Profit Maximization by Integrating Production Planning, Maintenance Scheduling, Quality Aspects and Sale Decisions

Hossein Beheshti Fakher
Laval University, Mechanical Engineering Department
Quebec (QC.) G1K 7P4, Canada

Mustapha Nourelfath
Industrial & Management Systems Engineering Department,
College of Engineering & Petroleum, Kuwait University
P.O. Box 5969 Safat 13060, Kuwait
On leave from Mechanical Engineering Department, Faculty of Science and Engineering, Laval University, Quebec (QC.), Canada

Michel Gendreau
Department of Mathematics and Industrial Engineering (MAGI)
École Polytechnique of Montreal, Canada

Abstract

The objective of this paper is to improve decision systems by integrating production and sales planning with preventive maintenance scheduling taking into account quality aspects of the production system. The concept of age based imperfect maintenance with multiple PM levels and multi-state machine is taken into account. Reduction in the age of machine corresponds to the cost of scheduled PM and also, the cost of machine restoration is linked to the delay in detecting deteriorated condition. In this paper, the mathematical model of the problem and related formulations are presented and evaluation of the various cost components and interacting factors as well as the different aspects of the model complexity are explained. The objective is to maximize total profit. Total cost of the system is the sum of production, inventory, backorder, and set up cost as well as the cost of corrective and preventive maintenance and the income is acquired by selling perfect and imperfect products in different prices. A numerical example is presented and sensitivity of the model to multiple parameters is tested. Based on the results obtained from solving several problems, integrated approach shows between 0.5 to 20 percent improvements compared to the disintegrated models. Such level of increase in the total profit especially in large companies demonstrates the importance of integration.

Keywords
Production, Maintenance, Quality, Multi State Machines, Imperfect Preventive Maintenance

1. Introduction
In spite of vigorous links between production and sales planning, maintenance scheduling and design of quality system in the organizations, these important concepts are mainly regarded individually in the literature and joint models are scarce. Failure in establishment of such important interactions influences on the quality of decisions made in the disintegrated systems and the solutions can be non-optimal. The main objective of this paper is to deal with this issue and to improve planning and scheduling activities by integrating production and sales planning with preventive maintenance scheduling taking into account the quality aspects of the production system. Coordination among maintenance, production and quality is essential for the success especially in the case of modern companies. However in many cases these subjects are conflictive because of their shared resources (Nourelfath, 2012). Quality competitors have more pro-active maintenance policies, better planning and control systems, decentralized maintenance organization structures and more effective maintenance management (Pinjala et al, 2006). There is actually little work that addresses integration of maintenance, production, selling plans, and quality, compared to many existing work in each of these areas.

In this paper, the production planning includes economical determination of the batch sizes that lies in the context of short term production planning and tries to satisfy customer demands by addressing the tradeoffs between various costs in the system. Capacitated lot-sizing problem (*CLSP)* involves balancing set-up costs against storage costs, under a common capacity constraint, over a finite time horizon, minimizing total costs (Haugen, 2007). Introduction of the flexibility of a pricing option or *PCLSP* problems are easier to solve from a numerical point of view (Haugen 2007, Salvietti 2008). Brojeswar et al. (2013) inspected the economical lot sizing problem with quality aspects in an imperfect production system. They presented mathematical model of the problem by considering in-control and out-of-control conditions of the machine with different non-conformity rates. Bouslah et al. (2013) studied the problem of joint determination of the optimal lot sizing and production control policy for an unreliable and imperfect manufacturing system which uses a single acceptance sampling plan to control the quality of lots.

Quality of the production concerns not only to efficiency of the quality system but also to the maintenance scheduling. In case of poor maintenance, age of the machine and probability of shifting to degraded states and consequently the imperfectness rate increases. Some papers are addressing the integration of maintenance scheduling with design of the quality system. Investigation of the optimal quality sampling intervals in presence of Weibull shocks with increasing hazard rate (Banerjee and Rahim, 1988) showed the efficiency of non-uniform sampling scheme. They suggest maintaining a constant hazard rate over all intervals. Rahim (1994) considered the problem of jointly determination of EPQ level, inspections schedule, and $\bar{x}$-chart parameters of an imperfect production system and show that non-uniform inspections are more efficient. However, production of imperfect products is somehow unavoidable and a capable system will just keep its rate in the allowed ranges. These items then should be reworked or should be sold in lower process.

Integration of maintenance scheduling with production planning has recently been studied by several researchers. Lee et al. (1989) were the first on joint modeling of production and maintenance decisions. Cassady and Kutanoglu (2003, 2005) proposed an integrated model where its objective is to determine optimal age-based PM policies and to minimize weighted completion time of the jobs. Producing single product in a single-unit production system and integration of lot-sizing and PM decisions is also investigated (Nourelfath 2012, Hadidi et al. 2012, and Suliman 2012). Nourelfath considered general probabilities for sojourn and failure times and showed that introducing machine failures results in reduction of production lot sizes. Min Ji et al (2007) consider the problem of single machine scheduling with several PM periods and show that the LPT algorithm is the best possible solution method for minimizing the total completion time. Aghezzaf (2007) studied coordinated production planning and preventive maintenance scheduling in a lot production system with several products in distinct periods. Kenne and Gharbi proposed an integrated model for an unreliable system with sale returns. They offered multiple threshold levels hedging point policy to solve related problems. Their work also justifies the importance of integrated approach with significant cost reductions. Sung and ock (1992) studied integrated production and maintenance scheduling in a system composed of single-machine single product with intermediate machine restorations in which when the machine shifts to out-of-control state, it starts to produce a given proportion of imperfect products. Simultaneous production and maintenance scheduling in presence of demands uncertainty increases the flexibility of production system and reduces the risk of shortage or overage of demand (Xiaoning Jin et al., 2009). The impact of process deterioration on production and maintenance policies in a single stage production system is examined by Kazaz and Sloan (2013).

There is no paper in the literature addressing the integration of all aforementioned concepts and just a few papers are engaged in joint production planning, maintenance scheduling and design of the quality systems. Preventive maintenance actions can prevent a production process from further deterioration and improve product quality in conjunction with statistical process control (Xiang, 2013). Pandey (2011) and Nourelfath and Ben-Daya (2012) are the first on integration of these three concepts. Their research revealed substantial economic benefits in joint scheduling. Rivera et al. (2013) proposed the model for joint production planning and overhaul scheduling in an imperfect production system in which the rate of non-conformity increases by time.

Notation

d_{pt} :	Demand of product *p* in period *t*;
h_{pt}:	Holding cost of product *p* in period *t;*
b_{pt}:	Backorder cost of product *p* in period *t*;
B_{pt}:	Backorder level of product p in period *t*;
s_{pt} :	Set-up cost of product *p* in period *t*;
Set_{pt}:	set up decision variable for product *p* in period *t*;
π_{pt} :	*P*roduction cost of product *p* in period *t*;
x_{pt}:	Production lot size of product *p* in period *t*;
g_p:	Nominal production rate of product *p*;
T,t:	Number and index of periods;

P,p: Number and index of products;
Q, q: Number and index of PM levels;
L: Fixed length of the periods;
α_p: Non-conformity rate for product p;
β: Cost of the machine inspection;
IC^t_p: Inventory level for conforming product p at the end of period t;
IN^t_p: Inventory level for nonconforming product p at the end of period t;
PC^t_p: Price of conforming item of product p in period t;
PN^t_p: Price of non-conforming item of product p in period t;
SC^t_p: Number of conforming items of product p sold in period t;
SN^t_p: Number of nonconforming items of product p sold in period t;
m: Number of PMs in the periods;
η: PM imperfectness factor;
w^t_k, y^t_k: Age of the machine at beginning and end of the k^{th} interval in period t;
ξ_1, ξ_2: Parameters of restoration cost;
CPM^t_k: Cost of the K^{th} PM in period t;
CPM^q: Cost of PM of level q;
CMR: Cost of minimal repair;

2. Problem Definition

2.1. The production and the maintenance system

Production system is composed of one machine that can produce multiple products. At the beginning of each period, the machine is in its perfect conditions (*in-control state*) with nominal production rate g_p. While time progresses, the machine may shift to the *out-of-control* state (*OOC* state) with increased nonconformity rate (α_p). Conforming items are assumed to satisfy the demands while nonconforming items can be sold by any level in a second market. Several PM plans with different cost and effect (and negligible time) can be performed on the machine. The planning horizon includes several production periods each of known length L. Number of PM per periods is constant m which divide the periods into $m+1$ equidistant inspection-PM intervals. At the end of each interval and before carrying out the PM, an error free inspection will be performed. If the machine has not shifted to out of control state, planned PM will be performed otherwise the machine will be stopped to be restored at the end of the period. Linear relationship between restoration cost and detection delay is taken into account and no PM or inspection will be performed at the end of the last interval in each period. The objective is to simultaneous determination of production lot sizes and preventive maintenance schedule which maximizes the profit. Since deterioration of the system increases non-conformity rate and the cost of machine failures, we are interested in optimal assignment of PM tasks to the PM events to maintain the age of machine in economical levels.

2.2. Links to the quality

Designing a complete quality control system concerns different decisions such as type of the control chart, sampling intervals, sample size, rejection and acceptation levels, etc. which are beyond the scope of this paper. When the machine shifts to OOC state, non-conformity rate increases and while not detected, the machine continues to produce non-conforming products. Machine shifts has close relation with age and conditions of the machine and quality and maintenance systems are interdependent. Here we just will use the expected non-conformity rate to be included in the model. Also we have considered that the sampling process is enough accurate and so distinct error types such as false signal when the process is in control or no signals when the process is out of control are negligible.

3. Evaluation of Sales and Costs

3.1. Evaluation of costs

The objective function maximizes the profit. Source of revenue is selling of conforming and nonconforming products and cost of the system is the sum of all production and maintenance.

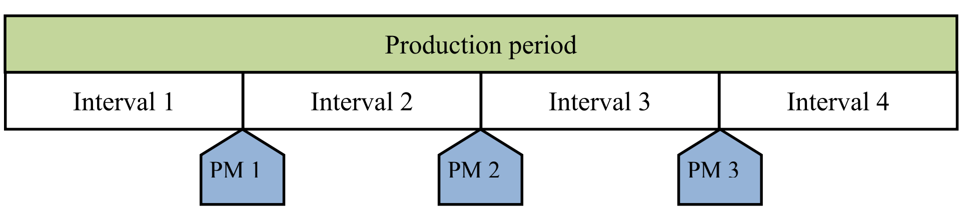

Figure 1: A production period with three PM and four intervals.

Number of PMs in the periods and lengths of inspection-PM intervals are known and constant. As illustrated in figure 1, preventive maintenance divides the period into several intervals. In this figure, *3* PMs have divided the period into four intervals. Length of all intervals is identical. At the end of each interval, a PM will be executed which reduces the age of machine. Machine age in k^{th} interval of period *t (k=1 to m+1)* changes from w^t_k to $y^t_{k.}$ ($k=1...m_t+1$) then the length of the k^{th} interval is $y^t_{k.}$ - $w^t_{k.}$ After each PM, the age of the machine will reduce. This reduction in the machine age is a function of the PM level and the rank of PM in the period. Preventive maintenance levels are indicated by q ($q = 1... Q$, which Q is the index of the lowest preventive maintenance level) and CPM^q refers to the cost of a PM of level q (CPM^1 is the cost of the highest PM level). Therefore the cost of the k^{th} PM in period t which its level is q will be $CPM_k^t = CPM^q$.

Preventive maintenances are imperfect and it is considered that the change in the age of machine PM is proportional to the cost of PM level (Case *C* of Nakagawa' imperfect maintenance model, 1980). So the ratio $\frac{CPM^q}{CPM^1}$ (a number between 0 and 1) is the efficiency of PM of type q. On the other hand, it is assumed that even for a given PM plan, its efficiency decreases while the rank of PM in the period increases. In this paper linear relationship between age reduction and rank of the PM is considered, *i.e.* $w_{k+1}^t = (1-\gamma_k^t).y_k^t$ where $\gamma_k^t = \eta^{k-1}\frac{CPM_k^t}{CPM^1}$ is the age reduction by performing k^{th} PM in the period t and η ($0 < \eta \leq 1$) is the imperfectness factor which implies the degradation in the effect of PM on the age of machine. As the formulation shows, a full PM brings the machine farther from the as good as new condition as more PMs are performed. Changes in the age of machine influences on the other cost parameters of the model including restoration cost, length of the production and allows joint scheduling of maintenance, production and quality aspects of the system. In this approach, it is possible to cancel any PM by assigning the lowest PM level (with cost *zero*). In this case, $\gamma_k^t = 0$ and $w_{k+1}^t = y_k^t$.

3.1.1. The cost of production, inventory holding, setup, and backorders

Production, inventory holding, backorder, and setup costs are respectively:

- The total production cost $\sum_{p\in P}\sum_{t=1}^{T}\pi_{pt}x_{pt}$, (1)
- The total inventory holding cost: $\sum_{p\in P}\sum_{t=1}^{T}h_{pt}\left(IC_p^t + IN_p^t\right)$, (2)
- The backorder cost : $\sum_{p\in P}\sum_{t=1}^{T}b_{pt}B_{pt}$, (3)
- The total setup cost $\sum_{p\in P}\sum_{t=1}^{T}s_{pt}Set_{pt}$, (4)

Where IC_p^t *and* IN_p^t are respectively inventory levels of conforming and nonconforming products at the end of period *t*.

3.1.2. The preventive maintenance cost

At the end of each interval and before performing the PM plan, a quick and error free inspection will detect the machine shifts. By detecting a shift, PM will be cancelled and the machine will be stopped until end of the period to be restored (complete overhaul). If inspection did not detect any shift, pre-scheduled PM will be executed and the next interval will be realized. Considering *F(t)* to be cumulative time to shift function, conditional probability of a shift in interval *k* of period *t* given the machine was in *in-control* state at the beginning of the interval is:

Pr{shift in the k^{th} interval | no shift at the beginning of the interval }= $Pr_k^t = \frac{F(y_k^t)-F(w_k^t)}{1-F(w_k^t)}$ (5)

Where w^t_k, and y^t_k are respectively the age of machine at the beginning and end of the interval. The k^{th} PM will be realized if no shift has detected in all the previous intervals. So the probability of realizing k^{th} PM is $\prod_{i=1}^{k-1}(1-Pr_i^t)$=$\prod_{i=1}^{k-1}\left(1-\frac{F(y_i^t)-F(w_i^t)}{1-F(w_i^t)}\right) = \prod_{i=1}^{k-1}\left(\frac{1-F(y_i^t)}{1-F(w_i^t)}\right)$ and total expected cost of preventive maintenance on the planning horizon will be

$$CPM = \sum_{t=1}^{T}\sum_{k=1}^{m}\left(CPM_k^t\cdot\prod_{i=1}^{k}(1-Pr_i^t)\right) = \sum_{t=1}^{T}\sum_{k=1}^{m}\left(CPM_k^t\cdot\prod_{i=1}^{k}\left(\frac{1-F(y_k^t)}{1-F(w_k^t)}\right)\right) \quad (6)$$

Where, CPM^t_k takes its values from existing PM levels of the machine and m is the number of PM performances per period. It should be mentioned that no inspection or PM is considered at the end of the last interval of each period.

3.1.3. The inspection cost

In each period, the first inspection will be certainly performed but fulfillment of the successive inspections depends on realization of the previous intervals. No inspection or PM will be executed at the end of last intervals. Considering realization probability of the k^{th} interval as proposed in the previous section and taking into account inspection cost β, total expected cost of inspection is given by:

Inspection cost $IC = \beta\cdot\sum_{t=1}^{T}\sum_{k=1}^{m}\prod_{i=1}^{k-1}(1-Pr_i^t)$. (7)

3.1.4. The cost of minimal repair

In addition to the shifts to out-of-control state, the machine also may randomly switch to non-operating or failed state according to general probability distribution function. When the machine fails, a minimal repair with known cost and negligible time brings the machine in operation without influencing on the age of the machine. Therefore failures occur according to a non-homogeneous Poisson process (NHPP) having intensity function $\zeta(y)$ as the failure rate of machine at age y. Then expected number of machine failures in the k^{th} interval of period t is given by $NF_k^t = \int_{w_k^t}^{y_k^t} \zeta(y)dy$ and so the number of failures in period t, taking into account realization probabilities of the intervals will be:

$$NF^t = \sum_{k=1}^{m} \left(NF_k^t \prod_{i=1}^{k-1} (1 - Pr_i^t) \right) \tag{8}$$

Having in hand the number of failures in a period, expected cost of minimal repairs in period t will be as follows

$CMR^t = CMR * NF^t$, where, CMR is the cost of minimal repair. (9)

3.1.5. Expected revenue

Sale levels can be different from sale possibilities i.e. it is likely to hold in-hand products to be sold in future periods. However, in each period, maximum sale level is limited to the sum of demand in that period plus backorders until that period. Taking into account sale prices, the revenue by selling product p in period t will be $\mathrm{RN}_p^t = SC_p^t.PC_p^t + SN_p^t.PN_p^t$ Where, SC_p^t is number of sold items of conforming product p and SN_p^t is the number of sold items of nonconforming product p in period t.

Then total revenue for all products in all periods would be:

$$RN = \sum_{t=1}^{T} \sum_{p=1}^{P} RN_p^t = \sum_{t=1}^{T} \sum_{p=1}^{P} \left(SN_p^t.PN_p^t + SC_p^t.PC_p^t \right) \tag{10}$$

3.1.6. The restoration cost

By occurrence of a shift, next inspection will certainly detect the shift and therefore the machine will be stopped to be restored at the end of that period however, it is considered that the restoration cost changes linearly by delay in detecting the shift. So if machine shifts at time t in interval k of period t, detection delay or the duration of time that machine operates in out of control state will be $y_k^t - t$. Since the conditional probability of a shift at time t is $f^c(t) = \frac{f(t)}{1-F(w_k^t)}$ then expected value of detection delay will be $\tau_k^t = \int_{w_k^t}^{y_k^t - TPM_k^t} (y_k^t - t) f^c(t)dt$. Taking into account the probability of machine shift at k^{th} interval, expected detection delay or expected duration of time that the machine operates in out of control state in period t is given by

$$\tau^t = \sum_{k=1}^{m+1} \left(\int_{w_k^t}^{y_k^t - TPM_k^t} (y_k^t - t) f^c(t)dt \cdot Pr_k^t \prod_{i=1}^{k-1} (1 - Pr_i^t) \right) \tag{11}$$

By supposing linear relationship between the restoration cost and the detection delay, the restoration cost in period t will be $R(\tau^t) = \xi_0 + \xi_1 \tau^t$ where ξ_0 and ξ_1 are given constants. Total expected restoration cost over the planning horizon is

Restoration cost $RC = \sum_{t=1}^{T} (\xi_0 + \xi_1 \tau^t) = T\,\xi_0 + \xi_1 \sum_{t=1}^{T} \tau^t$ (12)

3.1.7. Production possibilities and expected operational time

In order to prepare executable production schedules, expected available production time in the period should be estimated. Nominal length of the periods *(L)* and the number of the PM per periods *(m)* are given but occurrence (and detection) of a shift will result in stopping the machine until end of the period and therefore real available production time will be smaller than length of the period and it depends to the shift probabilities in the intervals. Therefore, the expected duration of time that the machine is operational in period t is

$$EPT^t = \sum_{k=1}^{m+1} \left(\prod_{i=1}^{k-1} (y_k^t - w_k^t)(1 - Pr_i^t) \right) \tag{13}$$

Total needed production time to produce scheduled production levels is bounded to the expected operational time given by (13).

3.1.8. Number of conforming and nonconforming items

By occurrence of a shift at time t, the machine will start to produce nonconforming items until end of the interval. Expected time that the machine operates in out of control state in period t is given by (11) and expected duration of time that the machine is operational in that period is given by (13).

Therefore, the expected proportion of time that the machine functions in out-of-control state would be:

$$\Gamma^t = \frac{\tau^t}{EPT^t} = \frac{\sum_{k=1}^{m+1}\left(\int_{w_k^t}^{y_k^t}(y_k^t-t)f^c(t)dt\cdot Pr_k^t\prod_{i=1}^{k-1}(1-Pr_i^t)\right)}{\sum_{k=1}^{m+1}\left(\frac{L}{m+1}\prod_{i=1}^{k-1}(1-Pr_i^t)\right)} \tag{14}$$

Considering x_{pt} as the planned production level for product p in period t, needed time for producing x_{pt} is $\frac{x_{pt}}{g_p}$ and taking into account average proportion of time that the machine is in out-of-control state, $\frac{x_{pt}}{g_p}.\Gamma^t$ indicates the time that machine operates in out of control state while producing x_{pt}. Then the expected number of nonconforming items produced during production of x_{pt} is,

$$XN_p^t = Time * Rate = \left(\frac{x_{pt}}{g_p}.\Gamma^t\right).\left(g_p.\alpha_p\right) = x_{pt}.\alpha_p\cdot\Gamma^t \tag{15}$$

And the number of conforming products will be $XC_p^t = x_{pt} - XN_p^t = x_{pt}\cdot\left(1-\alpha_p\cdot\Gamma^t\right)$ (16)

Aforementioned quantities are rough estimations of conforming and non-conforming products and to have more exact assessments one needs to have exact job-scheduling over each interval. Since determination of production orders or production time schedules are beyond the scope of this paper, we will just use the appraisals given by (15) and (16).

4. The Profit Maximization Model

The profit maximization model for integrated batch scheduling, PM planning and quality aspects of the production system is

$$\text{Maximize } Z = \sum_{t\in T}\sum_{p\in P}\left(SN_p^t.PN_p^t + SC_p^t.PC_p^t\right) - \sum_{t\in T}\sum_{p\in P}\left(h_{pt}\left(IC_{pt}+IN_{pt}\right) + b_{pt}B_{pt} + \pi_{pt}x_{pt} + s_{pt}Set_{pt}\right) - \sum_{t=1}^{T}\sum_{k=1}^{m}\left(CPM_k^t\cdot\prod_{i=1}^{k}\left(\frac{1-F(y_k^t)}{1-F(w_k^t)}\right)\right) - \beta\cdot\sum_{t=1}^{T}\sum_{k=1}^{m}\prod_{i=1}^{k-1}\left(\frac{1-F(y_k^t)}{1-F(w_k^t)}\right) - CMR\cdot\sum_{t=1}^{T}\sum_{k=1}^{m}\left(\int_{w_k^t}^{y_k^t}\zeta(y)dy\prod_{i=1}^{k-1}\left(\frac{1-F(y_k^t)}{1-F(w_k^t)}\right)\right) - T\xi_0 - \xi_1\sum_{t=1}^{T}\sum_{k=1}^{m+1}\left(\int_{w_k^t}^{y_k^t}(y_k^t-t)\frac{f(t)}{1-F(w_k^t)}\,dt\cdot\frac{F(y_k^t)-F(w_k^t)}{1-F(w_k^t)}\prod_{i=1}^{k-1}\left(\frac{1-F(y_k^t)}{1-F(w_k^t)}\right)\right) \tag{17}$$

Subject to

$$IC_{pt} = IC_{Pt-1} - SC_p^t + x_{pt}\left(1-\alpha_p\Gamma^t\right), p\in P, t\in T \tag{18}$$

$$IN_{pt} = IN_{Pt-1} - SN_p^t + x_{pt}\alpha_p\Gamma^t, p\in P, t\in T \tag{19}$$

$$B_{pt} = B_{pt-1} + d_{pt} - SC_{pt}, p\in P, t\in T \tag{20}$$

$$x_{pt} \le GM\cdot S_{pt}, p\in P, t = 1,2,\dots,T, \text{ where } GM \text{ is a big upper bound for production levels.} \tag{21}$$

$$\sum_{p\in P}\frac{x_{pt}}{g_p} \le EPT^t, t\in T \tag{22}$$

$$w_{k+1}^t = \left(1-\eta^{k-1}\frac{CPM_k^t}{CPM^1}\right)\cdot y_k^t \;, 0<\eta\le 1, k = 1,\dots,m, t = 1,2,\dots,T, q = 1,\dots,Q \tag{23}$$

$$y_k^t = w_k^t + \frac{L}{m+1} \;, k = 1,\dots,m+1, t = 1,\dots,T \tag{24}$$

$$w_1^t = 0 \;, t = 1,\dots,T \tag{25}$$

$$\Gamma^t = \frac{\sum_{k=1}^{m+1}\left(\int_{w_k^t}^{y_k^t}(y_k^t-t)f^c(t)dt\cdot\left(\frac{F(y_k^t)-F(w_k^t)}{1-F(w_k^t)}\right)\prod_{i=1}^{k-1}\left(\frac{1-F(y_i^t)}{1-F(w_i^t)}\right)\right)}{\sum_{k=1}^{m+1}\left(\frac{L}{m+1}\prod_{i=1}^{k-1}\left(\frac{1-F(y_i^t)}{1-F(w_i^t)}\right)\right)} \tag{26}$$

$$EPT^t = \frac{L}{m+1}\sum_{i=1}^{m+1}\prod_{j=1}^{k-1}\left(\frac{1-F(y_j^t)}{1-F(w_j^t)}\right) \tag{27}$$

$$CPM_k^t \in \{CPM^1, \dots, CPM^Q\}\;, t = 1, 2, \dots, T\;, k = 1, 2, , \dots, m \tag{28}$$

$$S_{pt}\in\{0,1\}, x_{pt}, IC_p^t, IN_p^t, b_{pt}, y_k^t, w_k^t, EPT^t, \Gamma^t \in \mathbb{R}^+ \tag{29}$$

The first term of the objective function (17) is related to the total income by selling conforming and nonconforming products, second term is the sum of production, inventory holding, backorders and setup costs for all products over the planning horizon. The other terms correspond respectively to cost of preventive maintenance, inspection cost, cost of minimal repairs, and the restoration cost. The flow constraint of conforming products (18) links inventory, backorder and selling amounts to the expected number of conforming items produced in each period. The last term of this constraint estimates expected number of conforming items acquired by producing x_{pt} in period t. As in the previous constraint, equation (19) links inventory and selling amounts of nonconforming items to the expected number of nonconforming products in the periods. It is considered that any desired level of nonconforming items can be sold in the second market. Constraint (20) links the values of backorders in consequent periods to the values of demands and sales in the period. Constraint (21) forces $x_{pt} = 0$ if $S_{pt} = 0$, and frees $x_{pt} \geq 0$ if $S_{pt} = 1$. The quantity GM is an upper bound for x_{pt}. Equation (22) states that the total production time in the period is bounded to the length of the period L. Equations (23) to (25) link PM plans to the age of the machine in the intervals in periods. Constraints (26) and (27) are respectively expected proportion of time that the machine operates in *out-of-control* state in period t and expected available production time in period t (as shown in sections 3.1.7 and 3.1.8). Constraint (28) pushes the PM plans to be selected from existing PM plans of the machine and finally (29) shows the bound and type of decision variables.

5. Complexity of the Model

The formulation of the model given by (17)-(29) corresponds to a very complex nonlinear mixed-integer programming model that is too difficult to be solved. Briefly, sources of its complexity are:

- Age of the machine in non-linearly linked to the PM plans
- Shift and failure probabilities are non-linearly linked to the age of machine
- Several terms of objective function and constraints need value of shift and failure probabilities
- Model requires to estimate expected number of conforming and non-conforming products for any production level
- PM levels should be selected from given list of existing PM levels(discreet optimization)
- There are integrals in the objective and constraints

On the other hand, for a known PM levels, values of the machine age and therefore shift and failure probabilities can be calculated and then problem will reduce to a linear MIP that can be solved using existing solution methods or by commercial solvers.

In the literature of reliability, it is shown that the exponential distribution function, Rayleigh distribution function, or in the general form, Weibull distribution are the most appropriate fitting functions for machine shifts and failures. The theory explains why mortality rates increase exponentially with age (the Gompertz law) in many species and it also tells why technical devices usually fail according to the Weibull law (Gavrilov, 2001). Weibull distribution has comprehensive characteristics that can cover a wide range of (mainly, single failure mode) machines and components of different production systems. For example when $\varphi<1$ the hazard rate over the time is decreasing, $\varphi=1$ yields constant hazard rate (exponential distribution with hazard rate equal to λ) and $\varphi>1$ results in increasing hazard rate over the time. When $\varphi=2$, Weibull distribution returns Rayleigh probability function. Because of wide application of Weibull distribution for time to shift and time to failure probabilities and its applied characteristics and flexibility, this distribution (with increasing hazard rate over the time, i.e. $\varphi>1$) is used in the sample problems of section 8. With Weibull distribution, the probability of remaining in *in-control* state when the age of machine is t is given by $f(t,\lambda,\varphi|t,\lambda,\varphi>0) = \lambda\varphi t^{\varphi-1}e^{-\lambda t^{\varphi}}$ and its CDF is given by $F(t) = 1 - e^{-\lambda t^{\varphi}}$. Reliability of a component is defined as $R(t) = 1 - F(t) = e^{-\lambda t^{\varphi}}$ and hazard function is defined as $H(t) = \frac{f(t)}{R(t)} = \lambda\varphi t^{\varphi-1}$.

6. Solution Method and Illustrative Example

Considering that a given PM plan results in detailed and complete solution of the integrated problem, enumerating all possible PM plans and solving related MIP can be applied for small problems.
Let us consider a small problem composed of 2 products ($P=2$), 3 periods ($T=3$), length of each period is $L=1$ month and 3 PM per period ($m=3$). Tables 1 shows demand, production cost, backorder cost, holding cost, setup cost and sale price for conforming and nonconforming items in each of the periods and table 2 shows machine and maintenance data.

It is assumed that the time to shift to *out-of-control* state and the time to machine failure both follow Weibull distributions with different parameters i.e. CDF of time to shift is given by $F(t) = 1 - e^{-\lambda t^{\varphi}}$ and CDF of time to failure is given by $\varsigma(t) = 1 - e^{-\theta t^{\rho}}$. Values of the parameters, parameters of restoration cost, and cost of the machine inspection are given in table 2. Optimal solution (plan 3) (found by enumeration method) and two other PM plans are represented in table 3.

Table 1 - Products' data in three periods

		Demand		Production cost		Backorder cost		Holding cost		Setup cost		Price Conforming		Price Non-conforming	
Product		1	2	1	2	1	2	1	2	1	2	1	2	1	2
Period	1	50	50	70	130	25	43	4	6	500	830	120	230	25	55
	2	48	49	72	125	27	46	3	7	490	790	125	230	23	60
	3	48	50	67	120	20	47	3	5	510	810	115	225	29	57

Table 2: Maintenance data and PM levels of problem 1

PM plans	1	2	3	Minimal Repair	Inspection	Restoration	Shift Probability	
Cost	350	175	0	120	80	$\xi_1 + \xi_2 * \tau$	λ	4.5
							φ	3
Product	1	2	Imperfectness factor		Restoration parameters		Failure Probability	
Prod. Rate	150	150	H	0.85	ξ_1	170	θ	5
Non Conf. Rate	0,25	0,2			ξ_2	2700	ρ	2.5

Table 3: PM Schedules

Plan	Description of the plan	Total Profit	PM Levels In period 1			PM Levels In period 2			PM Level In period 3		
			1	2	3	1	2	3	1	2	3
Plan 1	Plan minimizing the maintenance cost	15365	2	2	0	2	2	0	2	2	0
Plan 2	Plan maximizing the sales	14394	0	0	0	0	0	0	0	0	0
Plan 3	Optimal solution found by enumeration	15608	2	0	2	2	2	0	2	2	0

As this table shows, the optimal solution (Plan 3 – with total profit 15608) is 1.6% better than the plan 1 that minimizes maintenance cost and 8.4% better than the plan 2 that maximizes the production and sales problem. Figure 2 shows how increasing the PM budget decreases the expected number of machine shifts and failures in a period. Each PM plan with known budget corresponds to a certain value of the number of machine failures and shifts in the period. It demonstrates that more investment on PM can significantly reduce the number of machine shifts and failures. Solution method is programmed in Visual Basic 2010 using CPLEX 12 to solve related MIP. Solution time was 132 minutes (on a 3.4GHz core i7 machine with 16GB of RAM).

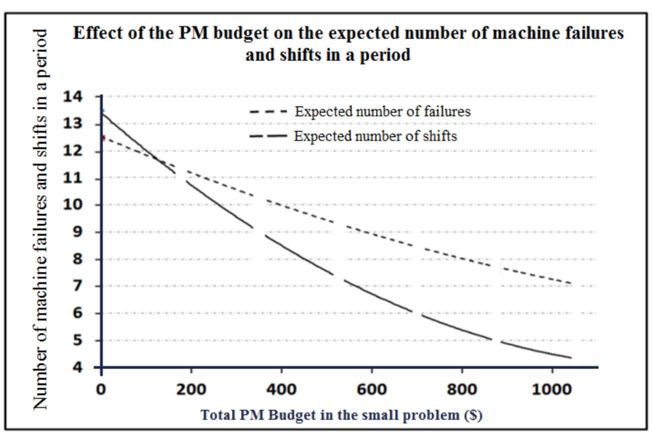

Figure 2: Effect of PM budget on expected number of shifts and machine failures in a period

In order to perform sensitivity analysis, a new small problem (problem 2) with two products, one period, four PM levels, and three PM in each period is defined (See, tables 4 and 5). The optimal solution of this problem is 0.3% to 16.6% better than disintegrated solutions obtained by solving the problems individually.

Each profit is linked to a PM schedule. Figure 3, represents how investment on PM first improves the total profit by decreasing non-conformity rate and increasing the available production time but additional budgets cannot be compensated by the savings therefore appropriate PM budget needs to be determined. Changes in the available production time with PM investment are shown in figure 4. It also represents variations of the time of in-control state.

Figure 5 represent how the profit and the optimal PM budget are influenced by the price of non-conforming items. In this graph, the price of imperfect product is considered as a percentage of prices of conforming items. For example when the price is increased from 25% to 50% of conforming product, total profit is increased by 10% and optimal PM budget is decreased by about 30%. This indicates that when the price of non-conforming items is relatively high, PM budget can be caught. Finally as represented in figure 6, shorter length of quality inspection intervals, increases the flexibility of decision system and results in higher profits but since the inspection contributes in the total cost, optimal determination of the number of quality inspections can be found by various what-if analysis and solving the problem for multiple cases of quality inspection intervals.

Table 4: Production data and sale prices

PM plans	1	2	3	4	Shift parameters	
PM Cost	360	240	120	0	λ	4.5
					φ	3
Minimal Repair cost	120	Restoration parameters			Failure parameters	
Inspection cost	80	ξ_1	170		θ	5
Imperfectness factor	0.85	ξ_2	2700		ρ	2.5

Table 5: Maintenance data

	Product			Product	
	1	2		1	2
Demand	60	40	Price of conforming item	120	200
Production Cost	75	130	Price of non-conforming item	30	50
Backorder Cost	25	43	Production rate	190	140
Holding cost	4	6	Non-conformity rate	25%	20%
Setup cost	500	830			

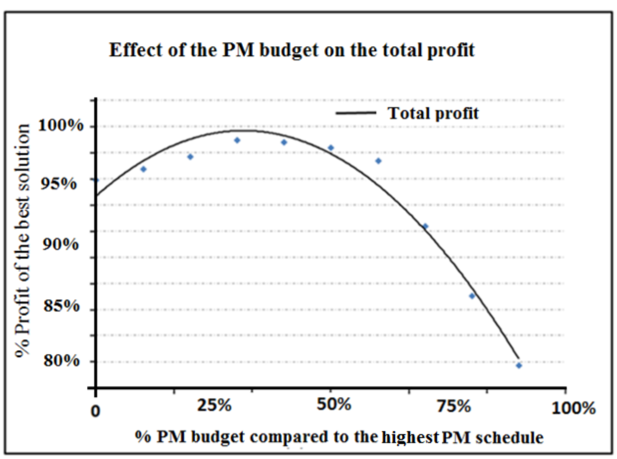

Figure 3: Maximum profit as a function of PM investment

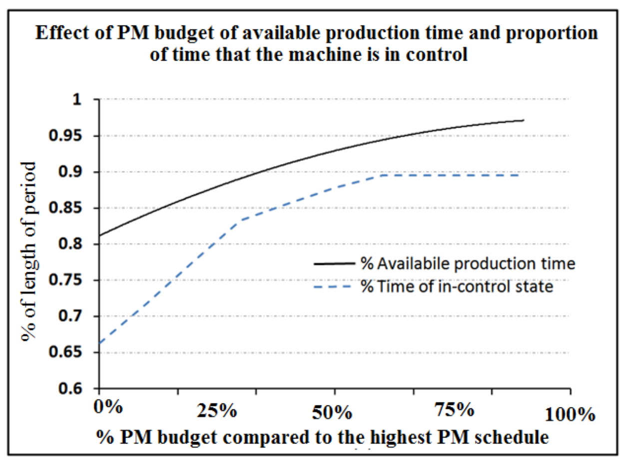

Figure 4: Effect of PM budget of relative conformity rate and available production time

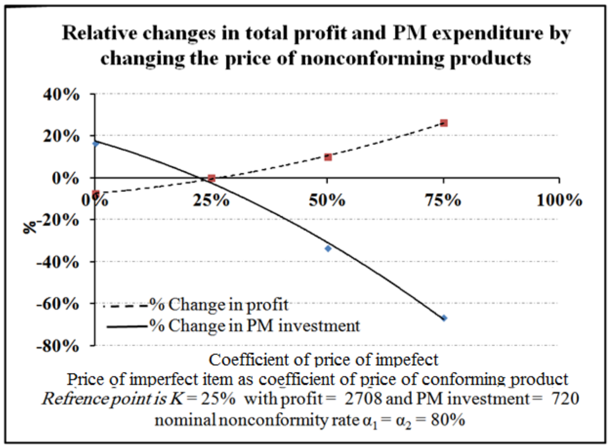

Figure 5: Relative changes of total profit and PM investment by sale price of nonconforming items

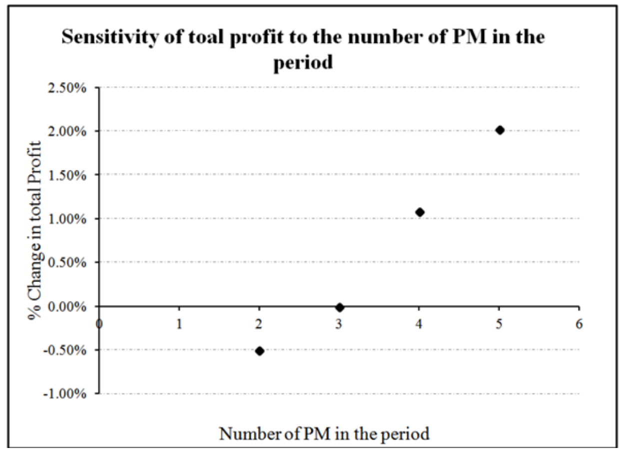

Figure 6: Sensitivity of the total profit to the number of PM in the period

7. Conclusions

One substantial issue of the advanced decision systems is dealing with complex interactions and challenges between multiple parts of the system. Production and sales planning, maintenance scheduling, and design of the quality system are conflicting issues because of the shared resources and there are strong links between them. The main contribution of this paper is mathematical modeling and formulation of the profit maximization model which integrates lot sizing, sales planning, and preventive maintenance scheduling with certain aspects of the quality system in the same decision structure. Formulations and evaluation of the costs and interacting factors are explained and complexity of the model is discussed. The notion of age based maintenance with linear age reduction and delay dependent restoration cost as well as the concepts of multi-state system are considered to decrease the gap between the theory and the real production systems. Taking into account time varying costs has increased the flexibility of the model to handle multiple choices in selecting different production and warehousing opportunities in the various periods. The problem is NP-hard and even for small problems the solution space is too large requiring solution of numerous MIP problems with complicated evaluations. Illustrative examples are presented and sensitivity of the solution to multiple parameters of the model is tested. Based on these results, integrated model significantly contributes in improvement of the planning and scheduling activities and increase the total profit by 0.5% to 20%. Such notable saving supports future studies on the joint models and justifies the need for more contributions in this field. Sensitivity analysis demonstrated that the solution is highly sensitive to the PM schedules and financial outcomes of the machine failure. Bearing in mind the complexity of the model, our next contribution will generally focus on developing efficient solution methods for the proposed model and to consider several decisions concerning the design of the quality system. We will also extend the model for the case of more complex systems with multiple machines.

References

Aghezzaf, E.H., Jamali, M.A., and Ait-Kadi, D. (2007), An integrated production and preventive maintenance planning model, European Journal of Operational Research, 181, 679-685.

Banerjee, P.K., and Rahim, M.A. (1988), Economic design of $\bar{x}$-chart under Weibull shock models, Technometrics, 30, 407–414.

Belmansour, A-T., and Nourelfath M. (2010), An aggregation method for performance evaluation of a tandem homogenous production line with machines having multiple failure modes, *Reliability Engineering and System Safety*, 95, 1193-1201.

Bouslah, B., Gharbi, A., and Pellerin, R. (2013), Joint optimal lot sizing and production control policy in an unreliable and imperfect manufacturing system, *International Journal of Production Economics*, 144, 143–156.

Cassady, C.R., and Kutanoglu, E. (2003), Minimizing job tardiness using integrated preventive maintenance planning and production scheduling, *IIE transactions*, 35-2, 503-513.

Cassady, C.R., and Kutanoglu, E. (2005), Integrating preventive maintenance planning and production scheduling for a single machine, *IEEE Transactions on Reliability*, 54-2, 304-309.

Dhouib, K., Gharbi, A., and BenAziza, M.N. (2012), Joint optimal production control/preventive maintenance policy for imperfect process manufacturing cell, *International Journal of Production Economics*, 137, 126-136.

Fitouhi, M.Ch., and Nourelfath, M. (2012), Integrating noncyclical preventive maintenance scheduling and production planning for a single machine, *International Journal of Production Economics*, 136, 344-351.
Gavrilov, L.A., Gavrilova, N.S. (2001), The reliability theory of aging and longevity, Journal of Theoretical Biology, 213, 527-545.
Hadidi, L.A., Al-Turki, U.M., and Abdur Rahim, M. (2012), Integrating production planning and maintenance: Practical implications, *International conference of industrial engineering and operations research, Istanbul, Turkey*,2012 , July 3-6.
Haugen, K.K., Olstad, A., and Pettersen, B.I. (2007), The profit maximizing capacitated lot-size (PCLSP) problem, *European Journal of Operational Research*, 176, 165-176.
Ji, M., He, Y., and Cheng, T.C.E. (2007), Single-machine scheduling with periodic maintenance to minimize makespan, *Computers & Operations Research*, 34, 1764-1770.
Kazaz, B. and Sloan, Th.W. (2013), The impact of process deterioration on production and maintenance policies, European Journal of Operational Research, 227, 88–100.
Kenne, J.P., Gharbi, A., and Beit, M. (2007), Age-dependent production planning and maintenance strategies in unreliable manufacturing systems with lost sale, *European Journal of Operational Research*, 178, 408-420.
Nakagawa, T. (1980), A summary of imperfect preventive maintenance policies with minimal repair, *Operations research*, 14-3, 249-255.
Nourelfath, M. , and Ben-Daya, M. (2012), An integrated model for production, maintenance and quality, *4^{th} International Conference on Information Systems, Logistics and Supply Chain, ILS 2012 Quebec (Canada),* August 26-29.
Pal, B., Sana, S.S., and Chaudhuri, K. (2013), A mathematical model on EPQ for stochastic demand in an imperfect production system, *Journal of Manufacturing Systems,* 32, 260– 270.
Pandey, D., Kulkarni, M.S., and Vrat, P. (2011), A methodology for joint optimization for maintenance planning, process quality and production scheduling, *Computers & Industrial Engineering*, 61, 1098-1106.
Percy, D.F., and Kobbacy, K.A.H. (2000), Determining economical maintenance intervals, *International Journal of Production Economics*, 67, 87-94.
Pinjala, S.K., Pintelon, L., and Vereecke, A. (2006), An empirical investigation on the relationship between business and maintenance strategies, *International Journal of Production Economics*, 104, 214-229.
Rahim, M.A. (1994), Joint determination of production quantity, inspection schedule and control chart design*, IIE Transactions*, 26-6, 2–11.
Rivera-Gomez, H., Gharbi, A., and Kenné, J.P. (2013), Joint production and major maintenance planning policy of a manufacturing system with deteriorating quality, *International Journal of Production Economics*, http://dx.doi.org/10.1016/j.ijpe.2013.08.006.
Salvietti, L., and Smith, N.R. (2008), A profit-maximizing economic lot scheduling problem with price optimization, *European Journal of Operational Research*, 184, 900-914.
Suliman, S.M.A., and Jawad, S.H. (2012), Optimization of preventive maintenance schedule and production lot size, *International Journal of Production Economics*, 137, 19-28.
Sung, C.S., and Ock, Y.S. (1992), Optimal production policy for a single-product single machine problem with intermediate machine inspection allowed, *International Journal of Production Economics*, 28, 85-94.
Wang, W.Y. (2013), Optimum production and inspection modeling with minimal repair and rework considerations, Applied Mathematical Modelling 37, 1618–1626.
Xiang, Y. (2013), Joint optimization of X control chart and preventive maintenance policies: A discrete-time Markov chain approach, *European Journal of Operational Research*, 229, 382–390.

Biography

Hossein Beheshti Fakher currently is a Ph.D. student in industrial engineering in department of mechanical engineering at Laval University, Quebec, Canada. He earned his B.S. in Industrial Engineering from Iran University of Science and Technology, Tehran, Iran, and his Masters in Systems and Productivity Management from Sharif University of Technology, Iran. He has done research projects with Iran Khodro Company (Iranian Automaker Company). His research interests include Production and maintenance scheduling, reliability, Layout planning and optimization, reliability, Multi State Machines, and scheduling in complex manufacturing systems. He is member of Interuniversity Research Center on Enterprise Networks, Logistics and Transportation (CIRRELT).

Mustapha Nourelfath is currently a Visiting Professor at Kuwait University in Industrial and Management Systems Engineering Department. He is on leave from Mechanical Engineering Department in Laval University (Canada). From 1999 to 2005, he was Professor at the University of Quebec (UQAT, Canada). After graduating from ENSET-Mohammedia (Morocco), Professor Nourelfath obtained a M.Sc. and a Ph.D. in industrial automation and industrial engineering from the National Institute of Applied Science of Lyon (France). Dr. Nourelfath is a member of CIRRELT (Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation). His specific topics of interest are operations research and artificial intelligence applications in reliability, logistics and manufacturing. He has published about 50 papers in well-known international journals and presented in more than 100 international conferences. He is member of the Editorial Board of Reliability Engineering and Systems Safety, and International Journal of Performability Engineering. He is also member of the organizing and scientific committees of different international conferences, and regularly acts as a referee for more than 30 scientific journals.

Michel Gendreau is Professor of Operations Research at the Department of Mathematics and Industrial Engineering of École Polytechnique de Montréal (Canada). He received his Ph.D. from University of Montreal in 1984. From 1999 to 2007, he was the Director of the Centre for Research on Transportation. Since September 2009, he is the Chairholder of the NSERC/Hydro-Québec Industrial Chair on the Stochastic Optimization of Electricity Generation. His main research area is the application of operations research to energy, transportation and telecommunication systems planning and management. Dr. Gendreau is the Editor in chief of Transportation Science and member of several other editorial boards. Until December 2009, he was also Vice-President of the International Federation of Operational Research Societies (IFORS) and of The Institute for Operations Research and Management Science (INFORMS). Dr. Gendreau has received several research grants and awards including the Merit and the Service Award of the Canadian Operational Research Society.