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MIP-based Models vs. Heuristics Algorithms for Solving Dynamic Network Flow Problems

Mojahid Saeed Osman
Department of Systems Engineering
King Fahd University of Petroleum & Minerals, Dhahran 31261, Saudi Arabia

Bala Ram
Department of Industrial & Systems Engineering
North Carolina A&T State University, Greensboro, NC 27411, United States

M. I. M. Wahab
Department of Mechanical and Industrial Engineering
Ryerson University, Toronto, ON M5B 2K3, Canada

Abstract

Standard network flow problems are structured problems that can be solved with known general approaches. However, because of the underlying dynamic structure special methods have been developed for solving dynamic network flow problems. In this paper we present a comparison of the implementation and results of integer programming model and heuristics algorithm developed for solving dynamic network flow problems. The results came from the developed approach show that the selected MIP-based model is competitive with heuristic methods in terms of the resulting routes and schedules.

Keywords
Network flows; Integer programming; Heuristics

1. Introduction

Dynamic flow, flow over time, generalizes standard network flows by introducing an element of time. The previous research on dynamic flow problems are classified into two approaches. One approach models time in discrete time steps. The other approach models time continuously. Research of the first type typically uses the time-expanded network, either explicitly in the algorithms, or implicitly in the proofs, to produce efficient algorithms [1, 2]. The quickest transshipment problem is defined on a dynamic network with a set of sources and sinks; each source has a specified supply of flow, and each sink has a specified demand. The quickest flow problem is the single-source single-sink version of the quickest transshipment problem.

The maximum dynamic flow problem generalizes the standard maximum flow problem by introducing time. The objective is to send as much flow from source to sink in a particular time interval as possible, and capacities are interpreted as an upper bound on the rate of flow entering an arc. A related problem is the universally maximum flow, which is to send a flow from source to sink that maximizes the amount of flow arriving at the sink by a certain time. Hoppe and Tardos [3] considered a further generalization of this problem that allows arc and node capacities to change over time; they showed how to compute a flow that maximizes the amount of flow reaching the sink in all time intervals. Burkard et. al. [4] showed that the quickest flow problem is closely related to the maximum dynamic flow problem and to linear fractional programming problems. Based on these relationships they developed several polynomial algorithms and a strongly polynomial algorithm for the quickest flow problem.

The approach that models time continuously has considered networks with time-varying capacities and costs, and has focused on proving the existence of optimal solutions while further generalizing the model [1,2]; Hajek and Ogier [5] provided the first polynomial time algorithm to solve a continuous dynamic network flow problem. They considered a network with zero transit times, constant upper bounds on the flow rates, and infinite node storage, and constructed a provably polynomial time algorithm to empty the network of excess supply in minimum time. Ogier [6] generalized this result to solve the problem where the edge and storage capacities are piecewise constant functions of time. Fleischer and Skutella [7,8] described faster algorithms to solve the Hajek and Ogier problems, and related dynamic flow problems and introduced a condensed variant of time-expanded networks which is based on a rougher discretization of time and therefore leads to networks whose size is polynomially bounded in the input size [9].

Evacuation problems can be modeled as flow problems on dynamic networks. Most evacuation models are based on dynamic network flow. The common idea of these models is to represent the attributes of a building or region components in a static network G. The modeling of evacuation over time is then done in a dynamic network G_T which is the time expanded version of G and in which the network flows correspond to evacuation processes [10, 11].

The purpose of this paper is to provide a comparative study of the implementation and comparative analysis of the results of two specialized methods, MIP-based model and heuristics algorithm used or solving dynamic network flow applied to evacuation problems.

2. Selected Approaches

1) *MIP-based Model:* Two-Phase Evacuation Routing (2PER): Saeed Osman & Ram [12] proposed a Two-Phase Evacuation Routing (2PER) approach to solving the dynamic flow of emergency evacuation problem. The two main steps are as follows:

- *Determine and assign evacuation routes*: A set of routes is determined for individual object/evacuee, which includes assigning pathways in the building and road network using the solution for the un-capacitated integer multi-commodity routing (UIMR) model.
- *Compute total evacuation time and largest start time:* A capacity-constrained evacuation scheduling (CCES) algorithm is developed for calculating the total evacuation time and the largest start time. This algorithm requires a predetermined upper bound on the number of evacuees (that can traverse every pathway) per unit time, and the time needed to travel along each pathway. It assumes staggering of the departure of individual objects from their source points, but with no delay at the intermediate nodes along the path; the largest of these departure times is termed “largest start time”.

2) *Heuristic Algorithm:* Capacity Constrained Route Planning (CCRP) is a well-known heuristic approach that proposed by Lu, George, and Shekhar makes use of well known shortest path algorithms and extends them by incorporating capacity constraints [13]. It models capacity as a time series to account for the time dependent nature of the networks. It uses only the original evacuation network instead of the time-expanded network used by the LP based approach and thus requires less memory. This algorithm may provide generally good results, however; heuristic solutions are usually near optimal solutions.

The capacity constrained routing planner algorithm (CCRP) is one of the best algorithms for solving the capacity constrained evacuation planning problem. The CCRP uses a heuristic approach to solve the evacuation route planning problem. The basic idea behind the heuristic is to send the largest possible number of evacuees on the shortest path to the nearest destination [13]. CCRP method uses Dijkstra‟s shortest path algorithm; in addition, it employs a time-series to account for node and edge capacity constraints of the network. The algorithm does not always yield an optimal solution with minimum evacuation time, as shortest paths are computed repeatedly at different snapshots of time.

The CCRP algorithm generates an evacuation plan that consists of origin-destination (OD) routes and a scheduling of evacuees on each route. Although the CCRP method has been effectively used in medium-sized networks, it might not scale up efficiently to an evacuation scenario that involves millions of evacuees and a transportation network of hundreds of thousands of nodes. We implement CCRP algorithm using two modules: Shortest-Paths (SP) module and Evac-Scheduling (ES) module. We report this implementation in the next section. Here, we present the implementation of 2PER approach by comparing it to the CCRP algorithm.

3. Implementation of CCRP

We implement CCRP algorithm using two modules: Shortest-Paths (SP) module and Evac-Scheduling (ES) module as shown in Figure 1. We report this implementation here in this paper as Lu et al. [13] do not provide any implementation details. The algorithm as stated in [13] could have variations in implementation.

The SP module constructs the shortest route between all starting-nodes and destination-node pairs. Every arc between two nodes has a certain travel time, and the CCRP algorithm finds the path for each start-destination pair with the lowest travel time. Using the shortest paths generated from the SP module, the EP module searches for routes minimizing the total evacuation time when considering the capacities of arcs and nodes on the evacuation routes.

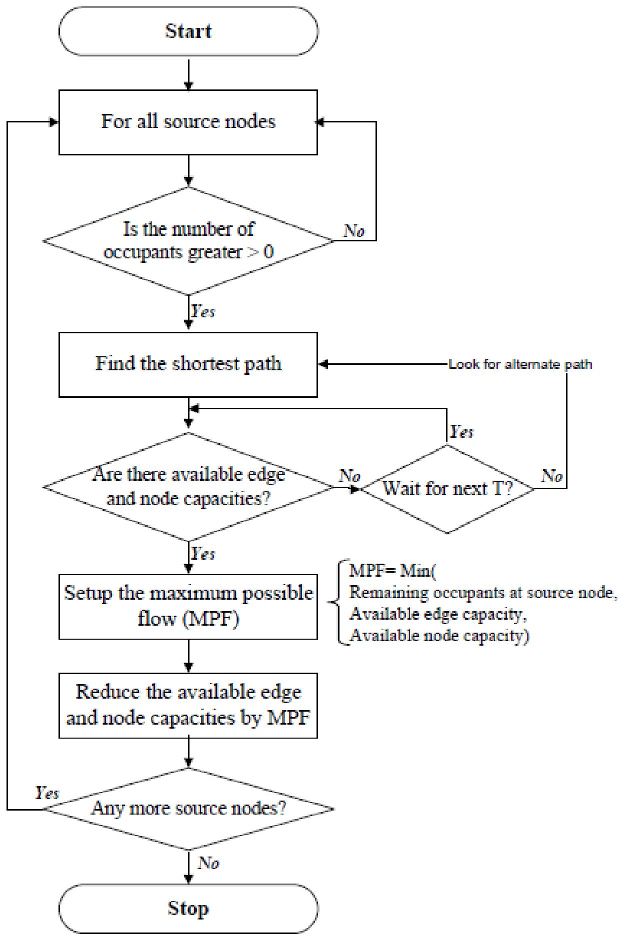

Figure 1: Heuristic Algorithm Implementation

The SP algorithm assembles Dijkstra‟s algorithm that is the most widely cited algorithm. SP constructs the shortest path between every source-gateway node pairs with the lowest travel time. Given the nodes input data file which includes the initial occupancy of each node and the edges input data file that contains the source-destination nodes for each edge, upper bound of the number of evacuees per time unit that can traverse the edge, and the edge travel time, this algorithm finds the shortest path between every source node (has initial occupancy greater than zero) and gateway node (has initial occupancy of „9999‟) with the lowest travel time. Once a source node with initial occupancy greater than zero and unequal to „9999‟ is found, SP determines the path travel time by accumulating the travel time along the edges to reach every node with initial occupancy of „9999‟and stores the source and the gateway ID, travel time, and the edges along the shortest path in an output data file, to be used as an input data file for the ES implementation. The SP module is implemented in Microsoft Visual Basic 2008.

Using the shortest paths generated from the SP module, The ES algorithm searches for routes minimizing the total evacuation time when considering the capacities of arcs and nodes on the evacuation routes, thus the SP output file is an essential input data file for the ES. The nodes and the edges input data file with edge and node capacities, travel time on edges, and initial occupancies at source nodes are also essential in order to account for capacity constraints. The CCRP algorithm uses a time expanded network for capacity constraint evaluation and object evacuation. In ES, this time expanded network is implemented via arrays for nodes availability and edges availability, which hold edge and node capacities at every time-slot (the number of time slots are as many as the upper bound of evacuation time). The availability of nodes and edges are initialized with the nodes and edges maximum capacities. The implementation begins with selecting the source node with earliest arrival time to a gateway node. The number of objects that can flow via the shortest path is determined using the minimum number of evacuees at the source node, the available capacity of each edge along the route, and the available capacity at each node on the route. If the number of objects that can flow via the shortest path is zero (there is no available capacity at a node or an edge along the selected shortest path) at the current time T, a decision on whether to select another path or to keep objects waiting for the next time T has to be made. Selecting another path or keeping objects waiting depends on a comparison of path travel times. If an object will arrive at a gateway node earlier using the selected shortest path by waiting one time unit than by selecting another path, then the start time of that particular object would be updated by incrementing one time unit. If an object will arrive to any gateway node earlier by selecting another path than by waiting one time unit on the selected shortest path then an alternate path will be chosen and the start time will remain the same.

Once an object is evacuated through the selected path the edge availabilities and node availabilities along that path are reduced by the appropriate number of objects, preparing the network for the next iteration. The ES module is implemented in Microsoft Visual Basic 2008.

The integration of the SP and ES modules outputs an evacuation path for individual objects with their evacuation start time.

4. Experiments and Results

The selected MIP-based model and heuristic algorithm were used for solving 16 dynamic network flow problems [12]. The flow time metric is used for comparing the selected approach. The Flow-Time obtained as results of solving the sixteen problems are given in Table 1.

Table 1 Solution metric for 16 Dynamic Network Flow Problems

No.	Problem Size (*node × arcs × objects flowing*)	Flow Time (sec)	
		MIP-based Model	Heuristic Algorithm
1	169×338×339	124	124
2	202×416×85	59	59
3	233×472×114	74	74
4	232×472×110	49	49
5	169×338×339	124	124
6	366×764×85	122	122
7	594×1246×114	152	152
8	821×1728×110	137	137
9	202×416×339	923	912
10	398×842×85	980	980
11	626×1324×114	953	953
12	853×1806×110	918	917
13	202×416×339	923	912
14	398×842×424	980	980
15	626×1324×538	980	980
16	853×1806×649	980	980

5. Conclusion

The results obtained using the heuristic algorithm is in close agreement with the MIP-based model that provides marginally better evacuation times in three out of the sixteen problems. While the selected heuristic algorithm is reported to be the best approach in terms of both solution quality and computational efficiency, it is only a heuristic and cannot yield an optimal solution. The MIP-based model approach uses an uncapacitated routing model as routing models with capacity constraints explode in size due to the time-expanded network they employ. The MIP-based model with its objective function ensures good time-efficient flows for all objects and serves to stagger the dynamic network flows from the source nodes to satisfy the flow capacity restrictions. So the results are not surprising.

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Biography

Mojahid F. Saeed Osman is an Assistant Professor of Industrial and Systems Engineering in the department of Systems Engineering at King Fahd University of Petroleum and Minerals, Saudi Arabia. He received the Bachelor degree in Mechanical Engineering from Sudan University of Science and Technology, and MS degree in Industrial & Systems Engineering and Ph.D. degree in Industrial Engineering both from North Carolina A&T State University, USA. His research interests include modeling and analysis of production systems, applied operations research, dynamic network flow modeling, and linear and nonlinear optimization. He has published and presented papers in leading international conferences in his research area. He has also published his research efforts in highly-reputed peer reviewed international journals.

Bala Ram is a Professor in the department of Industrial & Systems Engineering at North Carolina A&T State University. He is currently an Interim Associate Dean for Graduate Programs & Research of the College of Engineering. He earned BS degree in Mechanical Engineering and MS in Industrial Engineering from Indian

Institute of Technology, and Ph.D. degree from State University of New York at Buffalo. He is ABET evaluator for Industrial Engineering program. His research interests include manufacturing engineering, materials handling, industrial simulation, applied operations research and information systems. He has published over 30 papers in well known international journals, and also attended and presented papers at numerous international conferences. He worked closely with local industry on industrial engineering projects. He is a founding member of the Board of the Piedmont Triad Center for Advanced Manufacturing in NC, USA. He is a member of IIE, ASEE, and SME.

Mohamed Wahab Mohamed Ismail is an Associate Professor of Mechanical and Industrial Engineering at Ryerson University, Toronto, Ontario, Canada. He is currently a visiting Associate Professor at KFUPM. His research interests lie in intersections between operations research and finance, supply chain, and manufacturing system. He has published a number of journal and conference papers and book chapters. He received the B.Sc.Eng. degree in Mechanical Engineering from the University of Moratuwa, Sri Lanka, the M.Eng. degree in Industrial Engineering from Asian Institute of Technology, Bangkok, Thailand, and Ph.D. degree in Industrial Engineering from the University of Toronto, Canada. Dr. Wahab is a Registered Professional Engineer in the Province of Ontario, Canada.