A Heuristic Approach to Solve Production Planning Problem with Machine Failure in a Series Parallel Multi-State System

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Abstract

We consider a production system containing a set of machines which are arranged according to a series-parallel configuration. The machines are unreliable and the failure rate of machine depends on the load assigned to the machine. The expected performance of the system is considered to be a non-monotonic function of its load. The issue of machine failure rate depending upon machine load brings the issue of load distribution problem to optimize total production output. This paper considers the integration of load distribution decisions with production planning decision for a given time horizon. The product demands during a specified time horizon are known in advance. The objective is to minimize the sum of capacity change costs, unused capacity costs, setup costs, holding costs, backorder costs, and production costs. The main constraints consist in satisfying the demand over the specified time horizon, and not exceeding available repair resources to repair the breakdown machine. The paper proposes a three phase heuristic approach to solve the integrated load distribution and production planning problem. The efficiency of the proposed heuristic is tested through several numerical examples.

Keywords
Heuristics; Load distribution; Failure rate; Multi-state systems; Production planning

1. Introduction

This paper develops a heuristic to solve the problem of integrated load distribution and production planning problem in a series-parallel machine configuration. This problem was recently proposed by Mustapha and Yalaoui (2011). Many empirical studies of mechanical systems and computer systems have proven that the workload strongly affects the failure rate (Iyer and Rosetti (1986)). In industry often machines are run at different production rate to meet the production requirements. When machines are over-utilized, more failures and interruptions are observed. In industry, the production systems are often run at overloaded conditions. For example, when the demand is too high, the machines are overloaded to avoid backorders. However, the overload of machines increases their failure rate which ultimately reduces the production rate. In this situation often managers faces the problem of optimal load distribution while planning their production systems. It is therefore important to consider load versus failure rate relationship while performing production planning optimization.

Mustapha and Yalaoui (2011) considered a series-parallel multi-state production system containing a set of non-identical machines (also called components). Each machine can be assigned to discrete loads which corresponds to a possible production rate of the machine. In general, the failure rate of the machine increases with the assigned load. When machines are assigned to a higher load their production rates becomes high. However, the higher machine load also increases the failure rate which ultimately reduces the average production rate over a long horizon. Therefore, the expected machine performance behaves as a non-monotonic function of its assigned load. Levitin and Amari (2009) developed a model that determines the optimal load, on each machine of a series-parallel multi-state system, to maximize the expected performance. The objective of the present paper is to develop the solution technique for such integrated system in which load distribution decisions and tactical production planning are combined. The system produces a set of products during a given planning horizon. For each product, a demand is to be satisfied at the end of period. The integrated plan should determine the quantities of items (lot-sizes) to be produced for each period, and the optimal load on each machine. The available repair resources are limited and thus the loads are assigned in such a way that the required repair resources should not exceed available repair resources. The objective is to minimize the sum of setup costs, holding costs, backorder costs, production costs, capacity change costs, and unused capacity costs while satisfying the demand for all products over the entire horizon.

In reliability engineering, there are many papers that deal with optimal load distributions. One can distinguish between static and dynamic problems to consider the effects of load on machine failures. In the static load distribution models, the machine load is constant for a given time period. In the dynamic load distribution models, the load on a given component may vary with time (Kuo and Zuo (2003)). Although the dynamic load distribution approach has a wide range of applications, they require more information to capture the information on the system state. On the other hand, the static load distribution problem requires less information In this paper, we consider static load distribution problem while considering planning issues in an integrated way.

A number of research papers can be found in literature dealing with the tactical production planning decision (See Shapiro (1993) and Sipper and Bulfin (1997) for the advancement in this area). A comparison of lot sizing methods considering capacity change costs can be found in Collier (1980). Sebestyen and Juhasz (2003) study the impact of the unused capacity cost on production planning of manufacturing systems. Eiamkanchanalai and Banerjee (1999) developed a mathematical model for production lot-sizing with variable production rate and explicit idle capacity cost. In general, solution methodologies for multi-product capacitated lot-sizing problems vary from traditional linear mixed integer programming, and associated branch and bound exact methods to the heuristic methods. See Wolsey (2002) for a survey on solution methods for lot sizing problems.

To the best of our knowledge, the only paper which addresses the issue of combining load distribution and production planning is by Mustapha and Yalaoui (2011). The problem addressed by Mustapha and Yalaoui (2011) is important since it considers the load versus failure rate relationship while optimizing planning of production systems. The benefit of integrated model was illustrated using an exhaustive evaluation of all the load distribution method by Mustapha and Yalaoui (2011). The exhaustive enumeration approach in not suitable for large problem instances because the computer CPU time increases exponentially with the problem size. Therefore, we present a heuristic approach for solving the integrated production and load distribution planning model for multi-state series-parallel systems.

The remainder of the paper is structured as follows. Section 2 presents some definitions and preliminaries. Section 3 describes the integrated load distribution and production planning model. Section 4 presents our three-phase heuristic approach. Numerical examples are presented in section 5 followed by the conclusions in section 6.

2. Definitions and preliminaries

Before developing the integrated model in section 3, we define the load-failure relationship model, and the method used to evaluate the system production rate. We use following acronyms and notations in this paper.

Acronyms

AFTM	accelerated failure-time model
LD	Load distribution
MIP	mixed-integer program
MSS	multi-state system
UMGF	universal moment generating function

Notation

H	planning horizon
T	number of periods
t	period index ($t = 1, 2, \ldots, T$)
Λ	length of period t (all periods have the same length)
P	set of products
p	product index ($p \in P$)
d_{pt}	demand of product p by the end of period t
n	number of components
j	component index ($j = 1, 2, \ldots, n$)
L_j^t	load on machine j during period t

$\overline{L}_j$	maximum allowed load on component *j*
$\underline{L}_j$	minimum allowed load on component *j*
L_{j0}	baseline load of component j
λ_j^t	failure rate of component *j* during a period *t*
α_φ	parameter of component *j* power law
A_j^t	steady-state availability of machine *j* during a period *t*
μ_j	repair rate of machine *j*
g_j^t	production rate of component *j* during a period *t*
G_{MSS}	average production rate of the MSS
h_{pt}	inventory holding cost per unit of product *p* by the end of period *t*
b_{pt}	backorder cost per unit of product *p* by the end of period *t*
s_{pt}	fixed set-up cost of producing product *p* in period *t*
π_t	variable cost of producing one unit of product *p* in period *t*
u	unitary cost of unused capacity
c_{jt}	cost of changing load distribution for machine *j* from period *t*-1 to period *t*
δ_{ij}	Kronecker delta function; δ_{ij} = 1 if i = j, and δ_{ij} = 0 otherwise
RT	required repair resource
RT_0	available repair resource

Decision variables

x_{pt}	quantity of product *p* to be produced in period *t*
I_{pt}	inventory level of product *p* at the end of period *t*
B_{pt}	backorder level of product *p* at the end of period *t*
y_{pt}	binary variable, which is equal to 1 if the setup of product *p* occurs at the end of period *t*, and 0 otherwise
L^t	vector of loads of all the machines for a given period *t*, $L^t = \left\{L_1^t, L_2^t, \ldots, L_n^t\right\}$

2.1. The load-failure rate relationship model

Let L_j^t be the load on machine *j* during period *t*, where $L_j^t \in \left[\underline{L}_j, \overline{L}_j\right]$. It is assumed that the failure rate of machine *j* depends on the load it is carrying, and the function that describes the relationship between the load and failure rate $\lambda_j^t\left(L_j^t\right)$ is known. We use accelerated failure-time model (AFTM) used by Nelson (1990) to express the relationship between the load and the failure behavior of a machine. It was first proposed by Pike (1966), and widely applied after it was used by Nelson (1990) . In AFTM, the failure rate for a given period *t* is expressed as follows:

$$\lambda_j^t\left(L_j^t\right) = \lambda_j\left(L_{j0}\right)\left(L_j^t / L_{j0}\right)^{\alpha_j}, \tag{1}$$

Where L_{j0} is the initial load of machine *j*, $\lambda_j\left(L_{j0}\right)$ is the failure rate at the initial load and α_j is the parameter of component *j* power law. Equation (1) can be simplified as follows:

$$\lambda_j^t\left(L_j^t\right) = k_j \; {L_j^t}^{\alpha_j}, \text{ where } k_j = \frac{\lambda_j\left(L_{j0}\right)}{{L_{j0}}^{\alpha_j}}. \tag{2}$$

2.2. Machine performance model

To evaluate the average production rate of the system during each period, it is necessary to estimate the average availability of each machine j (j = 1, 2, …, n) per period. Let $A_j^t\left(L_j^t\right)$ denote the steady-state availability for machine j at time period t for machine load L_j^t and let μ_j denote the repair rate of machine j then the availability can be expressed as:

$$A_j^t\left(L_j^t\right)=\frac{\mu_j}{\lambda_j^t\left(L_j^t\right)+\mu_j}. \tag{3}$$

2.3. MSS performance model

Let $\mathbf{L}^t=\left\{L_1^t, L_2^t, ..., L_n^t\right\}$ be the vector of loads of all the machines for a given period t. The average production rate of the entire MSS is denoted by $G_{MSS}(\mathbf{L}^t)$ which is a function of the load vector $\mathbf{L}^t$. It depends on the performance of its machines and their availability. Once the average availability is calculated for each period, and for each machine, an appropriate evaluation method can be used to determine $G_{MSS}(\mathbf{L}^t)$. A detailed study of these evaluation methods can be found in Lisnianski and Levitin (2003) and Levitin (2005).

3. The integrated model

The problem under study considers an integrated load distribution, and production planning model for a multi-state system. The production planning decisions involve determining the production quantities of items (lot-sizes) to be produced in each time period. Tactical planning bridges the transition from the strategic planning level (long-term) to the operational planning level (short-term) where time horizons are usually considered to be one week or one month. The integrated model is mathematically formulated as follows:

Minimize

$$\sum_{p\in P}\sum_{t=1}^{T}\left(h_{pt}I_{pt}+b_{pt}B_{pt}+\pi_{pt}x_{pt}+s_{pt}y_{pt}\right)+\sum_{j=1}^{n}\sum_{t=1}^{T}c_{jt}\left(1-\delta_{L_j^tL_j^{t-1}}\right)+\sum_{t=1}^{T}u\left(\Lambda G_{MSS}\left(\mathbf{L}^t\right)-\sum_{p\in P}x_{pt}\right) \tag{4}$$

subject to

$$I_{pt}-B_{pt}=I_{pt-1}-B_{pt-1}+x_{pt}-d_{pt},\quad p\in P,\ t=1,\ 2,\ ...,\ T, \tag{5}$$

$$x_{pt}\le\left(\sum_{q\ge t}d_{pq}\right)y_{pt},\quad p\in P,\ t=1,\ 2,\ ...,\ T, \tag{6}$$

$$\frac{\sum_{p\in P}x_{pt}}{\Lambda}\le G_{MSS}(\mathbf{L}^t),\ t=1,\ 2,\ ...,\ T, \tag{7}$$

$$RT(\mathbf{L}^t)\le RT_0,\ t=1,\ 2,\ ...,\ T, \tag{8}$$

$$\mathbf{L}^t=\left\{L_1^t, L_2^t, ..., L_n^t\right\},\ t=1,\ 2,\ ...,\ T, \tag{9}$$

$$L_j^t\in\left\{\underline{L_j},\ \underline{L_j}+1,\ \underline{L_j}+2,\ ...,\ \overline{L_j}-1,\ \overline{L_j}\right\},\ j=1,\ 2,\ ...,\ n;\ t=1,\ 2,\ ...,\ T, \tag{10}$$

$$x_{pt}, I_{pt}, B_{pt}, \underline{L_j}, \overline{L_j}\in\mathbf{N};\ y_{pt}\in\left\{0,\ 1\right\}. \tag{11}$$

The first term in the objective function (4) represents the sum of inventory cost, back order cost, production cost and setup cost. The second term represents the capacity change cost for changing machine load between period t-1 to period t. Here, δ_{ij} is the Kronecker delta function, which takes value 1 if $i = j$ and zero otherwise. The third term represents the cost of unused capacity where parameter u designates unitary cost of unused capacity, and Λ designates the period length. The first constraint (5) ensures the flow of inventory between two consecutive periods. The second constraint (6) forces $x_{pt} = 0$ if $y_{pt} = 0$, and frees $x_{pt} \geq 0$ if $y_{pt} = 1$. Equation (7) corresponds to the available production capacity constraint. Equation (8) specifies the constraint on the total repair resources that should not exceed RT_0. Following the same reasoning as in Levitin and Amari (2009), the total expected repair time can be expressed as the following additional constraint

$$RT(\mathbf{L}^t) = \sum_{j=1}^{n} \frac{\lambda_j^t\left(L_j^t\right)}{\mu_j},\ t = 1,\ 2,\ ...,\ T\ . \tag{12}$$

Constraints (9)-(11) restricts the decision variables to take values between the minimum and the maximum allowed loads. This integrated model allows us to determine jointly the optimal values of production plan, and the load distribution (LD) decisions. For each product p, and for each period t, the decision variables are x_{pt}, I_{pt}, B_{pt}, y_{pt} and $\mathbf{L}^t$. For a period (t = 1, …, T), each possible load distribution is represented by a vector $\mathbf{L}^t = \left\{L_j^t\right\}$ with j = 1, …, n.

4. The proposed heuristic

The integrated load distribution and production planning problem is solved using a three phase heuristic approach. In the first phase, an optimal production rate of components is determined irrespective of the time period. The optimal production rate imposes restriction on the production plan as an upper limit for the available production capacity for a given time period. The second phase solves a production planning problem by imposing the optimal production rate obtained in phase 1 as an available production capacity. In the third phase, a load distribution problem is solved to determine the machine load which satisfied the production quantity obtained in phase 2. The second phase minimizes the production planning cost alone. The third phase minimizes the unused capacity cost and load changing cost while respecting the production plan obtained in phase 2. The detailed description of the three phase approach is described in the following sub sections.

4.1. Phase 1: Determine optimal production rate

This phase determines the feasible optimal production rate that can be achieved by n machines in a given time period while respecting all the machines related constraints. Since the machines parameters are same for all time periods, we solve the load distribution problem irrespective of the time period. The load distribution problem aims to maximize the production rate in such a way that the total repair resources used is less than the available repair resources. We seek to solve following load distribution problem in the first phase. Let call this problem as a problem P1.

Problem P1:

Maximize $\quad G_{MSS}(L) \quad (13)$

Subject to

$$RT(L) = \sum_{j=1}^{n} \frac{\lambda_j(L_j)}{\mu_j} \tag{14}$$

$$RT(L) \le RT_0 \tag{15}$$

$$L = \{L_1, L_2, ..., L_n\} \tag{16}$$

$$L_j \in \{\underline{L_j}, \underline{L_j} + 1, ..., \overline{L_j} - 1, \overline{L_j}\},\ j = 1, 2, ..., n \tag{17}$$

This problem cannot be solved exactly because of non-linear objective functions and constraints. Therefore, we propose a heuristic which solves problem P1in two steps. The first step finds the optimal production rate by relaxing the available resource capacity constraint ($RT(L) \le RT_0$) and the second step finds the feasible solution by decreasing the components load until the required repair resources is within the available repair resources.

Step 1: Finding Initial Solution

In this paper, we consider systems where machines are arranged in a parallel machines configuration. In case of parallel machine configuration, the total production rate $G_{MSS}(L)$ is expressed as a sum of individual production rate. Thus the problem is de composed in a *n* sub problems where the j^{th} sub-problem can be formulated as:

Problem P2:

Maximize $$G_{MSS}(L_j) \quad (18)$$

Subject to $$L_j \in \{\underline{L_j}, \underline{L_j}+1, ..., \overline{L_j}-1, \overline{L_j}\} \quad (19)$$

The production rate for machine j $G_{MSS}(L_j)$ in parallel machine system can be stated as a $A_j(L_j)\times L_j$. After substituting the expression for $A_j(L_j)$, problem P2 is expressed as:

Maximize $$\frac{\mu_j \times L_j}{\mu_j + \lambda_j(L_j)} \quad (20)$$

Subject to $$L_j \in \{\underline{L_j}, \underline{L_j}+1, ..., \overline{L_j}-1, \overline{L_j}\} \quad (21)$$

The objective function is a simple continuous non linear function of machine load which maximizes at load $L_j^{free} = \left(\frac{\mu_j}{k_j(\alpha_j - 1)}\right)^{1/\alpha_j}$. With maximum and minimum load constraint the optimal load is expressed as a $L_j^{continious} = \min(\max(L_j^{free}, \underline{L_j}), \overline{L_j})$. If the value of $L_j^{continious}$ is integer then this solution is considered as an optimal load, otherwise two adjacent integer points are checked for the optimal load.

We will use notation $L_j^{Glob-opt}$ and $L_{MSS}^{Glob-opt}$ to represent the load on component *j* and optimal production rate obtained by solving problem P2. We use notation $RT^{Glob-opt}$ to represent the total resources used when components loads are set to $L_j^{Glob-opt}$.

Step 2: Finding feasible solution

The solution obtained in step 1 could be infeasible with respect to resource capacity constraint (15). If the problem is infeasible then an attempt is made to find the feasible solution by minimizing the components load iteratively to bring the required repair resources below the available repair resources. When a component load is reduced from current load, the average production rate and the required repair resource reduces. In order to find the feasible solution, we start with the optimal load $L_j^{Glob-opt}$ and seek to find the component which reduces the average production rate least and repair resources most when component load is reduced by one. Let the optimal load find in this step is denoted by L_j^{Opt} and the corresponding production rate is denoted by G_{MSS}^{Opt}.

4.2. Second Phase: Solving production planning problem

The second phase of the proposed heuristic solves the production planning problem by imposing an optimal production rate as a production capacity constraint on the problem. We solve following production planning problem which is referred as problem P3.

Problem P3:

Minimize $$\sum_{p\in P}\sum_{t=1}^{T}\left(h_{pt}I_{pt}+b_{pt}B_{pt}+\pi_{pt}x_{pt}+s_{pt}y_{pt}\right) \tag{22}$$

subject to $$I_{pt}-B_{pt}=I_{pt-1}-B_{pt-1}+x_{pt}-d_{pt},\ \ p\in P,\ \ t=1,\ 2,\ ...,\ T, \tag{23}$$

$$x_{pt}\le\left(\sum_{q\ge t}d_{pq}\right)y_{pt},\ \ p\in P,\ \ t=1,\ 2,\ ...,\ T, \tag{24}$$

$$\frac{\sum_{p\in P}x_{pt}}{\Lambda}\le G_{MSS}^{Opt},\ t=1,\ 2,\ ...,\ T, \tag{25}$$

$$x_{pt},\ I_{pt},\ B_{pt}\in\mathbb{N};\ y_{pt}\in\{0,\ 1\}. \tag{26}$$

We use CPLEX to solve the problem P3 exactly. This phase determines the production quantity x_{pt}, the back order quantity B_{pt}, and inventory quantity I_{pt}. Let $X_t=\sum_{p\in P}x_{pt}$ denote the total quantity produced in period *t* determined after solving problem P3. The quantity X_t is now a known quantity. The third phase tries to minimize the unused capacity cost by finding the machine load close to the value X_t.

4.3. Third Phase: Solve load distribution problem

The third phase solves the modified load distribution problem to minimize the unused capacity cost by setting the machine load in such a way that the total production quantity is close to the required production quantity X_t obtained in second phase. Our aim in third phase is to solve following problem called problem P4.

Problem P4:

Minimize $$\sum_{j=1}^{n}\sum_{t=1}^{T}c_{jt}\left(1-\delta_{L_j^tL_j^{t-1}}\right)+\sum_{t=1}^{T}u\left(\Lambda G_{MSS}\left(\mathbf{L}^t\right)-X_t\right) \tag{27}$$

Subject to $$G_{MSS}(L^t)=\sum_{j=1}^{n}\frac{\mu_j\times L_j}{\mu_j+\lambda_j(L_j)},t=1,2,...,T, \tag{28}$$

$$G_{MSS}(L^t)\ge X_t,t=1,2,...,T, \tag{29}$$

$$L_j^t\in\{\underline{L_j},\underline{L_j}+1,...,\overline{L_j^{Opt}}-1,\overline{L_j^{Opt}}\},\ j=1,2,...,n;t=1,2,...,T. \tag{30}$$

This problem cannot be solved exactly because of non linearity, therefore, we use heuristic to solve the problem. Our heuristic starts with the optimal load L_j^{Opt}, for all the machines. Then we improve the solution by decreasing their load to bring the average production rate $\Lambda G_{MSS}(L^t)$ close to the required production quantity X_t.

The three phase approach described above determines the complete solution for integrated load distribution production planning problem. The second phase determines the production quantity x_{pt}, the back order quantity B_{pt}, and Inventory quantity I_{pt} while the third phase determines the load of component *j* for period *t*. The total cost of integrated problem is the simply the sum of the objective function of problem P3 and P4.

5. Numerical experiments

The effectiveness of the proposed heuristic is evaluated using randomly generated problem instances. We have generated two types of data sets on the basis of effectiveness of the available resources (i.e., effectiveness of constraint $RT(L^t)\le RT_0$). These data sets are denoted by SetT and SetL. The available resources are very tight in

SetT while it is loose and do not impose restriction in SetL. Each data set consists of 5 problem sets, A, B, C, D and E where numbers of machines are 5, 10, 15, 20 and 25 respectively. Each data set is divided in two sets, indexed 1 and 2 where index 1 and 2 has time period of 5 and 10 respectively. For each problem set 10 problem instances are generated. Set of product P is set to 2 and length of period Λ is set to 1 for all problem sets. Other parameter values related to machine are generated randomly. For a particular instance, all the values for SetT and SetL are same except the available resources.

The proposed heuristic is coded in C and implemented on AMD Opteron 2.3 GHz with 16 GB of RAM. We used lower bound solution to evaluate the performance of our heuristics. We obtained lower bound value by solving the production planning formulation while ignoring the load distribution related constraints and objective function component. We used CPLEX 12.4.0.0 version to solve exact problem in phase 2 and to calculate lower bound formulation.

Table 1: Comparison of Heuristic Solution for Data Type SetT, and SetL

Problem Set	N	T	SetT			SetL		
			Lower Bound Cost	Heuristic Cost	% Gap	Lower Bound Cost	Heuristic Cost	% Gap
A1	5	5	76281.70	79835.99	4.43	30501.15	77150.42	1.30
A2	5	10	193749.50	210389.20	7.54	59672.85	195622.80	1.23
B1	10	5	116841.00	121785.22	4.22	48650.76	118298.19	1.42
B2	10	10	241425.00	260434.90	7.11	75156.06	244560.80	1.49
C1	15	5	180174.40	191048.70	5.41	66295.37	182009.30	1.12
C2	15	10	316494.60	341121.60	8.02	196986.6	319349.90	0.86
D1	20	5	258671.50	267757.40	3.40	127733.1	260898.00	0.93
D2	20	10	491834.40	519274.60	5.40	226803.1	496507.00	1.01
E1	25	5	286653.90	301970.90	4.96	128283.1	289070.20	0.87
E2	25	10	593923.50	615295.00	3.43	280312	599407.80	0.97
Average			275604.95	290891.35	5.39	122244.1	278287.44	1.12

Table 1 reports the lower bound value, the average absolute value produced by heuristic solution and their % gap from lower bound solution. For each problem set the average value of 10 problem instances are reported.

The results reported in Table 1 shows that the proposed heuristic is on average 5.39 % away from the lower bound solution for SetT and 1.12 % for SetL. The percentage gap for problem set A1, B1, C1, D1 and E1 is 4.43, 4.22, 5.41, 3.40 and 4.96 for SetL. These results show that the proposed heuristic is able to produce consistently good solution irrespective of the problem size. This consistency is supported by the results of SetL as well. Comparing the results between problem sets A1 &A2, B1 & B2, C1 & C2, D1 & D2 and E1 & E2 indicate that when the number of time period increases the optimal percentage gap increases. Note that the number of time period of index 1 is 5 while it is 10 for index 2. These results gives an impression that the heuristic performance decrease when number of time period increases, however, this is not always true. Comparison of lower bound percentage gap between problem sets E1 & E2 from SetL, B1&B2, and A1 & A2, C1 & C2 from SetL shows improvement of heuristic performance with the number of time period increase.

The results reported in Table 1 shows that the heuristic solution is on average 5.39 %, and 1.12 % away from the lower bound solution for SetT and SetL respectively. The percentage gap from lower bound for SetT is maximum because the available repair resource constraint is too tight. The percentage gap from lower bound for SetL is low because the available repair resources constraint becomes redundant for SetL. Note that the tightness of our current lowerbound depends on gap between production rate for infinite resources and the production rate for given resources (i.e., the gap between $G_{MSS}^{Glob-opt}$ and G_{MSS}^{opt}). When gap between $G_{MSS}^{Glob-opt}$ and G_{MSS}^{opt} increases, calculation of our lowerbound value deteriorates. The gap between $G_{MSS}^{Glob-opt}$ and G_{MSS}^{opt} is zero for SetL and therefore the lowerbound calculation for SetL is considered to be close to the optimal solution for SetL. The percentage gap from lower bound for SetL is just 1.12 %. This result shows that our proposed heuristic is able to produce the results close to the optimal solution for problem instances varying from 5 machines to 25 machines.

6. Conclusions

This paper considers production planning problem with unreliable machine in the context of multi state system. The problem is important because it considers the load versus failure rate while optimizing production planning system. The proposed model is motivated by the fact that combining load distribution decisions with production planning may reduce the total expected cost. The objective of the production planning is to minimize the total production cost while the objective of the load distribution planning is to maximize the production planning rate which might lead to the high cost of unused capacity. If the load distribution and production planning decisions are performed sequentially then it might lead towards the non-optimal decisions. Therefore, the integrated load distribution and production planning decisions are considered to find the best tradeoff between the different parameters of load distribution and production planning.

We propose partition based heuristic approach to solve the problem. The proposed heuristic solves the problem in three phases. In the first phase a load distribution problem for a single period is solved to find the possible maximum production rate that is possible in a given time period. The second phase solves a production planning problem by respecting the upper limit of production rate obtained in phase 1. In the third phase, a load distribution problem is solved for the complete planning period. The effectiveness of proposed heuristic solutions is evaluated by solving the randomly generated problem instances. The lower bound value is obtained to evaluate the performance of proposed heuristic. The comparison with lower bound solution shows that the proposed heuristic produces good quality results for randomly generated problem instances considered in this paper. The proposed heuristic is used to solve the MSS with parallel arrangement of machines; however, it can be easily extended for series-parallel arrangement of machines.

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