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Designing a Variables Two-Plan Sampling System with Process Loss Consideration

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Abstract

Acceptance sampling had become an essential practical tool for quality assurance application, closely related with incoming inspection. Acceptance sampling plans provide the decision rule for product acceptance to meet the preset product quality required by the vendor and the buyer. As today's modern paradigm of quality improvement, reduction of variance from the target value is the guiding principle as well as reduction on fraction of defective. The C_{pm} index particularly adopts the loss function approach, which distinguishes product quality by increasing penalty to products deviating from the target. This paper proposes variables two-plan sampling system based on process capability index C_{pm} that takes into account departures from target value as well as changes in the process variation. The operating procedure and the conditions to determine the plan parameter are provided. The plan parameter of the proposed sampling plans determine effectively by solving the two nonlinear equations simultaneously. The behavior of the proposed two-plan sampling system is also discussed. For practical purpose, table of the required sample size and the corresponding critical acceptance value under various parameter settings is provided.

Keywords
Product acceptance determination; process capability analysis; variables two-plan sampling system; process loss

1. Introduction

Inspection of incoming products is an integral part of the order and quality contract between vendor and buyer. For this purpose, acceptance sampling had become one of the essential statistical tools that provide vendor and buyer decision rules for product acceptance determination to meet their quality requirement. Acceptance sampling majorly classified as variables and attributes. It has been investigated that variables sampling has more advantage over attributes sampling. Under the same protection both to the vendor and the buyer, variables sampling would require less sampling compare to attributes sampling (Montgomery 2009). Even the inspection in variable sampling is costly, the significant reduction on sample number will more than offset the increasing cost due to time consuming especially when employed the destructive test.

Sampling system is a collection of two or more sampling plans together with criteria by which appropriate plans may be chosen and the switching rules as well (Schiling and Neubauer 2009). A sampling system involve only normal and tightened sampling plan consider as tightened-normal-tightened (TNT) system, simply called two-plan system. Tightened inspection will be used when the supplier's recent quality has deteriorated and normal inspection will be used when the quality is found to be very good.

Calvin (1977) firstly utilized two single-sampling plans for attribute—TNT-$(n_{\mathrm{T}}, n_{\mathrm{N}}; 0)$, take n_{T} sample size for tightened inspection, n_{N} sample size for normal inspection, and apply zero acceptance number. Several studies have made on variables two-plan sampling system, say Muthuraj and Senthilkumar (2006), Senthilkumar and Muthuraj (2010), and Balamurali and Jun (2009). However, the existing variable TNT systems are designed specifically for process with one-sided specification limit. All are applied approximation approach by assuming the quality characteristic follow normal distribution.

Modern's paradigm of quality improvement has championed by Taguchi (Taguchi, 1986) in which reduction of variance from the target value is the guiding principle as well as reduction on fraction of defective (Boyle, 1991 and Pearn and Wu, 2006). The index C_{pm} measures the ability of the process to cluster around the target and reflect the degree of process centering. The C_{pm} index particularly adopts the loss function approach, which distinguishes product quality by considering cost of deviating quality characteristic from the target value. This C_{pm} index also called loss-based index or Taguchi index.

In this paper, we developed a two-plan sampling system based on C_{pm} index for lot sentencing using measurement data required by variables sampling. The proposed sampling plan is based on exact sampling distribution rather than approximation. Hence, the decisions made are more accurate and reliable. The concept of C_{pm} index is reviewed, and the development of the proposed sampling system is discussed. The behaviors of the proposed method accompanied by the tabulated solved plan parameters are also provided.

2. Process Capability Index C_{pm}

Process capability indices (PCI's) include C_p and C_{pk} have been widely used in manufacturing industry as a unitless measure of the capability process to produce items meeting the quality/reliability requirement preset in factory (Kane, 1986) . Hsiang and Taguchi (1985) and Chan et al. (1988) separately introduced the C_{pm} index, and further note as first-generation modification. The C_{pm} index incorporates two components—the target value and the process variability.

The C_{pm} index is defined as $C_{pm} = (\mathrm{USL} - \mathrm{LSL}) / 6\tau = (\mathrm{USL} - \mathrm{LSL}) \Big/ 6\sqrt{\sigma^2 + (\mu - T)^2}$. Where USL and LSL are the upper and lower specification limits of the process, τ is standard deviation from target, σ^2 is the process variance, μ is the process mean, T is the process target, and $(\mu - T)^2$ is the deviation of the process mean from the target. Boyle (1991) provides the estimator of C_{pm} − $\hat{C}_{pm}$ defined as $\hat{C}_{pm} = (USL - LSL) \Big/ 6\sqrt{s_n^2 + (\bar{X} - T)^2}$. Where $\bar{X} = \sum_{i=1}^{n} x_i / n$ and $s_n^2 = \sum_{i=1}^{n} (X_i - \bar{X})^2 / n$ are the maximum likelihood estimators of μ and σ^2, respectively.

Under normally distributed process, the CDF of $\hat{C}_{pm}$ is expressed as mixture of the chi-square and normal distributions as follow (Chan et al., 1988)

$$F_{\hat{C}_{pm}}(y) = 1 - \int_0^{b\sqrt{n}/(3y)} G\left(\frac{b^2 n}{9y^2} - t^2\right) \times \left[\phi\left(t + \xi\sqrt{n}\right) + \phi\left(t - \xi\sqrt{n}\right)\right] \mathrm{d}t \tag{1}$$

for $y \geq 0$, where $b = 3C_{pm}\sqrt{(1+\xi^2)}$, $\xi = (\mu - T)/\sigma$, $G(.)$ is the CDF of the chi-square distribution with degree of freedom $n-1$, χ^2_{n-1} . $\phi(.)$ is the probability density function of the standard normal distribution $N(0,1)$. For fixed values of y and n , we would obtain the identical equation if ξ is substituted by $-\xi$.

3. Designing two-plan system based on C_{pm}

Consider the quality characteristic of interest has both the lower and the upper specification limit as the acceptable value and follows normal distribution, the operating procedure of the proposed two-plan system as follows:

Step 1: Conduct inspection under tightened single sampling plan with sample size n_{T} and critical acceptance value k. Accept the lot if $\hat{C}_{pm} \geq k$. When t lots in a row are accepted then switch to normal inspection.

Step 2: Conduct inspection under normal single sampling plan with sample size n_{N} and critical acceptance value k. Reject the lot if $\hat{C}_{pm} < k$. When an additional lot is rejected in the next s lots after rejection, switch to tightened inspection.

Thus, the plan parameters include n_{T}, n_{N}, k, t, and s. Where n_{T} and n_{N} are sample size under tightened and normal inspection respectively, k is acceptance criteria, t and s are the switching criteria to normal and tightened inspection respectively.

The variables two-plan system undergoes as long as production is steady. The lots are submitted as the order of production and expected to have the same quality. If the quality of the submitted lot is C_{pm}, the performance measure in terms of the probability of acceptance for a single inspection, known as operating characteristic (OC) function based on the C_{pm} index is:

$$P_a(C_{pm}) = P\left(\hat{C}_{pm} > k\right) = 1 - P\left(\hat{C}_{pm} < k\right) = 1 - F_{\hat{C}_{pm}}\left(y\right) \tag{2}$$

According to Calvin (1977), for the two-plan system, the cumulative probability of acceptance, known as the eventual probability of acceptance, $\pi_A(C_{pm})$, is given by

$$\pi_A(C_{pm}) = \frac{P_{\mathrm{T}}(C_{pm})\left(1-\left[P_{\mathrm{N}}(C_{pm})\right]^s\right)\left(1-\left[P_{\mathrm{T}}(C_{pm})\right]^t\right)\left[1-P_{\mathrm{N}}(C_{pm})\right] + P_{\mathrm{N}}(C_{pm})\left[P_{\mathrm{T}}(C_{pm})\right]^t\left[1-P_{\mathrm{T}}(C_{pm})\right]\left(2-\left[P_{\mathrm{N}}(C_{pm})\right]^s\right)}{\left(1-\left[P_{\mathrm{N}}(C_{pm})\right]^s\right)\left(1-\left[P_{\mathrm{T}}(C_{pm})\right]^t\right)\left[1-P_{\mathrm{N}}(C_{pm})\right] + \left[P_{\mathrm{T}}(C_{pm})\right]^t\left[1-P_{\mathrm{T}}(C_{pm})\right]\left(2-\left[P_{\mathrm{N}}(C_{pm})\right]^s\right)} \tag{3}$$

where $P_{\mathrm{T}}(C_{pm})$ and $P_{\mathrm{N}}(C_{pm})$ are the probability of lot acceptance based on C_{pm} under tightened and normal sampling plans, respectively.

The proposed sampling system is determined by contract between vendor and buyer with certain desirable risk. In order to design the two-plan system with specified OC curve, a commonly used approach is to require that the OC curve pass through two specified points e.g $\left(C_{\mathrm{AQL}}, 1-\alpha\right)$ and $\left(C_{\mathrm{RQL}}, \beta\right)$. Where C_{AQL} and C_{RQL} are the capability requirements corresponding to the AQL (acceptable quality level) and RQL (rejectable quality level) based on the C_{pm} index, respectively, AQL is the lowest level of quality for vendor's process that buyer consider acceptable as process average. RQL represents the lowest level of quality that buyer is willing to accept. The probability of acceptance must be greater than $100(1-\alpha)\%$ if the quality level of submitted lot is at AQL. While the quality level of submitted lot is at RQL, the probability of acceptance would be no more than $100\beta\%$. Hence, the plan parameters are the solution to the following two nonlinear equations simultaneously.

$$\pi_A\left(C_{\mathrm{AQL}}\right) \geq 1-\alpha \tag{4}$$

and

$$\pi_A(C_{\mathrm{RQL}}) \leq \beta \tag{5}$$

Under certain value of s and t, the three plan parameters $\left(n_T, n_{\mathrm{N}}; k\right)$ should be determined. There will be several possible solutions that satisfy the two conditions. Hence, the adjustment may consider that the sample size for tightened inspection n_{T} must greater than the sample size of normal inspection n_{N}. n_{T} can be considered as multiplication of n_{N} with a value of m $(m>1)$ i.e $n_{\mathrm{T}} = m \times n_{\mathrm{N}}$, as suggested by Soundararajan and Vijayaraghavan (1992). It is necessary to keep that number of sample for tightened inspection n_{T} always larger than number of sample for normal inspection n_{N}, $\left(n_{\mathrm{T}} > n_{\mathrm{N}}\right)$. Then, the only parameter left need to be determined is $\left(n_{\mathrm{N}}; k\right)$, under the given value of m, s, and t, The required sample size n_N is the least possible integer value of n_N satisfying Eqs. (4) and (5). Moreover, to illustrate how we solve the above two nonlinear simultaneous equations, let

$$S_1(n_N;k) = \pi_A(C_{AQL}) - (1-\alpha) \tag{6}$$

and

$$S_2(n_N;k) = \pi_A(C_{RQL}) - \beta \tag{7}$$

4. Computation and Discussion

The extensive calculations are performed to obtain the plan parameters under various combinations of the entry parameter for convenience application. In order to show how to solve the two nonlinear simultaneous equations, some figures are presented. Under $(C_{AQL}, C_{RQL}) = (1.50, 1.00)$, $(\alpha,\beta) = (0.05, 0.01)$, $(s,t) = (1,1)$, and $m = 1.5$, Fig.1(a) display the surface and (b) contour plots of Eqs. (6) and (7) simultaneously. From Fiq. 1 we determine the plan parameters as the intersection between $S_1(n_N;k)$ and $S_2(n_N;k)$ at level 0, which is result $(n_N;k) = (40;1.2693)$.

Regarding to the switching rule (s,t), we examine several possible pairs as suggested by Muthuraj and Senthilkumar (2006) – $(s,t) = (1,1),(1,2),(2,3)$, and $(4,5)$. The investigation of the effect of different switching rule can be explained by the OC curve as present in Fig.2. Under $(C_{AQL}, C_{RQL}) = (1.50, 1.00)$, $(\alpha,\beta) = (0.05, 0.01)$, and $m = 3$, it is clearly observed that the combination of $(s,t) = (4,5)$ have more steeper OC curve. This pattern also work for another plan under any value of m. Means that the proposed sampling system has more discriminatory power compare to traditional single sampling plan (Wu and Pearn, 2006) i.e when $(s,t) = (1,1)$ and $m = 1$. Therefore, we are focusing the analysis of the proposed two-plan sampling system under the switching rule $(s,t) = (4,5)$.

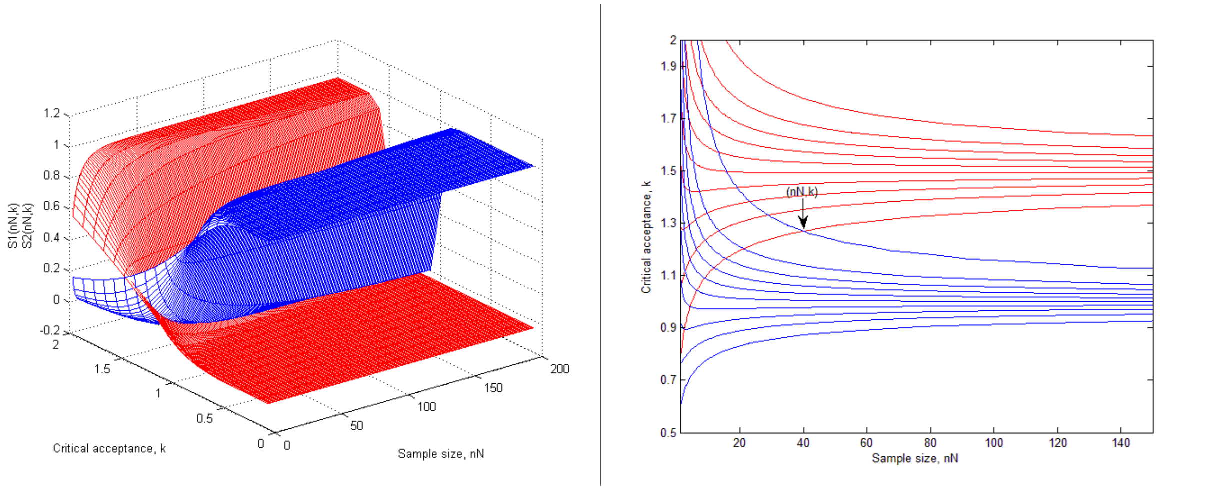

Fig. 1. (a) the surface and (b) contour plots of the simultaneously solution of $S_1(n_N;k)$ and $S_2(n_N;k)$

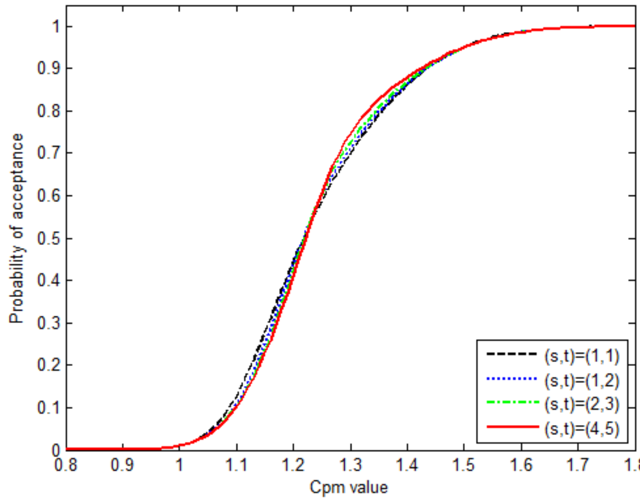

Fig. 2. OC curve for different (s,t) under $(C_{\mathrm{AQL}}, C_{\mathrm{RQL}}) = (1.50, 1.00)$, $(\alpha, \beta) = (0.05, 0.10)$, and $m = 3$

For convenience application of the proposed two-plan system to practitioners, we calculate and tabulate the plan parameters $(n_{\mathrm{N}}; k)$ under various quality level $(C_{\mathrm{AQL}}, C_{\mathrm{RQL}})$, producer's risk α, and consumer's risk β, multiplication number m, for $(s,t) = (4,5)$ as provided at Table 1. For instance, under $(C_{\mathrm{AQL}}, C_{\mathrm{RQL}}) = (1.50, 1.00)$, $(\alpha, \beta) = (0.10, 0.05)$, and $m = 2$, then $n_{\mathrm{N}} = 18$ and $n_{\mathrm{T}} = 36$ should be sampled for normal and tightened inspection respectively with critical value $k = 1.2522$. First conduct inspection under tightened single sampling plan by take 36 samples and accept the lot if $\hat{C}_{pm} \geq 1.2522$. If $t = 5$ lots in a row are accepted, then switch to normal inspection. Following inspection conduct by take 18 samples from the lot and reject the lot if $\hat{C}_{pm} < 1.2522$. If an additional lot is rejected in the next $s = 4$ lots after rejection, switch to tightened inspection. This sampling plan will protect the buyer from making the wrong decision to accept the bad lot with probability 5%, and protect the vendor from the wrong decision to reject the god lot with probability 90%.

Table 1. The plan parameter $(n_{\mathrm{N}}; k)$ under various value of $(C_{\mathrm{AQL}}, C_{\mathrm{RQL}})$, (α, β), and m

$(C_{\mathrm{AQL}}, C_{\mathrm{RQL}})$	α	β	$m = 1.5$		$m = 2$	
			n_{N}	k	n_{N}	k
(1.33, 1.00)	0.05	0.01	78	1.177726	66	1.16629
	0.1	0.05	42	1.174918	35	1.164718
(1.50, 1.00)	0.05	0.01	39	1.269418	33	1.252238
	0.1	0.05	21	1.26738	**18**	**1.252254**

5. Conclusion

This study attempt to construct two-plan sampling system based on capability index C_{pm}. The operating characteristic (OC) curve of the proposed sampling system is derived based on exact sampling distribution and it is required to pass two designed point. The plan parameters are determined by solving two non-linear equations in order to satisfy the producer's and consumer's risk simultaneously. For convenience practical application of the proposed sampling system, the table of the plan parameters is provided. The pattern and behavior of the proposed two-plan sampling system are discussed under several setting of parameter. The comparison between the proposed sampling system and traditional single sampling plan provide the advantage of the proposed plan. The proposed two-plan sampling system can be considered as generalization of the sampling plan. It will reduce to traditional single sampling plan for $(s,t) = (1,1)$ and $m = 1$.

References

Balamurali, S., Jun, C.H., (2009). Designing of a variables two-plan system by minimizing the average sample number. *Journal of Applied Statistics*, 36 (10), 1159－1172.

Boyle, R.A., (1991). The Taguchi capability index. *Journal of Quality Technology*. 23 (1), 17－26

Calvin, T.W., (1977). TNT zero acceptance number sampling. *American Society for Quality Control, Annual Technical Conference Transaction*. Philadelphia, Pennsylvania, 35－39.

Chan L.K., Cheng, S.W., Spiring, F.A., (1988). A new measure of process capability C_{pm}. *Journal of Quality Technology*, 20 (3), 162-175

Hsiang, T.C, Taguchi, G., (1985). A tutorial on quality control and assurance–the Taguchi methods. *ASA Annual Meeting*, Las Vegas, Nevada, USA

Kane, V.E., (1986). Process capability indices. *Journal of Quality Technology*, 18 (1), 41－52.

Montgomery, D. C., (2009). *Statistical Quality Control, A Modern Introduction*, sixth ed. John Wiley & Sons, NY

Muthuraj, D., Senthilkumar, D., (2006). Designing and construction of tightened- normal-tightened variable sampling scheme. *Journal of Applied Statistics*, 33, 101－111.

Pearn, W.L., Wu, C.W., (2006). Variables sampling plans with PPM fraction of defective and process loss consideration. *Journal of Operational Research Society*, 57, 450－459

Schilling, E.G., Neubauer, D.V., (2009). *Acceptance Sampling in Quality Control*, second ed. CRC Press, New York.

Senthilkumar, D., Muthuraj, D., (2010). Construction and selection of tightened- normal-tightened variable sampling scheme of type TNTVSS $(n_1, n_2; k)$. *Journal of Applied Statistics*, 37 (3), 375－390.

Soundararajan, V, Vijayaraghavan, R., (1992). Construction, selection of tightened-normal-tightened sampling inspection scheme of type TNT－ $(n_1, n_2; c)$. *In. Appl. Stat*. 19, 339－349

Taguchi, G., (1986). *Introduction to Quality Engineering.* Asian Productivity Organzation, Tokyo, Japan

Wu, C.W., Pearn, W.L., (2006). A variables sampling plan based on C_{pmk} for product acceptance determination. *European Journal of Operational Research*, 184, 549－560.

Biography

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