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Designing the Milk Collection Network using Integrated Location Routing Approach

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Abstract

Milk collection remains a challenging problem for dairy processors around the world. Previous work on milk collection network has not considered solving the location and routing problem simultaneously. The integrated location routing literature on the other hand remains silent on its application to the milk collection problem. We bridge this gap by considering the integrated location routing problem in the context of a milk collection network. The goal of this paper is to analyze the effect of simultaneously solving the location and routing problem vis-à-vis solving location first and routing second on the optimal solution and computational time. We develop a math formulation for the milk collection network location routing problem (MCNLRP) and for the milk collection network vehicle fleet mix problem (MCNVFMP). We then develop an algorithm for solving MCNLRP and compare the results of the algorithm with the optimal solution for various instances of the problem.

Keywords
Integrated location routing, milk collection network, optimization, algorithm

1. Introduction
The milk collection problem is a challenging logistics problem and is well known in rural areas around the world (Butler, Herlihy and Keenan, 2005; Caramia and Guerriero, 2010). The perishable nature of milk along with the problem of travelling short distances for the collection of milk makes the problem unique (Butler et al., 2005). The unique nature of the problem has generated special interest in milk collection network and we see several papers on the milk collection problem in the literature.

The early work in the area was conducted by Sankaran and Ubgade (1994). The authors study the milk collection problem at Etah Dairy, a unit belonging to Lipton India Ltd. They model the network as single-depot, multi-node, deterministic demand vehicle routing problem with time constraints and capacity restrictions. They develop a heuristic based DSS (decision support system) called CARS (computer aided routing system). The adoption of CARS at Etah brought annual savings of Rs. 1 million. The paper by Butler et al. (2005) discussed how a geographic information system (GIS) based DSS allows scheduler to interact with optimization algorithms to plan milk collection. Claassen and Hendriks (2007) presented an OR-based approach for supporting goat milk collection and proposed the development of a DSS for generating stable milk collection plans. Caramia and Guerriero (2010) studied a real life problem at an Italian dairy company. They developed a mathematical formulation for minimizing the number of vehicles to be routed and the tour length and compared their solution with the process that the company previously used.

Though these papers study the milk collection network their focus is on vehicle selection and routing (Claassen et al., 2007; Caramia et al., 2010). However, cost of vehicle routes in a milk collection network is closely linked to the number and location of chilling centers (the main points where the vehicles originate from and return to) and vice versa. It is therefore important to see the effect of simultaneously considering the location and routing on total milk collection cost. Previous literature on integrated location routing does not specifically address the case of milk collection networks. Moreover, the magnitude of the effect of considering the two together versus solving them separately on the optimal solution and computational time yet needs to be analyzed.

In this paper we study the milk collection system adopted by large dairy processors. The contributions of the paper are as follows:

1. The effect of integrating location and routing decisions versus solving location first and routing next in the context of milk collection network, on solution cost and computation time is analyzed.
2. Properties of MCNLRP are studied.
3. An algorithm is developed, based on the understanding of MCNLRP properties. The algorithm provides better computation time as compared to original problem formulation.

The milk collection problem addressed in this paper can be considered a special case of the integrated location routing problem (LRP). We therefore proceed with a brief discussion of the integrated location routing literature in Section 2. Next, in Section 3, we describe the milk collection network design problem and present its formulation. In section 4 we discuss some properties of the problem and problem relaxation that are useful in developing our algorithmic approach to solving problems of practical nature in the milk collection business. In Section 5, we present formulation for the milk collection network vehicle fleet mix problem. In Section 6, we discuss results from test runs using MCNLRP and MCNVFMP for the milk collection network problem. In Section 7, we present an algorithmic approach to solving practical size MCNLRP. Performance of the approach is discussed in Section 8. Section 9 concludes the paper and offers future research directions.

2. Literature Review

Location routing problems (LRP) are concerned with finding the number and location of facilities to serve an existing set of customers, allocation of customers to these facilities and vehicle tours that would minimize aggregate cost comprising of facility location cost and transportation cost from the serving facilities to customer locations. The problem though has its roots in both facility location and vehicle routing problem, it is different. In the traditional facility location problem direct delivery from the depot to the customers is considered whereas in the location routing problem the depots are linked to customers through vehicle tours (a tour consists of several customers that a vehicle originating at the depot, visits before returning to the depot). As far as the traditional vehicle routing problem is considered it, unlike the LRP, takes facility locations as given.

If in the LRP we consider full truck loads for all customers such that a vehicle starts at the depot and ends at a specific customer location or at the depot after serving only a particular customer, then we have a traditional facility location problem. If on the other hand we consider depot locations as fixed and a vehicle has to traverse a route consisting of several customers, before getting back to the depot then we have a vehicle routing problem.

Webb (1968) and Christofides and Eilon (1969) were among the first to note the error introduced in facility location modeling by considering a direct link between depot and location. Later on Rand (1976) observed that many practitioners are aware of the danger of sub-optimizing by considering depot location and vehicle routing separately. In a paper published in 1989, Salhi and Rand showed the effect of ignoring vehicle routing costs while designing facility locations. In 1999, Salhi and Nagy further investigated the location routing problem and found that the use of location routing together can reduce costs over a long planning horizon within which the routes are allowed to change.

Considering the importance of considering location and routing aspects together into one problem, we see research in this domain geared towards both theoretical development of the field and its practical application (for a list of papers see Nagy and Salhi, 2007). Several papers have been published in this area since 1970s. Chan and Hearn (1976) solved a deterministic location-routing problem, the round-trip location problem, by considering customers located on a tree graph. Drezner (1982) solved the round-trip location problem with up to 5000 pairs of demand points. Laporte and Nobert (1981) presented the first exact algorithm for the general location routing problem.

Following this brief description of the field, we discuss some papers in the LRP literature that are closely related to the work being done in this paper. We specifically talk about papers in LRP literature dealing with single stage deterministic problems, multi-echelon problems, multi-depot and heterogeneous fleet problems, and heuristic methods for LRP.

Jacobsen and Madsen (1980) formulate a problem and heuristic solution procedure for deciding number and locations of transfer points, designing tours from printing office to transfer points and designing tours from transfer points to retailers. The problem is solved using three constructive heuristics for 4500 customers in Denmark.

Bruns and Klose (1996, cited in Prins et al. 2007, pp. 471) 'develop a heuristic for a three-level general LRP with a maximum duration per route. They propose an iterative "location first-route second" heuristic in which cost coefficients for serving customers from a depot are used in the location phase and re-estimated after each routing phase. Hence, the location subproblem is solved independently using a Lagrangean heuristic based on the relaxation of the capacity constraints. From the obtained assignment of customers, a tour construction procedure is executed and followed by an iterative tour improvement heuristic. The overall procedure iterates until the estimate of the cost coefficients is stabilized or a maximum number of iterations is reached. The heuristic is evaluated on instances having up to 10 plants, 50 potential depots, and 100 customers.'

Wu, Low and Bai (2002) develop model and heuristic solutions for the multi-depot location-routing problem. The problem considers multiple depots, limited heterogeneous fleet. They use simulated annealing for solving their problem and compare the results with Perl (1983) and Hansen, Hegedahl, Hjortkjaer and Obel (1994) for the homogeneous fleet case. They furthermore generate and test their own cases for the problem with single and multiple vehicle types.

Barreto, Ferreira, Paixao and Santos (2007) use a cluster analysis based sequential heuristic algorithm to solve a capacitated location-routing problem. They consider a problem of homogeneous fleet with capacity limits. The algorithm starts by constructing group of customers considering the vehicle capacity limit. Then for each customer group (each cluster) the routing is determined optimally (by solving a TSP problem) if the group has 40 customers or less, otherwise a two-stage heuristic procedure is applied that uses a savings method and 3-opt local improvement procedure. Once the routes have been finalized a facility location problem is solved considering each route as a single customer. The authors compare the performance of four heuristic versions and six proximity measures. They find one phase hierarchical method and group average proximity to perform the best among test cases.

Berger, Coullard and Daskin (2007) provide a set-partitioning based formulation of an uncapacitated location-routing problem with distance constraints. The objective is to find the distribution centers to open and their associated vehicle routes. The authors find an alternate set of constraints that reduce the actual number of original constraints in the problem and improve the relaxation bound, are introduced. A branch and price algorithm is then developed to solve instances of the problem. Problems with 10 candidate facility locations and 100 customers with various distance constraints are solved to optimality.

Shittekat and Sorensen (2009) develop a tool for the design of a 3PL transport network i.e. selection of 3PL (3rd party logistics) providers and corresponding transport platforms. The tool generates a set of high quality solutions using tabu search and the Hamming distance metric. The show results from application of the tool on a real-life case study from Toyota. The tool is implemented by Toyota to improve the distribution of its spare parts in Germany. The weight, volume and number of vehicles are fixed.

Nguyen, Prins and Prodhon (2011) develop a two-echelon location-routing model and solve it by GRASP reinforced by a learning process and path relinking. They model the problem with one main depot at the highest echelon, a set of possible satellite locations at the second level and a set of customers at the next echelon. The satellites have fixed capacity and opening cost. A set of primary vehicles with fixed capacity and cost is available at the main depot and a set of secondary vehicles with fixed capacity and cost is shared by the satellites. Fleet size is a decision variable. The authors aim at solving problems with instances having up to 200 customers and 10 satellites.

Although several versions of the location-routing problem exist in literature, the exact problem that we address in relation to the upstream dairy supply chain network design i.e. milk collection network, has not been solved in literature. In our problem we consider a 1-echelon (dispatch points-collection centers) supply chain network with multiple dispatch points whose number and location has to be determined, and a set of collection centers at given locations. The objective is to find the number and location of dispatch points, allocation of collection centers to dispatch points, number and type of vehicles, vehicle routes and allocation of vehicles to routes that would minimize total milk collection cost comprised of dispatch points' operating cost and vehicle transportation cost. All collection center volumes must be collected. The vehicles must complete their tour within a specified time limit. The problem may be classified as deterministic, single stage LRP composed on uncapacitated facility location problem and capacitated heterogeneous vehicle routing problem with time constraints and unlimited number of vehicles.

3. Problem Description and Formulation

3.1 The Problem

Milk collection networks operated by large dairy processors demand milk collection from a large number of small farmers spread over a wide geographical region. Milk collection centers (MCCs) are established in pockets where several hundred liters of milk is available to justify the cost of building and operating these centers. Since milk is a perishable product it must be handled with care. Milk collected at the MCCs is hence chilled. Except for some milk areas close to the plant this milk is taken to chilling centers or dispatch points where milk is chilled again. The milk is transported to the dispatch center within a specified time frame owing to the perishable nature of the product. The milk is transferred to larger vehicles at the dispatch points and then transported to the plant to save transportation costs. The milk collected is transferred from dispatch points to the processing plant daily. Milk collection is done in vehicles of specific sizes. The vehicle on collection routes are smaller as compared to the tankers moving on main route (from dispatch centers to the plant) because 1. They need to travel to areas where larger vehicles cannot move, 2. The total travel is limited by time constraints and it does not make sense to hire a larger size vehicle if the total milk volume on a given route can be collected with smaller vehicles. The vehicles are only used for transporting milk.

See figure 1.1 below for pictorial representation of the milk collection chain.

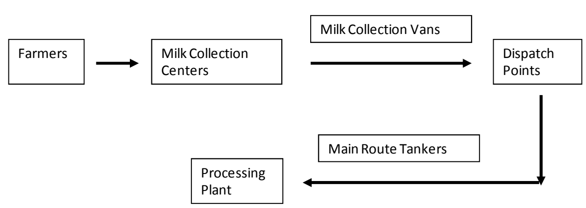

Figure 1.1: Milk Collection System

A milk collection network consists of a number of milk collection centers located at milk pockets, dispatch points and plants. A fleet of vehicles, comprising of vehicles with different capacities and different fixed and variable costs, is available at the dispatch points. The volume of milk at the collection centers varies from one day to another and varies with season. However, supply for a particular day is known before the vehicles leave the dispatch point. The vehicles collect milk from the milk collection centers daily and bring it to its respective dispatch points. All the milk at each milk collection center must be picked. All vehicles must complete their tours within a specified amount of time. We use distance as a proxy to time and introduce a distance-time constraint in our formulation.

The problem facing the dairy processors is basically related to collecting all the available milk at the lowest possible cost. Milk collection costs comprise of dispatch center operating costs and vehicle transportation costs and form a major portion of the total milk cost, other significant cost being the cost of milk itself.

The assumption is that the dispatch points have unlimited capacity which is a fair assumption since the capacity of the dispatch centers is not an issue for the milk processing companies as the total volume that may be collected from a given area is limited, considering the time constraints imposed on vehicles.

Our objective is to simultaneously find the number and location of dispatch points, number of each vehicle type and vehicle routes that would minimize facility operation costs and transportation costs which include vehicle fixed costs and mileage costs.

The constraints are:

1. The total supply from all supply points on a given route must not exceed the capacity of the vehicle assigned to that route
2. The vehicles must collect supplies from all supply points and in full
3. Each supply point must be visited by one vehicle only
4. Each vehicle makes one trip only and starts and ends at the central collection center or dispatch point
5. The vehicle must complete its tour on a route within a specified time

3.2 Milk Collection Network Location Routing Problem (MCNLRP) Formulation

We consider the milk collection network location problem described in the previous section. The problem consists of potential dispatch center locations, milk collection centers at given locations and a heterogeneous fleet of vehicles. The objective is to find the number and location of dispatch centers, the number and type of vehicles originating from each dispatch center, and vehicle routes that minimize the total dispatch center location, vehicle fixed and per mile transportation costs. We use the following notations to formulate our problem.

Notation related to the underlying network

Let m be the number of potential dispatch points, n be the number of milk collection centers, k be the vehicle type in the fleet and r be the number of daily cycles to be completed by a particular vehicle. Then we use the following notations for represent the milk collection network design problem:

J = Set of potential dispatch point locations, J = (n+1,…,n+m)
I = Set of supplier locations, I = (1,…n)

Input Parameters

F_j = Fixed cost of opening dispatch point j.
G_k = Fixed cost of operating vehicle k.
v_k = Variable cost of operating vehicle k.
d_{il} = Distance from node i to node l (a node may be a dispatch point or a collection center).
q_i = Supply (volume) available at the milk collection center i (i = 1,…n).
Q_k = Capacity of vehicle type k.
D = Maximum allowed vehicle travel distance.
R = Maximum number of cycles per vehicle allowed in a day.

Decision Variables

X_j = 1 if dispatch point j is selected
0 otherwise

V_{jk} = 1 if vehicle k originating from dispatch point j is selected
0 otherwise

Y_{jkril} = 1 if vehicle k originating from dispatch point j travelling along arc (i,l) (i = 1,…,n+m; l = 1,…,n+m) completing cycle r is selected
0 otherwise

If the vehicles are allowed single runs on any particular day then index r may be omitted from the binary variable Y_{jkril}.

The following formulation follows from the problem definition and the notations, parameters and decision variables:

$$\text{(P):} \quad \text{Min } Z = \sum_{j=n+1}^{n+m} F_j X_j + \sum_{j=n+1}^{n+m} \sum_{k\in K} G_k V_{jk} + \sum_{j=n+1}^{n+m} \sum_{k\in K} \sum_{r\in R} \sum_{i=1}^{n+m} \sum_{l=1}^{n+m} v_k d_{il} Y_{jkril}$$

Subject to:

$\sum_{l=1}^{n} Y_{jkril} - X_j \leq 0$ $\quad\forall\ i \in I, I= \{1,\ldots,n+m), \quad j \in J, k \in K$ (1)

$\sum_{k\in K} \sum_{r\in R} \sum_{i=n+1}^{n+m} Y_{jkril} = 0;$ $\quad\forall\ j \in J, i \neq j$ (2)

$\sum_{j=n+1}^{n+m} \sum_{k\in K} \sum_{r\in R} \sum_{i=1}^{n+m} Y_{jkril} = 1$ $\quad\forall\ l \in C$ (set of customers) (3)

$\sum_{j=n+1}^{n+m} \sum_{k\in K} \sum_{r\in R} \sum_{l=1}^{n+m} Y_{jkril} = 1$ $\quad\forall\ i \in C$ (set of customers) (4)

$\sum_{i=1}^{n+m} Y_{jkil} = \sum_{i=1}^{n+m} Y_{jkrli}$ $\quad j = n+1,\ldots, n+m; k \in K, l = 1,\ldots,n+m$ (5)

$\sum_{j=n+1}^{n+m} \sum_{k\in K} \sum_{r\in R} Y_{jkril} + \sum_{j=n+1}^{n+m} \sum_{k\in K} \sum_{r\in R} Y_{jkrli} <= 1$
$\forall$ I, $l \in C$ (set of customers) (6)

$Y_{jkrjl} - \sum_{l=1}^{n} Y_{jkrjl} \leq 0$ (7)

$\sum_{i=1}^{n} q_i \sum_{l=1}^{n} Y_{jkil} \leq Q_k V_{jk}$ $\quad j = n+1,\ldots, n+m; k \in K, l = 1,\ldots,n$ (8)

$\sum_{r\in R} \sum_{i=1}^{n+m} \sum_{l=1}^{n+m} d_{il} Y_{jkril} <= D$ $\quad\forall\ j \in J, k \in K$ (9)

Explanation:
Objective Function: Z is a function of number of depots, number of vehicles and distances travelled by the vehicles. The first component is the dispatch point fixed cost, the second component is vehicle fixed cost and the third component is vehicle variable cost. The first and the second components are increasing functions of number of dispatch points. The third component is a decreasing function of number of dispatch points. Note that Z is convex with a minimum at a particular value of number dispatch points (see § 4.1).
Constraints (1): An arc originating from dispatch point node i to supply node j may only be assigned to a vehicle type k originating from dispatch point j if the vehicle is selected (a vehicle may only leave a depot and go on a route if it is selected).
Constraints (2): Assign vehicles to dispatch points (vehicles may only exit and enter depots to which they are assigned).
Constraints (3): A node may only be visited once.
Constraints (4): Flow conservation (a vehicle that enters a node must leave).
Constraints (5): At most one vehicle from a dispatch point will cover an arc i-j.
Constraints (6): A vehicle should not get back to the supply point it has already covered.
Constraints (7): If a vehicle k originating from dispatch point j makes a trip r then it must leave its respective dispatch point j.
Constraints (8): Vehicle capacity (total supply from supply points that a vehicle visits should be less than or equal to vehicle capacity).
Constraints (9): Vehicles must return to their dispatch points within a specified time limit.

Constraints 1 through 7 ensure that vehicle routes are formed such that all supply points (collection centers) are covered, no two vehicles visit the same supply point and vehicles return to their respective dispatch points after collecting milk from all the supply points on their route. Constraints 8 and 9 are vehicle capacity and travel time constraints. Note that for cases where vehicles' capacities are much greater than the supply available at each supply point, vehicles may travel until filled if they are not limited by time constraints. Similarly if the vehicle capacities are small as compared to supply available at individual supply points then time constraints become redundant (given that distances between dispatch points and supply points are small and can be covered within the time limits). Since vehicles of different capacities are available and the volumes at the supply points varies greatly between winter and summer seasons both capacity and time constraints are jointly required.

4. MCNLRP Properties

In this section we discuss some properties of the milk collection network location routing problem that may be helpful in developing our algorithmic approach in the next section.

4.1 Properties based on distance constraints

The objective function in problem formulation (P) comprises of dispatch point fixed costs and vehicle fixed and variable costs. Both the dispatch point fixed costs F_j and vehicle fixed costs G_k are dependent on the number of dispatch points. It is obvious that fixed costs of operating the dispatch points increase as the number of dispatch points increase. The link of vehicle fixed cost with number of dispatch points becomes evident if we consider the functionality of the dispatch points. The vehicle fixed costs increase since there is an associated fleet with each dispatch point that could go out to collect milk and bring back to the dispatch point, otherwise dispatch points do not have a function to perform.

Since both F_j and G_k increase with the number of dispatch points, we combine the two terms in the objective function of (P). The objective function then may be written with a fixed cost component and a variable cost component as follows:

$$Z = FC(m) + VC(m) \tag{10}$$

It is easy to see that Z has a minimum value at the intersection of FC(m) and VC(m). A point of no difference occurs when FC(m) = VC(m) i.e.

$$F_j + \sum_{k \in Sj} G_k = \sum_{k \in Sj} \sum_{i \in Ij} \sum_{l \in Ij} v_k d_{il}$$

S_j = Set of vehicles at dispatch point j.
I_j = Set of collection centers visited by vehicles at dispatch point j.

Since $F_j >> v_k$ and $G_k > v_k$ it is easy to see that more than one dispatch points are required when $\sum_{i \in Ij} \sum_{l \in Ij} d_{il}$ is very large.

Since constraints (9) restrict the problem to take long distances, at least one dispatch point must be established such that all the collection points within that constrained distance are served. Hence, it follows that the collection centers in the larger milk collection network may be partitioned into clusters such that each cluster has no more than one dispatch point and all the collection centers within the cluster are covered.

Moreover, it is easy to see that the length of any route in a given cluster is at least equal to D/2 (straight line return distance of a vehicle).

4.2 Relaxation of (P)

It follows from §4.1 that constraints (1), (2) and (7) may be relaxed. The resulting problem is single facility MCNVFMP. We denote the relaxed formulation by RP.

5. Milk collection network vehicle fleet mix problem (MCNVFMP)

The objective of MCNVFMP is to find the number of vehicles of a given type to be located at each dispatch point and the associated route of each vehicle.

While developing MCNVFMP formulation we dropped the decision variable X_j and considered that the number of dispatch points is known. However, we included the fixed dispatch point operating cost in the objective function. This is to prevent the program from avoiding the fixed costs while selecting vehicles and routes. Constraints (1) that relate to establishing of dispatch points are also dropped.

Resulting formulation, along with notations, input parameters and decision variables is presented below.

Notation related to the underlying network

Let m be the number of potential dispatch points, n be the number of milk collection centers, k be the vehicle type in the fleet and r be the number of daily cycles to be completed by a particular vehicle. Then we use the following notations for represent the milk collection network design problem:

J = Set of dispatch point locations, J = (n+1,…,n+m)
I = Set of supplier locations, I = (1,…n)

Input Parameters

F_j = Fixed cost of opening dispatch point j.
G_k = Fixed cost of operating vehicle k.
v_k = Variable cost of operating vehicle k.
d_{il} = Distance from node i to node l (a node may be a dispatch point or a collection center).
q_i = Supply (volume) available at the milk collection center i (i = 1,…n).
Q_k = Capacity of vehicle type k.
D = Maximum allowed vehicle travel distance.

Decision Variables

V_{jk} = 1 if vehicle k originating from dispatch point j
is selected
0 otherwise

Y_{jkril} = 1 if vehicle k originating from dispatch point j travelling along arc (i,l) (i = 1,…,n+m;
l = 1,…,n+m) completing cycle r is selected
0 otherwise

$$\text{Min } Z = \sum_{j=n+1}^{n+m} F_j X_j + \sum_{j=n+1}^{n+m} G_k V_{jk} + \sum_{j=n+1}^{n+m} \sum_{k \in K} \sum_{i=1}^{n+m} \sum_{l=1}^{n+m} v_k d_{il} Y_{jkril}$$

Subject to:

$$\sum_{k \in K} \sum_{r \in R} \sum_{i=n+1}^{n+m} Y_{jkril} = 0; \qquad \forall\ j \in J, i \neq j \qquad (1)$$

$$\sum_{j=n+1}^{n+m} \sum_{k \in K} \sum_{r \in R} \sum_{i=1}^{n+m} Y_{jkril} = 1 \qquad \forall\ l \in C \text{ (set of customers)} \qquad (2)$$

$$\sum_{j=n+1}^{n+m} \sum_{k \in K} \sum_{r \in R} \sum_{l=1}^{n+m} Y_{jkril} = 1 \qquad \forall\ i \in C \text{ (set of customers)} \qquad (3)$$

$$\sum_{i=1}^{n+m} Y_{jkil} = \sum_{i=1}^{n+m} Y_{jkrli} \qquad j = n+1,\ldots, n+m;\ k \in K,\ l = 1,\ldots,n+m \qquad (4)$$

$$\sum_{j=n+1}^{n+m} \sum_{k \in K} \sum_{r \in R} Y_{jkril} + \sum_{j=n+1}^{n+m} \sum_{k \in K} \sum_{r \in R} Y_{jkrli} <= 1 \qquad \forall\ i, l \in C \text{ (set of customers)} \qquad (5)$$

$$Y_{jkrjl} - \sum_{l=1}^{n} Y_{jkrjl} \leq 0 \qquad (6)$$

$$\sum_{i=1}^{n} q_i \sum_{l=1}^{n} Y_{jkil} \leq Q_k V_{jk} \qquad j = n+1,\ldots, n+m;\ k \in K,\ l = 1,\ldots,n \qquad (7)$$

$$\sum_{r \in R} \sum_{i=1}^{n+m} \sum_{l=1}^{n+m} d_{il} Y_{jkril} <= D \qquad \forall\ j \in J, k \in K \qquad (8)$$

6. Comparison of MCNLRP and MCNVFMP

The two problems were coded in CPLEX and run on a Pentium 2 GHz machine with 2 GB RAM with various data sets. The values for the dispatch point fixed cost, vehicle fixed and variable costs, vehicle capacities were taken from the survey of the milk collection network of a large dairy processor. The data from the volume of milk at the collection centers was generated using uniform distribution U[100,500]. Uniform distribution has been used to generate similar data in previous studies (Uster, Keskin and Cetinkaya, 2008). The range has been selected by observing the fluctuations at the dairy processor during summer season. Distances between various facility nodes in our network have been generated similarly.

Six problem instances have been tested and the results reported in table 6.1.

The results show that MCNLRP's performance is generally better than that of MCNVFMP on both measures of solution cost and computation time. MCNLRP outperformed MCNVFMP on solution cost for all instances tested and performed better on computation time for three out of six test cases. MCNLRP is expected to perform better on cost since location and routing are interrelated. However, MCNVFMP was expected to perform better on computation time, since the decision regarding location of dispatch points is omitted from this formulation.

It was observed that the computation time increases if the time limit is tightened. This may be because with tight limits, the space of feasible options reduces and the best solutions alternative is difficult to find. A more careful analysis is however required to understand the underlying causes of this behavior that may relate to structure of the problem and hence provide valuable insight into the problem. We leave this analysis for future research work.

Another interesting observation is that even for the MCNVFMP the vehicles select which depots to depart from. This happens because the transportation costs are impacted by the number of dispatch points. If the dispatch points increase for a given set of collection centers, the transportation costs decrease as the distance travelled to collect milk decreases with the increase in dispatch points. However, since vehicle fixed costs may increase with the dispatch centers, the overall impact of increasing may not be a decrease in total transportation costs. It is clear though that number of dispatch points have an impact on transportation costs. More interesting results are observed if we lower the dispatch point fixed costs. Since these costs are part of the objective function, and the goal is to minimize the overall cost, lowering dispatch point fixed costs to a certain level makes it better for the vehicles to depart from all the available dispatch points. This would happen at the point of indifference between fixed costs and vehicle transportation costs.

Instance	Dispatch Points	Vehicles (per dispatch point)	Number of Collection Centers	Max. Distance		Solution Cost	Solution Time (secs)	Better on Cost	Better on Time
1	3	10	12	100	MCNLRP	23612.64	7	Yes	Equal
					MCNVFMP	59598.68	7	-	Equal
2	3	10	12	80	MCNLRP	31556.07	17	Yes	-
					MCNVFMP	61972.32	9	-	Yes
3	3	10	12	500	MCNLRP	22770.88	26	Yes	Yes
					MCNVFMP	58770.88	137	-	-
4	3	10	14	100	MCNLRP	25274.68	6.4	Equal	
					MCNVFMP	25274.68	5.6	Equal	Yes
5	3	10	17	150	MCNLRP	23715.16	98	Yes	Yes
					MCNVFMP	59687.24	158	-	-
6	1	10	17	100	MCNLRP	23527.61	776	Yes	Yes
					MCNVFMP	59527.68	1095	-	-

Table 6.1: MCNLRP and MCNVFMP Comparison

7. Algorithmic Approach to Solving MCNLRP

The algorithm is developed considering properties of MCNLRP discussed in §4.

We form clusters of collection centers with one dispatch point. Once the cluster are formed a single dispatch point MCNVFMP is solved.

Following notations are used in the algorithm:

D = Set of dispatch points (1,…,M).

S = Set of collection centers (1,…,N).
d_{ji} = Distance between dispatch point j and collection center i.
L = (l_1,…,l_m), l_1,…,l_m are sets of collections centers to be served by dispatch points 1,…,m'.
T = Temporary holding set, initially T $\leftarrow$ {Ø}.
C = Set of picked dispatch points, initially C $\leftarrow$ {Ø}.

RP Algorithm
Step 0: For each dispatch point m ∈ D, sort the collection centers in order of increasing distance from the dispatch point.

Step 1: Assign collection centers to dispatch points (form clusters of collection centers with one dispatch point).

do until D ≠ {Ø}
  do until S ≠ {Ø}
    while $d_{ji} \leq D/2$
    $l_j = l_j \cup \{i\}$
    S $\leftarrow$ S\{i}
    D $\leftarrow$ D\{j}

Step 2: Pick clusters with same elements i.e. collection centers.

do until L ≠ {Ø}
$T_j \leftarrow l_j$
for all $l_j = l_j'$ { l_j' ∈ L': L' L – {l_j} }
  $T_j \leftarrow T_j \cup \{l_j'\}$
  L $\leftarrow$ L\T_j

Step 3: Pick the best clusters such that all the collection centers are covered.

do until T ≠ {Ø}
  for all $l_j \in T_j$
  compute D = $\sum_{i' \in lj} d_{ji'}$
  pick l_j that gives lowest value of D
  C $\leftarrow$ C U {l_j}

Step 4: Solve RP

8. Performance of RP Algorithm

A network of thirteen collection centers and two dispatch points was selected. Maximum allowed distance was set to 40. The value was carefully selected such that two dispatch points get picked. The original problem formulation (P) achieved a solution cost of 42,345.96 and took 723 seconds to solve. The proposed algorithm got a solution with cost 42,886.8 in 13 seconds. The gap in solution cost was only 1.2% as compared to 98.2% improvement in computation time.

9. Conclusion and Future Research Directions

In this paper we consider a milk collection network design problem. We propose an integrated location routing formulation to the problem, MCNLRP. We study some useful properties of MCNLRP. Next we develop a formulation for the milk collection network vehicle fleet mix problem, MCNVFMP. We test the performance of MCNLRP and MCNVFMP on six instances and find that MCNLRP generally performs better than MCNVFMP on both measures of solution cost and computation time. Next we develop an algorithm to solve problems of practical size. The algorithm performs well on solution cost and computation time.

Future research may test the MCNVFMP on real data of dairy processors. It would be interesting to see the benefits achieved from using the algorithm in real life settings. Another interesting stream of research may be to look at the effect of decentralized planning (as compared to the centralized planning approach considered in this paper) on the both solution cost and computation time.

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Biography

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