Comparing the Median Run Length (MRL) Performances of the Max-EWMA and Max-DEWMA Control Charts for Skewed Distributions

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Abstract

The maximum double exponentially weighted moving average (Max–DEWMA) chart and the maximum exponentially weighted moving average (Max–EWMA) chart are applied to simultaneously monitor small and moderate shifts in the process mean and/or variability. The Max-DEWMA chart has lower Type I error rates compared to the Max-EWMA chart for all levels of skewnesses. Noting that the performances of both charts in the existing literature are based on average run length (ARL) measure. In this paper, the primary objective is to compare the performances of the two charts based on the median run length (MRL) measure. A Monte Carlo simulation was conducted using the Statistical Analysis Software (SAS) to study and compare the MRL performances for various magnitudes of process mean and/or variability shifts. The skewed distributions considered are the Weibull and gamma distributions. Overall, the results show that the Max-DEWMA chart has in-control MRLs which are closer to the specified value compared to that of the Max-EWMA chart, for all levels of skewnesses considered. Quality practitioners are recommended to apply the Max-DEWMA chart when the underlying distribution is skewed.

Keywords
Industry engineering, Median Run Length (MRL), Simulation, Skewed distributions, Statistical Process Control Chart.

1. Introduction
Every process has variations. Quality control is impossible without a tight implementation of a reliable system on a process. Control chart was introduced by Walter A. Shewhart in the 1920s. Since then, it has been widely used by quality practitioners for identification of assignable causes in a process (Montgomery, 2009). A control chart helps triggering the quality practitioners when a special cause occurred in a process thereby reducing variations in the process. When the level of variations is reduced to an acceptable range, the quality of the products is maintained and

the probability of producing defective units is reduced too. In short, a good control chart contributes to a tight process control which helps preventing the operation from making costly mistakes.

To date, the Maximum Double Exponentially Weighted Moving Average (Max-DEWMA) chart (Khoo et al. 2010) is one of the most efficient single EWMA charts in detecting a small or moderate shift in the process mean and/or variability simultaneously. Under the normality assumption, the single Max-DEWMA chart was shown to outperform the Maximum Exponentially Weighted Moving Average (Max-EWMA) chart proposed by Xie (1999) and Chen et al. (2001), for small and moderate shifts. The similar findings obtained by Teh and Khoo (2009) for skewed distribution. The two studies mentioned above were evaluated based on the average run length (ARL) measure.

However, according to Gan (1993), the median run length (MRL) measure provides a more meaningful explanation on the in-control and out-of-control performances of the charts as in-control run length distribution based on the ARL is highly skewed. Moreover, the MRL profile is also more readily understood by the practitioners compared to the ARL profile. Thus, Teh et al. (2013) further extended the work of Khoo et al. (2010) by comparing the performances of Max-EWMA and Max-DEWMA control charts based on MRL under normality assumption. The principal objective of this paper is to further extend the idea of Teh et al. (2013) in which the effect of skewness of the underlying distribution on the MRL performances of the Max-EWMA and Max-DEWMA charts are compared. The Weibull and gamma distributions are considered in this study because they are commonly used by researchers (see Khoo et al. 2008).

The study of skewed distribution is crucial because the normality assumption of the underlying distribution does not hold in many industrial processes. Montgomery (2009) discussed complicatedness in the application of statistical control charting methods to some real data in industries. For example, in reliability engineering, the lifetime and failure rate data for electrical and mechanical components and system follow a Weibull distribution. The failure rate of a product is divided into three phases. The failure rate is high at the introductory stage (early failure) and at the decline stage (wear out period) of the product life cycle. Random or spontaneous failures occur in between the product life cycle and this forms a bathtub curve for the failure rate function. Therefore, the Weibull distribution has been used in situations involving electronic devices, such as memory elements; mechanical components like bearings; and structural elements in aircrafts and automobiles. Many researchers studied control charts for skewed distributions (see Amhemad 2009; Khoo and Kassim 2008; Wang and Xu 2007).

The organization of the remainder of this paper is in the following order: First, a review of the method in constructing the Max-EWMA and Max-DEWMA control charts are described. Some statistical properties and design strategies are presented in Section 3. Section 4 is the evaluation and comparison of the MRL performances of the Max-EWMA and Max-DEWMA charts to be followed by conclusions in Section 5.

2. A Review of the Max-EWMA and Max-DEWMA Control Charts

Xie (1999) and Chen et al. (2001) proposed a single EWMA−type control chart, namely the Max−EWMA chart. The Max−EWMA control chart plots the maximum absolute values of the two independent EWMA statistics; one statistic controls the process mean and the other statistic controls the process variability. Hence, the Max−EWMA chart is capable of jointly monitoring the process mean and variability by using a single plotting variable. When an out-of-control signal is indicated, the chart is able to identify the source and direction of the shift. The findings of Xie (1999) and Chen et al. (2001) indicated that the ARL-based Max−EWMA chart is effective in detecting small changes in the process.

Assume a series of random observations, $X_{ij} \sim N\left(\mu + a\sigma, b^2\sigma^2\right)$, for i = 1, 2, 3, … and j = 1, 2, 3, …, n_i, where i is the sample number, j is the observation number; while a and b (>0) are constants. Note also that μ_0 and σ_0 represent the target values of the mean and standard deviation, respectively. The process is in-control when (a, b) = (0, 1); otherwise, the process has shifted. Let the sample mean and variability for i^{th} sample labeled as

$$\bar{X}_i = \frac{X_{i1} + X_{i2} + ... + X_{in_i}}{n_i} \quad (1)$$

and

$$S_i^2 = \frac{\sum_{j=1}^{n_i}\left(X_{ij} - \bar{X}_i\right)^2}{n_i - 1}, \tag{2}$$

respectively. These two statistics are the uniformly minimum variance of the unbiased estimators, μ_0 and σ_0^2, respectively, but they follow different distributions. The following two independent statistics are defined as follows:

$$U_i = \frac{\bar{X}_i - \mu}{\sigma/\sqrt{n_i}} \tag{3}$$

and

$$V_i = \Phi^{-1}\left\{H\left[\frac{(n_i - 1)S_i^2}{\sigma^2}; n_i - 1\right]\right\}. \tag{4}$$

Note that in Equation (4), $\Phi^{-1}(\,\cdot\,)$ denotes the inverse standard normal cumulative distribution function and $H(\,\cdot\,; n_i - 1)$ denotes the chi-square distribution function with degrees of freedom $n_i - 1$. Both U_i and V_i in Equations (3) and (4), respectively, are independent statistics having a common standard normal distribution that do not depend on the sample size n_i, when the process is in-control.

The two EWMA statistics computed from U_i and V_i are defined as follows:

$$Y_i = (1-\lambda)Y_{i-1} + \lambda U_i, \quad \text{for } i = 1, 2, \ldots \tag{5}$$

and

$$Z_i = (1-\lambda)Z_{i-1} + \lambda V_i, \quad \text{for } i = 1, 2, \ldots\ . \tag{6}$$

Here, $0 < \lambda \le 1$ is the smoothing constant; while Y_0 and Z_0 which are equal to zero, are the initial values of Y_i and Z_i, respectively. A single plotting statistic of the Max–EWMA chart to combine Y_i and Z_i is defined as (Xie, 1999 and Chen et al. 2001)

$$M_i = \max\left\{|Y_i|, |Z_i|\right\}. \tag{7}$$

Since $M_i \ge 0$, the Max–EWMA chart only needs an upper control limit (UCL_{ME}) which is given by

$$\begin{aligned} UCL_{ME} &= E(M_i) + K_{\mathrm{ME}}\sqrt{V(M_i)} \\ &= \left(1.128379 + 0.602810K_{ME}\right)\sqrt{\frac{\lambda\left[1-(1-\lambda)^{2i}\right]}{2-\lambda}}, \end{aligned} \tag{8}$$

for i = 1, 2, …, where $E(M_i)$ and $V(M_i)$ are the mean and variability of M_i, respectively, when the process is in-control; while K_{ME} is a multiplier that controls the width of the limit. An out-of-control signal is indicated by the Max–EWMA chart when $M_i > UCL_{\mathrm{ME}}$.

Khoo et al. (2010) explored the idea of using double exponential smoothing on the Max–EWMA chart. From the two EWMA statistics, Y_i and Z_i given in Equations (5) and (6), respectively, they introduced the following two corresponding DEWMA statistics:

$$W_i = (1-\lambda)W_{i-1} + \lambda Y_i, \quad \text{for } i = 1, 2, \ldots \tag{9}$$

$$Q_i = (1-\lambda)Q_{i-1} + \lambda Z_i, \quad \text{for } i = 1, 2, \ldots \tag{10}$$

Here, W_0 and Q_0 which are equal to zero, are the initial values of W_i and Q_i, respectively. Khoo et al. (2010) used a simple technique of setting same values of smoothing constants for Y_i, Z_i, W_i and Q_i, in Equations (5), (6), (9) and (10), respectively. Then, the two DEWMA statistics in Equations (9) and (10) are combined into the following single statistic for the proposed Max–DEWMA chart, i.e.

$$L_i = \max\left\{|W_i|, |Q_i|\right\}. \tag{11}$$

The interpretation of the value of L_i is similar to that of the Max-EWMA chart. Similarly, the Max–DEWMA chart only has an upper control limit, given by Khoo et al. (2010), i.e.

$$UCL_{MD} = E(L_i) + K_{MD}\sqrt{V(L_i)},$$

$$= \left(1.128379 + 0.602810 K_{MD}\right) \times$$

$$\sqrt{\frac{\lambda^4}{\left[1-(1-\lambda)^2\right]^3}\left\{1+(1-\lambda)^2-\left(i^2+2i+1\right)(1-\lambda)^{2i}+\left(2i^2+2i-1\right)(1-\lambda)^{2i+2}-i^2(1-\lambda)^{2i+4}\right\}}, \quad (12)$$

for i = 1, 2, …, where $E\left(L_i\right)$ and $V\left(L_i\right)$ are the mean and variability of L_i, respectively; while K_{MD} is a multiplier that controls the width of UCL_{MD}.

3. Statistical Properties and Design Strategies

Skewness is a measure of the degree of asymmetry for a distribution. A distribution is symmetric if the median divides the left side and the right side into two identical regions. The sample skewness is measured with the following equation (Kenney and Keeping 1962):

$$\text{Skewness} = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^3}{(n-1)S^3}, \quad (13)$$

where n is the sample size and S is the sample standard deviation. The skewness for a symmetric distribution has a value of zero. Left tail is longer relative to the right tail, indicating a negative skewed; while right tail is longer relative to the left tail, indicating a positive skewed.

The Weibull and gamma distributions are considered in this work because these distributions are very flexible. By appropriately selecting the parameters, they can represent a wide variety of shapes, ranging from nearly symmetric to highly skewed. For convenience, a scale parameter of one is selected for both the Weibull and gamma distributions. It is worth noting that the skewness is independent from the parameters of these distributions.

For a Weibull distribution, with a location parameter zero and scale parameter one, its cumulative distribution function (cdf) is given by Johnson at al. (1995) as

$$F(y) = 1 - e^{-(\omega y)^{\beta}}, \quad \textit{for } y \geq 0, \quad (14)$$

where ω >0 is the scale parameter and β >0 is the shape parameter. Note that when β = 1, the Weibull distribution reduces to the exponential distribution with mean ω. Letting ω = 1 and $P_Y = Pr(Y \leq \mu)$, where μ is the target mean value of Y, we have

$$P_Y = 1 - exp\left\{-\left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^{\beta}\right\}, \quad \textit{for } y \geq 0. \quad (15)$$

For a gamma distribution with a location parameter zero and scale parameter one, its cdf is given as (Johnson and Kotz 1970)

$$F(y) = \frac{\Gamma_Y(\alpha)}{\Gamma(\alpha)}, \quad \textit{for } y \geq 0, \ \alpha \geq 0, \quad (16)$$

where $\Gamma_Y(\alpha) = \int_0^y m^{\alpha-1} e^{-m}\, dm$ and $\Gamma(\alpha) = \int_0^{\infty} m^{\alpha-1} e^{-m}\, dm$. Then, for this case

$$P_Y = F(\alpha) \quad (17)$$

since $\mu = \alpha$. Here, α denotes the shape parameter of the gamma distribution. Similar to the Weibull distribution, when α = 1, the gamma distribution reduces to the exponential distribution with mean 1.

For the sake of comparison, besides the Weibull and gamma distributions, the normal distribution is also considered. Note that the skewness coefficient, γ, is unique for a given value of β or α. The shape parameters, β for the Weibull distribution and α for the gamma distribution, are determined so that the skewness coefficient, $\gamma \in$ {0.5, 1.0, 1.5, 2.0, 2.5, 3.0}. A skewness coefficient of zero indicates symmetry. The skewness coefficient, γ = 0.5 and 1.0 represent low levels of skewness; γ = 1.5 and 2.0 represent moderate levels of skewness; and γ = 2.5 and 3.0 represent high levels of skewness. A shift in the mean is represented by $\mu_{Y,1} = \mu_{Y,0} + \delta\sigma_{Y,0}$, where δ > 0 is the magnitude of a shift, in terms of the number of standard deviation units; while $\mu_{Y,0}$ and $\sigma_{Y,0}$ represent the in-control mean and in-control standard deviation, respectively. Note that we only consider the in-control process, i.e., when δ = 0. For a random variable, *Y*, from the Weibull and gamma distributions, their in-control means are

$$\mu_{Y,0} = \Gamma\left(1+\frac{1}{\beta}\right), \tag{18}$$

and

$$\mu_{Y,0} = \alpha , \tag{19}$$

respectively; while their in-control standard deviations are

$$\sigma_{Y,0} = \sqrt{\Gamma\left(1+\frac{2}{\beta}\right) - \left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^2} , \tag{20}$$

and

$$\sigma_{Y,0} = \sqrt{\alpha} , \tag{21}$$

respectively (Khoo et al. 2008).

4. A Comparison of the Performances of the Max-EWMA and Max-DEWMA Charts

Monte Carlo simulations are conducted using the Statistical Analysis Software (SAS) version 9.2 to compare the performances of the Max-EWMA and Max-DEWMA charts based on MRL, for monitoring the process mean and/or variability. These are made by setting MRL_0 of both charts at a fixed level of 250. The sample size, n = 5 and $\lambda \in \{0.05, 0.10, 0.20, 0.30, 0.50, 0.60, 0.80, 1\}$ are considered. As expected, other MRL_0s and sample sizes will also give similar results. Here, the exact limits of both charts are considered. A chart with an MRL_0 value closer or equal to 250 for most of the cases is considered as the best chart. The normal, Weibull and gamma distributions are considered in the computation of the MRL_0s of the Max-EWMA and Max-DEWMA charts.

Table 1 gives the MRL_0s for the Max-EWMA and Max-DEWMA charts when n = 5 and ARL_0 = 250. The various combinations of (λ, K_{ME}) for the Max-EWMA chart and (λ, K_{MD}) for the Max-DEWMA chart were obtained from Teh et al. (2013). Generally, the Max-DEWMA chart gives MRL_0 values closer or equal to 250 compared to its Max-EWMA counterpart in most of the cases, when the underlying population is skewed. For both of the Max-DEWMA and Max-EWMA control charts, when the level of positive skewness, γ, increases from 0.0 to 3.0, the probability of a sample point exceeding the control limit increases. Thus, the Type I error rate increases or the MRL_0 decreases. This condition is true for all the Weibul and gamma distributions. For the normal population, all the MRL_0s of both charts are equal to 250. When the skewness level, γ equals to zero, the distribution of the data is symmetric; therefore, the MRL_0s for the Weibull and gamma distributions approach the MRL_0 value of a normal distribution, i.e. 250. It is worth noting that all the entries for both charts are the same when λ = 1.00. This is because the Max-DEWMA chart reduces to the Max-EWMA chart when λ becomes unity.

5. Conclusions

A good understanding of a control chart is important as this helps to increase the confidence of quality practitioners. Therefore, the MRL is chosen as the design measure in this study because it is more readily comprehensible than the ARL by the quality practitioners. This study of the weighted control charts indicates that the Max-DEWMA chart is superior to the Max-EWMA chart for a simultaneous monitoring of the process mean and/or variability, for skewed populations (i.e., the Max-DEWMA chart produces MRL_0 values closer or equal to 250 than the Max-EWMA chart). This is true irrespective of the skewness, γ, smoothing constant, λ, and the type of skewed distribution.

The findings are based on the MRL criterion instead of relying on the ARL. Hence, the MRL can act as a favourable substitute to the existing ARL in the evaluation of the speed of a chart to detect shifts in a process. This is further supported by Teoh et al. (2013), where the shape of the run length distribution changes with the process mean shifts, ranging from highly skewed when the process is in-control to almost symmetric when the process mean shift is large. The MRL provides an insightful and a fair representation of the central location tendency.

Among the potential future works on this topic that are worthy of pursuing are as follow:

(i) To evaluate the performances of the Max-EWMA and Max-DEWMA charts, based on heavy tailed distributions, such as the Student-*t* or Cauchy distributions.

(ii) To measure the performances of the Max-EWMA and Max-DEWMA charts when their smoothing constants, λ's, have different weights.

(iii) To compare the performances of the Max-EWMA and Max-DEWMA charts for skewed populations, in terms of the standard deviation of the run length (SDRL) and percentage points of the run length distribution.

Table 1: The MRL$_1$s for the Max-EWMA and Max-DEWMA charts when n = 5 and MRL$_0$ 250

Parameters		λ	0.05		0.10		0.20		0.30	
			K_{ME}	K_{MD}	K_{ME}	K_{MD}	K_{ME}	K_{MD}	K_{ME}	K_{MD}
			2.757	1.930	3.038	2.327	3.250	2.743	3.343	2.998
	Distribution	γ	M-E	M-D	M-E	M-D	M-E	M-D	M-E	M-D
	Normal		250	250	250	250	250	250	250	250
	Weibul									
	3.6286	0.0	284	**280**	302	**284**	308	**282**	319	**298**
	2.2266	0.5	263	269	264	264	253	257	247	260
	1.5688	1.0	143	**153**	142	**154**	131	**150**	120	**145**
β	1.2123	1.5	52	**56**	54	**60**	54	**60**	52	**59**
	0.9987	2.0	22	22	23	**26**	24	**26**	24	**25**
	0.8598	2.5	12	11	14	14	14	**15**	14	**15**
	0.7637	3.0	8	7	9	9	9	**10**	9	**10**
	Gamma									
	38000	0.0	250	258	253	262	251	248	256	**252**
	15.4	0.5	221	**236**	219	**226**	198	**216**	191	**210**
	3.913	1.0	128	**138**	131	**140**	117	**134**	105	**134**
α	1.788	1.5	55	**57**	57	**62**	55	**62**	53	**62**
	0.983	2.0	22	**23**	23	**26**	24	**26**	24	**26**
	0.648	2.5	11	11	12	**13**	12	**14**	12	**14**
	0.442	3.0	6	5	7	7	7	**8**	7	**8**

Table 1: Continuous

Parameters		λ	0.50		0.60		0.80		1.00	
			K_{ME}	K_{MD}	K_{ME}	K_{MD}	K_{ME}	K_{MD}	K_{ME}	K_{MD}
			3.404	3.280	3.416	3.354	3.428	3.414	3.427	3.427
	Distribution	γ	M-E	M-D	M-E	M-D	M-E	M-D	M-E	M-D
	Normal		250	250	250	250	250	250	250	250
	Weibul									
	3.6286	0.0	332	**308**	345	**316**	354	342	349	349
	2.2266	0.5	233	**251**	230	**241**	224	**228**	218	218
	1.5688	1.0	103	**128**	95	**117**	85	**94**	82	82
β	1.2123	1.5	46	**56**	43	**52**	39	**42**	37	37
	0.9987	2.0	24	**26**	23	**25**	23	23	23	23
	0.8598	2.5	14	14	14	14	15	14	16	16
	0.7637	3.0	9	9	9	9	10	9	11	11
	Gamma									
	38000	0.0	252	**251**	249	252	250	246	256	256
	15.4	0.5	167	**192**	159	**184**	148	**157**	141	141
	3.913	1.0	87	**114**	78	**102**	70	**77**	66	66
α	1.788	1.5	46	**57**	43	**53**	40	**43**	37	37
	0.983	2.0	23	**25**	22	**24**	23	23	22	22
	0.648	2.5	12	**13**	12	**13**	13	13	13	13
	0.442	3.0	7	**8**	7	7	8	7	8	8

Remarks: M-D: M-E: Max-EWMA; Max-DEWMA; Boldfaced MRL$_0$s are closer to 250

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Biography

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