Introduction

In the global competition, integration is regarded as a prerequisite for winning performance. The environment has forced companies to increase their efficiency in order to reduce cost. Companies realize that inventories between buyer and vendor can be more efficiently managed through better coordination (Ben Daya et al, 2008). Many studies have investigated this relationship in order to achieve the minimal total inventory cost through optimal economic order quantity, such as Goyal (1776), Banerjee (1986), Affisco et al (1993), Hill (1999), Hoque and Goyal (2000), Kim and Ha (2003), Sucky (2005), Chan and Kingsman (2007), and Ohta et al (2009).

According to Applegate et al (2009) manufacturer has three options when determining how to structure the relationship with the buyer; transaction, contract and partnership. Transaction occurs on the exchange of goods and services in a simple way. Contractual relationship is done at a certain time period provisions are written for each party in terms of exchange of goods and services. It is clearly states when starting and ending the duration of an interaction. When the activities become complex, uncertain and crucial to the performance of the company, then partnership is preferable. In partnership the integration of activities across the organization is required.

Vendor Managed Inventory (VMI) is one of the most effective and popular approaches in the partnership (De Toni and Zamolo, 2005). With VMI, manufacturer will monitor and maintain the buyer's inventory replenishment. It shows that the buyer has delegates the ordering and replenishment decision to the manufacturer (Marques, et al, 2010). The effect of VMI on total cost in long-term and short-term planning horizons has shown that in certain conditions VMI's may decrease the manufacturer's revenue (Dong and Xu, 2002). The other inventory policy is defined as consignment policy. Consignment is one of the integration tools that based on the collaboration between buyer and the manufacturer. In consignment the product is still owned by the manufacturer until it used by the buyer. It shows that the buyer doesn't have the capital cost in the inventory (Gümüş et al, 2008).

At the beginning, this paper will consider the manufacturer-buyer relationship based on the agreement under VMI policy (Saraswati et al, 2013), and the relationship will continue by proposing VMI with consignment. It is assumed that the deliveries from the manufacturer to the buyer are executed during the production run, as soon as the quantity is equal to the batch size (Saraswati et al, 2008). Furthermore, this paper demonstrates the different way in approaching the solution of minimizing inventory cost compare to Gümüş et al (2008).

This paper is organized as follows. The problem descriptions, the notation used as well as the mathematical modeling are presented in Section 2. In section 3, the algorithms are developed to determine the continuation of the partnership between manufacturer and buyer by using VMI and consignment. Section 4 provides a numerical example to ilustrate the benefit of using VMI and consignment. Finally, section 5 provide a conclusions.

2. Problem Descriptions

Considering a single manufacturer with a production rate, P, produces a single product and delivers to a single buyer to meet the annual demand rate, D. It is assumed that production rate is greater than the demand rate. The manufacturer starts the production process with setup cost, S, and the production cost per item, C_v. In this case, the transportation cost from the manufacturer to the buyer location, F_v, is paid by the manufacturer. The item that wait to be delivered incurs inventory carrying cost, r. Therefore, the holding cost for each item at the manufacturer becomes $h_v = rC_v$. For buyer, the cost of issuing an order, A_p, plus the cost of receiving an order A_r, become the ordering cost, A. Based on the purchase price per item, C_b, and the cost of having an item in the storage, C_w, the holding cost for each item at the buyer becomes $h_b = r(C_b+C_w)$.

In typical inventory model with collaboration, EOQ model gives the buyer an optimal solution and EMQ model for the manufacturer. As a result, the optimal solution of the buyer is always unacceptable to the manufacturer and the EMQ of the manufacturer usually gives different value from the EOQ. This condition shows that buyer and manufacturer do not have the same optimal solution. Therefore, manufacturer and buyer have to agree to collaborate in order to optimize the availability of items at minimal cost of both parties.

As individually, the buyer begins the transaction with issuing an order at economic lot size, z_1, to the manufacturer with cost A_p. Manufacturer will process the order and gave the bill to the buyer for payment. Buyer will process the receiving items at cost A_r per unit. Therefore, the buyer's total ordering plus receiving cost become $((D/z_1)/(A_p + A_r))$, and the buyer's holding cost is $((z_1/2)/(C_b + C_w))$. In other words, the buyer's total inventory cost is given by

$$TC_{b1}(z_1) = ((D/z_1)/(A_p + A_r)) + (z_1/2)(rC_b + rC_w) \tag{1}$$

Since $A_p + A_r = A$, and $rC_b + rC_w = h_b$, then equation (1) can be re-write as follows

$$TC_{b1}(z_1) = ((D/z_1)/(A)) + (z_1/2)(h_b) \tag{2}$$

The first derivative of equation (1) with respect to z_1 is made equal to zero, and the value of buyer's economic lot size is given by

$$z_1^* = \sqrt{2AD/h_b} \tag{3}$$

Substituting (3) into (2), the optimal buyer's total inventory cost is

$$TC_{b1}(z_1^*) = \sqrt{2ADh_b} \tag{4}$$

Buyer will issue an order to the manufacturer when buyer's inventory level is at z_1. Thus it can be assumed that manufacturer will begin production when manufacturer receives an order from the buyer. In this case, manufacturer can still make the deliveries to the buyer even if the production period, $T_p = Q_v/P$, has not been completed yet as soon as there is enough items to meet the quantity of the delivery batch-size (Kim and Ha, 2003). Manufacturer's inventory cycle time is $T = Q_v/D$, and the period where manufacturer does not produce any product is $((Q_v/D) - (Q_v/P))$. Furthermore, the total accumulated inventory level in manufacturer is $(Q_v/2n)((2-n)D/P + n - 1)$ (Joglekar, 1988), where n is the number of deliveries per cycle. Actually manufacturer has begun the production before buyer's inventory equals to z_1. Therefore the average total inventory system that consists of the average inventory level for a single manufacturer and a single buyer becomes $(Q_v/2n)((2-n)D/P + n - 1) + z_1$ (Gümüş et al, 2008). Since the average inventory level for manufacturer is $(Q_v/2n)((2-n)D/P + n - 1) + z_1/2$ and for buyer is $(z_1/2)$. The manufacturer's total inventory cost is composed of setup cost, $(D/Q_v)S$, holding cost for product at the manufacturer's storage, $(Q_v/2n)((2-n)D/P + n - 1)h_v$, holding cost for product that being shipment to the buyer but not yet arrived, $(z_1/2)h_v$, and transportation cost $(D/z_1)F_v$. The manufacturer's total inventory cost is stated as follows

$$TC_{v1} = (D/Q_v)S + (Q_v/2n)((2-n)D/P + n - 1)h_v + (z_1/2)h_v + (D/z_1)F_v \tag{5}$$

This equation (5) contains controllable and uncontrollable variables. The controllable variables are setup cost and holding cost at the manufacturer's storage. Since z_1 is determined by buyer, then the holding cost of shipping's product and transportation cost become the uncontrollable variables. In this condition, manufacturer only produces items according to buyer demand, and manufacturer cannot influence the ordering lot size. For $z_1^* = \sqrt{2AD/h_b}$, the manufacturer's total inventory cost is given by $TC_{v1} = (D/Q_v)S + (Q_v/2n)((2-n)D/P + n - 1)h_v + ((\sqrt{2AD/h_b})/2)h_v + (D/(\sqrt{2AD/h_b}))F_v$. If equation (5) is simplified by defining $\gamma = F_v/A$ and $\phi = h_v/h_b$, then manufacturer's total inventory cost is

$$TC_{v1} = (D/Q_v)S + (Q_v/2n)((2-n)D/P + n - 1)h_v + (\gamma + \phi)\sqrt{2ADh_b/2} \tag{6}$$

Since, manufacturer cannot influence the ordering lot size, the manufacturer desire to participate in determining the ordering lot size by proposing Vendor Managed Inventory with Consignment approach. According to VMI approach, manufacturer monitors the buyer's inventory level and will make the deliveries when the inventory level reaches a certain level. As a result, the buyer doesn't need to place an order. In consignment the buyer own the products only when it is used for productions, thus buyer doesn't have to pay for the holding cost, but merely provides a storage area facilities.

When manufacturer implements VMI, the manufacturer delivers its products without waiting for an order from the buyer, thus the ordering process can be more efficient with an efficiency factor of $(1-\alpha)$, for $0 < \alpha < 1$. Therefore, the manufacturer's total inventory cost using VMI can be written as follows

$$TC_{v2} = (D/Q_v)S + (Q_v/2n)((2-n)D/P + n - 1)h_v + (z_2/2)h_v + (D/z_2)F_v + (D/z_2)\alpha A_P \tag{7}$$

The first partial derivatives of equation (7) to z_2 , yielding

$$z_2^* = \sqrt{(2D(F_v + \alpha A_p))/h_v} \tag{8}$$

Letting $\lambda_1 = A_p/A$ and $\lambda_2 = A_r/A$, with $\lambda_1 + \lambda_2 = 1$, $\gamma = F_v/A$, and $\phi = h_v/h_b$, the lot size equation (8) can be written as follows

$$z_2^* = \sqrt{(\gamma + \alpha\lambda_1/\phi)}\sqrt{2AD/h_b} \tag{9}$$

By substituting equation (9) into equation (7), it will provide the manufacturer's total cost as

$$TC_{v2} = (D/Q_v)S + (Q_v/2n)((2-n)D/P + n - 1)h_v + \sqrt{\phi(\gamma + \alpha\lambda_1})\sqrt{2ADh_b} \tag{10}$$

Manufacturer is willing to continue with VMI if $TC_{v2} < TC_{v1}$ and $\Delta TC_{v1-2} > 0$ for $\Delta TC_{v1-2} = TC_{v1} - TC_{v2}$.

Proposition 1: $TC_{v2} < TC_{v1}$, if $(\gamma + \phi) > 2\sqrt{\phi(\gamma + \alpha\lambda_1)}$

Proof: $TC_{v1} = Y + (\gamma + \phi)\sqrt{\dfrac{DAh_b}{2}}$ or $TC_{v1} = Y + \sqrt{2DAh_b}\,\dfrac{(\gamma + \phi)}{2}$, and

$TC_{v2} = Y + \sqrt{2DAh_b}\sqrt{\phi(\gamma + \alpha\lambda_1)}$. $TC_{v1} - TC_{v2} > 0$ for $\sqrt{2DAh_b} > 0$;

subsequently $\sqrt{2DAh_b}\,\dfrac{(\gamma + \phi)}{2} > \sqrt{2DAh_b}\sqrt{\phi(\gamma + \alpha\lambda_1)} \longrightarrow (\gamma + \phi) > 2\sqrt{\phi(\gamma + \alpha\lambda_1)}$.

Since the cost of issuing an order, A_p , is removed to the manufacturer's total cost, thus the buyer's total inventory cost with VMI is given by

$$TC_{b2} = \sqrt{\frac{DAh_b}{2}}\left(\frac{(1-\lambda_1)\phi + (\gamma + \alpha\lambda_1)}{\sqrt{(\gamma + \alpha\lambda_1)\phi}}\right) \tag{11}$$

Buyer will accept the proposal from manufacturing for implementing VMI, if $TC_{b2} < TC_{b1}$ and $\Delta TC_{b1-2} > 0$ for $\Delta TC_{b1-2} = TC_{b1} - TC_{b2}$.

Proposition 2: $TC_{b2} < TC_{b1}$, if $(1-\lambda_1)\phi + (\gamma + \alpha\lambda_1) < 2\sqrt{(\gamma + \alpha\lambda_1)\phi}$.

Proof: $\Delta TC_{b1-2} = TC_{b1} - TC_{b2} > 0$

$$\sqrt{2DAh_b} - \sqrt{\frac{DAh_b}{2}}\left(\frac{(1-\lambda_1)\phi + (\gamma + \alpha\lambda_1)}{\sqrt{(\gamma + \alpha\lambda_1)\phi}}\right) > 0 \text{ or } 2 - \left(\frac{(1-\lambda_1)\phi + (\gamma + \alpha\lambda_1)}{\sqrt{(\gamma + \alpha\lambda_1)\phi}}\right) > 0$$

subsequently $(1-\lambda_1)\phi + (\gamma + \alpha\lambda_1) < 2\sqrt{(\gamma + \alpha\lambda_1)\phi}$.

Finally, the saving cost for the system is given by the joint total inventory cost between manufacturer and buyer, for $TC_{v2} + TC_{b2} < TC_{v1} + TC_{b1}$ and $\Delta TC_{s1-2} = (TC_{v1} + TC_{b1}) - (TC_{v2} + TC_{b2}) > 0$.

Proposition 3: $TC_{v2} + TC_{b2} < TC_{v1} + TC_{b1}$, if $(\gamma + \alpha\lambda_1)(2\phi + 1) + (1-\lambda_1)\phi < (\gamma + \phi + 2)\sqrt{(\gamma + \alpha\lambda_1)\phi}$.

Proof: $\Delta TC_{s1-2} = (TC_{v1} + TC_{b1}) - (TC_{v2} + TC_{b2}) > 0$

$$TC_{v1} = Y + (\gamma + \phi)\sqrt{\frac{DAh_b}{2}} \text{ and } TC_{v2} = Y + 2\sqrt{\frac{DAh_b}{2}}\sqrt{\phi(\gamma + \alpha\lambda_1)}$$

$$TC_{b1} = 2\sqrt{\frac{DAh_b}{2}} \text{ and } TC_{b2} = \sqrt{\frac{DAh_b}{2}}\left(\frac{(1-\lambda_1)\phi + (\gamma + \alpha\lambda_1)}{\sqrt{(\gamma + \alpha\lambda_1)\phi}}\right)$$

$$\Delta TC_{s1-2} = \sqrt{\frac{DAh_b}{2}}\left((\gamma + \phi + 2) - \left(\left(\frac{2(\gamma + \alpha\lambda_1)\phi}{\sqrt{(\gamma + \alpha\lambda_1)\phi}}\right) + \left(\frac{(1-\lambda_1)\phi + (\gamma + \alpha\lambda_1)}{\sqrt{(\gamma + \alpha\lambda_1)\phi}}\right)\right)\right)$$

$$2(\gamma + \alpha\lambda_1)\phi + (1-\lambda_1)\phi + (\gamma + \alpha\lambda_1) < (\gamma + \phi + 2)\sqrt{(\gamma + \alpha\lambda_1)\phi}$$

$$(\gamma + \alpha\lambda_1)(2\phi + 1) + (1-\lambda_1)\phi < (\gamma + \phi + 2)\sqrt{(\gamma + \alpha\lambda_1)\phi}.$$

Once manufacturer decides to continue the partnership with consignment, the payment is made when the product is discharge from the storage to be used by the buyer in the production process. It means, buyer do not have to pay the opportunity cost for having the product (C_b), since the product still own by the manufacturer until the product utilized by the buyer. Since buyer's total holding cost (h_b) consists of opportunity cost, C_b, plus cost of having an item in the storage, C_w, hence $h_b = rC_b + rC_w$. Under consignment the buyer's total holding cost is $h_b = rC_w$.

Optimal ordering lot size is determined by the manufacturer with z_3, thus the buyer's total cost is $TC_{b3} = (D/z_3)A_r + (z_3/2)h_w$, where $h_w = rC_w = h_b$. Letting $\varepsilon_1 = h_w/h_b$ and $\varepsilon_2 = rC_b/h_b$, for $\varepsilon_1 + \varepsilon_2 = 1$. The buyer's total inventory cost can be re-write as follows

$$TC_{b3} = (DA(1-\lambda_1/z_3) + ((z_3\varepsilon_1 h_b)/2) \tag{12}$$

Therefore, the manufacturer's total inventory cost becomes

$$TC_{v3} = (D/Q_v)S + (Q_v/2n)((2-n)D/P + n - 1)h_v + (z_3/2)h_v + (D/z_3)F_v + (D/z_3)\alpha A_P + (z_3/2)rC_b \text{ or}$$

$$TC_{v3} = (D/Q_v)S + (Q_v/2n)((2-n)D/P + n - 1)h_v + (D/z_3)(\gamma + \alpha\lambda_1)A + (z_3/2)(\phi + \varepsilon_2)h_b \tag{13}$$

As a result, the manufacturer's production lot size is

$$z_3^* = \sqrt{((\gamma + \alpha\lambda_1)/(\phi + \varepsilon_2))}\sqrt{(2AD)/h_b} \tag{14}$$

By substituting equation (14) into equation (13), it will provide the manufacturer's total cost under consignment as follows

$$TC_{v3} = (D/Q_v)S + (Q_v/2n)((2-n)D/P + n - 1)h_v + \sqrt{(\gamma + \alpha\lambda_1)}\sqrt{(\phi + \varepsilon_2)}\sqrt{2ADh_b} \tag{15}$$

Manufacturer is willing to continue with consignment if $TC_{v3} < TC_{v1}$ and $\Delta TC_{v1-3} > 0$, for $\Delta TC_{v1-3} = TC_{v1} - TC_{v3}$.

Proposition 4: $TC_{v3} < TC_{v1}$, if $(\gamma+\phi) > 2\sqrt{(\gamma+\alpha\lambda_1)}\sqrt{(\phi+\varepsilon_2)}$ or $(\gamma-\varphi)^2 > 4(\gamma\varepsilon_2 + \alpha\lambda_1(\varphi+\varepsilon_2))$

Proof: $TC_{v1} = C' + \sqrt{2DAh_b}\dfrac{(\gamma+\varphi)}{2}$, and $TC_{v3} = C' + \sqrt{2DAh_b}\sqrt{(\gamma+\alpha\lambda_1)}\sqrt{(\varphi+\varepsilon_2)}$

Since $\sqrt{2DAh_b} > 0$, then $\dfrac{(\gamma+\varphi)}{2} > \sqrt{(\gamma+\alpha\lambda_1)}\sqrt{(\varphi+\varepsilon_2)}$ atau $(\gamma+\varphi) > 2\sqrt{(\gamma+\alpha\lambda_1)}\sqrt{(\varphi+\varepsilon_2)}$

$(\gamma-\varphi)^2 > 4(\gamma\varepsilon_2 + \alpha\lambda_1(\varphi+\varepsilon_2))$.

Since the opportunity cost for having the product (C_b), is removed to the manufacturer's cost, thus the buyer's total inventory cost with consignment is given by

$$TC_{b3} = \sqrt{\frac{DAh_b}{2}}\left(\frac{(1-\lambda_1)(\phi+\varepsilon_2)+\varepsilon_1(\gamma+\alpha\lambda_1)}{\sqrt{(\gamma+\alpha\lambda_1)}\sqrt{(\phi+\varepsilon_2)}}\right) \tag{16}$$

Buyer will have the benefit with the implementation of consignment, if $TC_{b3} < TC_{b1}$ and $\Delta TC_{b1-2} > 0$ for $\Delta TC_{b1-3} = TC_{b1} - TC_{b3}$.

Proposition 5: $TC_{b3} < TC_{b1}$, if $(1-\lambda_1)(\phi+\varepsilon_2)+\varepsilon_1(\gamma+\alpha\lambda_1) < 2\sqrt{(\gamma+\alpha\lambda_1)}\sqrt{(\phi+\varepsilon_2)}$

Proof: $TC_{b1} = 2\sqrt{\dfrac{DAh_b}{2}}$; $TC_{b3} = \sqrt{\dfrac{DAh_b}{2}}\left(\dfrac{(1-\lambda_1)(\varphi+\varepsilon_2)+\varepsilon_1(\gamma+\alpha\lambda_1)}{\sqrt{(\gamma+\alpha\lambda_1)}\sqrt{(\varphi+\varepsilon_2)}}\right)$; $\Delta TC_{b3} = TC_{b1} - TC_{b3} > 0$

$2\sqrt{\dfrac{DAh_b}{2}} > \sqrt{\dfrac{DAh_b}{2}}\left(\dfrac{(1-\lambda_1)(\varphi+\varepsilon_2)+\varepsilon_1(\gamma+\alpha\lambda_1)}{\sqrt{(\gamma+\alpha\lambda_1)}\sqrt{(\varphi+\varepsilon_2)}}\right)$; $(1-\lambda_1)(\varphi+\varepsilon_2)+\varepsilon_1(\gamma+\alpha\lambda_1) < 2\left(\sqrt{(\gamma+\alpha\lambda_1)}\sqrt{(\varphi+\varepsilon_2)}\right)$

The joint total inventory cost can get the benefit if $TC_{v3} + TC_{b3} < TC_{v1} + TC_{b1}$ and $\Delta TC_{s1-3} = (TC_{v1} + TC_{b1}) - (TC_{v3} + TC_{b3}) > 0$.

Proposition 6: $TC_{v3} + TC_{b3} < TC_{v1} + TC_{b1}$, if

$$2+\gamma+\phi > 2((\gamma+\alpha\lambda_1)(\phi+\varepsilon_2)) + ((1-\lambda_1)(\phi+\varepsilon_2)) + (\varepsilon_1(\gamma+\alpha\lambda_1)) \Big/ \sqrt{(\gamma+\alpha\lambda_1)(\phi+\varepsilon_2)}$$

Proof: $TC_{v1} = C' + \sqrt{2DAh_b}\dfrac{(\gamma+\varphi)}{2}$; $TC_{v3} = C' + \sqrt{2DAh_b}\sqrt{(\gamma+\alpha\lambda_1)}\sqrt{(\varphi+\varepsilon_2)}$; $TC_{b1} = \sqrt{2DAh_b}$

$TC_{b3} = \sqrt{\dfrac{DAh_b}{2}}\left(\dfrac{(1-\lambda_1)(\varphi+\varepsilon_2)+\varepsilon_1(\gamma+\alpha\lambda_1)}{\sqrt{(\gamma+\alpha\lambda_1)}\sqrt{(\varphi+\varepsilon_2)}}\right)$; $\Delta TC_{s3} = (TC_{v1} + TC_{b1}) - (TC_{v3} + TC_{b3}) > 0$

$$\sqrt{2DAh_b} + \sqrt{2DAh_b}\frac{(\gamma+\varphi)}{2} > \sqrt{2DAh_b}\sqrt{(\gamma+\alpha\lambda_1)}\sqrt{(\varphi+\varepsilon_2)} + \sqrt{\frac{DAh_b}{2}}\left(\frac{(1-\lambda_1)(\varphi+\varepsilon_2)+\varepsilon_1(\gamma+\alpha\lambda_1)}{\sqrt{(\gamma+\alpha\lambda_1)}\sqrt{(\varphi+\varepsilon_2)}}\right)$$

$$2\sqrt{\frac{DAh_b}{2}} + 2\sqrt{\frac{DAh_b}{2}}\frac{(\gamma+\varphi)}{2} > 2\sqrt{\frac{DAh_b}{2}}\sqrt{(\gamma+\alpha\lambda_1)}\sqrt{(\varphi+\varepsilon_2)} + \sqrt{\frac{DAh_b}{2}}\left(\frac{(1-\lambda_1)(\varphi+\varepsilon_2)+\varepsilon_1(\gamma+\alpha\lambda_1)}{\sqrt{(\gamma+\alpha\lambda_1)}\sqrt{(\varphi+\varepsilon_2)}}\right)$$

$$2+\gamma+\varphi > 2\sqrt{(\gamma+\alpha\lambda_1)}\sqrt{(\varphi+\varepsilon_2)} + \left(\frac{(1-\lambda_1)(\varphi+\varepsilon_2)+\varepsilon_1(\gamma+\alpha\lambda_1)}{\sqrt{(\gamma+\alpha\lambda_1)}\sqrt{(\varphi+\varepsilon_2)}}\right)$$

$$2+\gamma+\varphi > \frac{2((\gamma+\alpha\lambda_1)(\varphi+\varepsilon_2)) + (1-\lambda_1)(\varphi+\varepsilon_2)+\varepsilon_1(\gamma+\alpha\lambda_1)}{\sqrt{(\gamma+\alpha\lambda_1)(\varphi+\varepsilon_2)}}$$

3. Solution Procedure

The following procedure is developed to determine the continuation of the partnership between manufacturer and buyer by using VMI. Algortihm 1:

Step 1 Calculate the value of $\gamma = F_v / A$, $\lambda_1 = A_p / A$, and $\phi = h_v / h_b$

Step 2 Substituting the value of γ, λ_1, and ϕ into Proposition 3.

Step 3 Examining Proposition 3, whether it meets the criteria? If YES, then go to step 4, otherwise go to step 9.

Step 4 Substituting the value of γ, λ_1, and ϕ into Proposition 1

Step 5 Examining Proposition 1, whether it meets the criteria? If YES, then go to step 6, otherwise go to step 5a.

Step 5a Is the manufacturer intends to evaluate the manufacturer's total inventory cost? If YES, then go to step 5b, otherwise go to step 9.

Step 5b Do improvements to the constraints that affect the manufacturer's total inventory cost, then go to step 4.

Step 6 Substituting the value of γ, λ_1, and ϕ into Proposition 2.

Step 7 Examining Proposition 2, whether it meets the criteria? If YES, then go to step 8, otherwise go to step 7a.

Step 7a Is the manufacturer intends to evaluate the buyer's total inventory cost? If YES, then go to step 7b, otherwise go to step 9.

Step 7b Do improvements to the constraints that affect the buyer's total cost, then go to step 6.

Step 8 Partnership between manufacturer and buyer using VMI is advised to continue.

Step 9 Partnership between manufacturer and buyer using VMI is advised not to continue.

After learning that the partnership can be continue, the manufacturer determines the optimal delivery lot size, numbers of delivery, manufacturer's total inventory cost, buyer's total inventory cost, and overall system's total inventory cost with the following algorithm:

Step 1 Calculate γ, ϕ, and λ_1.

Step 2 Substituting γ, λ_1, and ϕ into equation (8), to get the optimal ordering lot-size with VMI (z_2^*).

Step 3 Set $n = 1$.

Step 4 Substituting n and z_2^* into equation (9), to obtain the manufacturer's total cost (TC_{v2}).

Step 5 Is $n = 1$? If YES, then go to step 5a, otherwise go to step 6.

Step 5a Set $n = n + 1$, then repeat step 4 and step 5, to get the new value of TC_{v2}.

Step 6 Is $TC_{v2}(n) \le TC_{v2}(n-1)$? If YES then go to step 5a, otherwise go to step 7.

Step 7 Set $TC_{v2}(n) = TC_{v2}(n-1)$.

Step 8 Substituting γ, λ_1, and ϕ into equation (10), to get the buyer's total cost (TC_{b2}).

Step 9 Calculate $TC_{v2}(n) + TC_{b2}(n)$ as the total inventory cost for the overall system.

The following procedure is developed to determine the partnership by using VMI and consignment. Algortihm 2:

Step 1 Calculate the value of $\gamma = F_v / A$, $\lambda_1 = A_p / A$, $\phi = h_v / h_b$, $\varepsilon_1 = h_w / h_b$ and $\varepsilon_2 = rC_b / h_b$.

Step 2 Substituting the value of γ, λ_1, ϕ, ε_1, and ε_2 into Proposition 6.

Step 3 Examining Proposition 6, whether it meets the criteria? If YES, then go to step 4, otherwise go to step 9.

Step 4 Substituting the value of γ, λ_1, ϕ, ε_1, and ε_2 into Proposition 4.

Step 5 Examining Proposition 4, whether it meets the criteria? If YES, then go to step 6, otherwise go to step 5a.

Step 5a Is the manufacturer intends to evaluate the manufacturer's total inventory cost? If YES, then go to step 5b, otherwise go to step 9.

Step 5b Do improvements to the constraints that affect the manufacturer's total inventory cost, then go to step 4.

Step 6 Substituting the value of γ, λ_1, ϕ, ε_1, and ε_2 into Proposition 5.

Step 7 Examining Proposition 5, whether it meets the criteria? If YES, then go to step 8, otherwise go to step 7a.

Step 7a Is the manufacturer intends to evaluate the buyer's total inventory cost? If YES, then go to step 7b, otherwise go to step 9.

Step 7b Do improvements to the constraints that affect the buyer's total cost, then go to step 6.

Step 8 Partnership between manufacturer and buyer using consignment is advised to continue.
Step 9 Partnership between manufacturer and buyer using consignment is advised not to continue.

If the partnership can be continue, the manufacturer determines the optimal delivery lot size, numbers of delivery, manufacturer's total inventory cost, buyer's total inventory cost, and overall system's total inventory cost with the following algorithm:

Step 1 Calculate γ, λ_1, ϕ, ε_1, and ε_2.
Step 2 Substituting γ, λ_1, ϕ, ε_1, and ε_2 into equation (14),
to get the optimal ordering lot-size with consignment (z_3^*).
Step 3 Set n = 1.
Step 4 Substituting n and z_2^* into equation (15), to obtain the manufacturer's total cost (TC_{v3}).
Step 5 Is n= 1? If YES, then go to step 5a, otherwise go to step 6.
Step 5a Set $n = n+1$, then repeat step 4 and step 5, to get the new value of TC_{v3}.
Step 6 Is $TC_{v3}(n) \leq TC_{v3}(n-1)$? If YES then go to step 5a, otherwise go to step 7.
Step 7 Set $TC_{v3}(n) = TC_{v3}(n-1)$.
Step 8 Substituting γ, λ_1, ϕ, ε_1, and ε_2 into equation (16), to get the buyer's total cost (TC_{b3}).
Step 9 Calculate $TC_{v3}(n) + TC_{b3}(n)$ as the total inventory cost for the overall system.

4. Numerical Examples

To illustrate the algorithms described earlier, it considers an inventory system with data as follows; for manufacturer: P = 15,000 (unit/period), C_v = 20,000 (IDR/unit), h_v = 2,000 (IDR/unit/period), S = 360,000 (IDR/setup), F_v = 30,000 (IDR/trip), and α = 0.2; for buyer: D = 10,000 (unit/period), C_b = 25,000 (IDR/unit), h_b = 3,500 (IDR/unit/period), A= 100,000 (IDR/order), A_p = 10,000 (IDR/order), D_v = 10,500 (unit/period), C_w = 10,000 (IDR/unit), and r = 0.1. In this example, the values are fixed for $\phi = h_v / h_b$ = 0.571, $\lambda_1 = A_p / A$ = 0.1, $\gamma = F_v / A$ = 0.3, $\varepsilon_1 = h_w / h_b = 0.286$, and $\varepsilon_2 = rC_b / h_b = 0.714$. Referring to Eq. (3) and (4), the buyer's economic lot size is z_1^* = 756 units and the buyer's total inventory cost is $TC_{b1}(z_1^*)$ = 2,646,000. The minimum manufacturer's total inventory cost is obtained at four times delivery with $TC_{v1}^*(z_1^*, = 756, n^* = 4)$ equals to 3,603,000, and for the system is $TC_{s1} = TC_{v1}^*(z_1^*, = 756, n^* = 4)$ + $TC_{b1}(z_1^*)$= 6,249,000. It is showned that both parties has different optimal value for the total inventory cost. Therefore, manufacturer would like to offer a partnership with buyer using Vendor Managed Inventory (VMI). In this case, both parties has already agreed to investigate whether the relationship beneficial. For algorithm 1, substituting γ, ϕ, and λ_1 into Proposition 3, 2, and 1 respectively. The calculation results show that all the citeria are met, hence the relationship between manufacturer and buyer can proceed to VMI. Further by VMI, the manufacturer determines the lot size and the number of deliveries that minimize the total inventory cost for both parties. Next algorithm is substituted the value of γ, ϕ, and λ_1 for 0.3, 0.571, and 0.1 respectively into Eq. (9), to get $z_2 \approx 566$ unit. Moreover, set $n = 1$, substituting z_2 and n into Eq. (10), to get $TC_{v2}(z_2 = 566, n = 1)$ =7,869,000. Repeat step 4 to 6, until the value of $TC_{v2}(n)$ is greater than $TC_{v2}(n-1)$.

With VMI, the results are $TC_{v2}(z_2 = 566, n = 6)$ =3,512,000, TC_{b2} =2,581,000, and TC_{s2} = 6,093,000.
Table 1 shows the saving cost for the manufacturer, buyer, and the overall system with VMI.

Table 1. Saving cost for manufacturer, buyer, and overall system ($F_v = 30{,}000$)

	Without VMI	With VMI	Saving cost
Manufacturer	$TC_{v1} = 3{,}603{,}000$	$TC_{v2} = 3{,}512{,}000$	91,000
	$z_1 = 756, n = 4$	$z_2 = 566, n = 6$	

Buyer	$TC_{b1} = 2,646,000$	$TC_{b2} = 2,581,000$	65,000
System	$TC_{s1} = 6,249,000$	$TC_{s2} = 6,093,000$	156,000

After knowing that the partnership with VMI can be resumed, then the manufactur intends to continue to implement the consignment. Apparently after examined by algorithm 2, the partnership can not proceed when using consignment, because it is not profitable. Therefore, the scenario needs to be changed by altering the transportation cost, F_v, to 400,000 (IDR/trip) and this changes will alter the proportion value of γ from 0.3 to 4.

Finally, under consignment with F_v = 400,000 , the result shows that $TC_{v3}(z_3 = 1337, n = 3)$ = 8,695,000, TC_{b3}=1,342,000, and TC_{s3}= 10,037,000. Consequently for comparing with VMI, then it should use the same scenario to determine which one is more profitable.

With VMI, the results are $TC_{v2}(z_2 = 2005, n = 2)$=6,913,000, TC_{b2}=3,958,000, and TC_{s2} = 10,871,000.

Table 2 shows the saving cost for the manufacturer, buyer, and the overall system with VMI and consignment.

Table 2. Saving cost for manufacturer, buyer, and overall system ($F_v = 400,000$)

	Present (IDRx1,000)	With VMI (IDR x 1,000)		With Consignment (IDRx1,000)	
		Cost	Saving	Cost	Saving
Manufacturer	$TC_{v1} = 8,947$	$TC_{v2} = 6,913$	2,034	$TC_{v2} = 8,695$	252
	$z_1 = 756, n = 4$	$z_2 = 2005, n = 2$		$z_3 = 1337, n = 3$	
Buyer	$TC_{b1} = 2,646$	$TC_{b2} = 3,958$	-	$TC_{b2} = 1,342$	1,034
System	$TC_{s1} = 11,593$	$TC_{s2} = 10,871$	722	$TC_{s2} = 10,037$	1,556

The calculations show F_v = 30,000 for transportation costs is not worth continuing partnership with the consignment, because it doesn't provide the cost savings. Otherwise, the calculations show that the cost of transportation F_v = 400,000, the partnership worth continuing to consignment. In this case, buyer does not obtain any benefit from a partnership with consignment. Analysis solution in partnership with VMI has shown the influence of changing the ratio of manufacturer's transportation cost to the buyer's ordering cost. The results of calculations using interpolation indicates that manufacturer will benefit from cost savings if γ <0.39 and γ >0.78. Thus, the manufacturer will not get this benefit if the value of γ is in the interval (0.39; 0.78). In the partnership with consignment, manufacturer will benefit from the cost savings if γ >3.94, which means that the cost savings would achieved if manufacturer's transportation cost approximately four times the buyer's ordering cost. This indicates that the suitability of the partnership with VMI followed by consignment determined by the value of the parameters used.

5. Conclusions

In this paper, a strategy for partnership with VMI and consignment is modeled. In VMI, the manufacturer has to monitor the buyer's inventory level and make the deliveries. In consignment, the manufacturer still own the product until the product is discharge from the storage to be used by the buyer in the production process. Algorithms are developed to determine manufacturer's optimal strategy. A numerical example was conducted to understand the influence of changing the ratio of manufacturer's transportation cost to the buyer's ordering cost. The numerical study shows that implementation of VMI and consignment must consider the value of parameter used, therefore the benefits of the costs saving can be achieved.

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