An Exact Solution for a Class of Green Vehicle Routing Problem

Mandy Taha, M. Nashat Fors and Amin A. Shoukry
Faculty of Engineering
Alexandria University
Egypt

Abstract

A mathematical model for presenting an exact solution for the Green Vehicle Routing Problem (G-VRP) is developed in this paper. G-VRP is concerned with minimizing the travel distance while maintaining less emission of carbon dioxide by using alternative sources of fuel. The solution aims to aid organizations that operate a fleet of alternative fuel-powered vehicles to overcome challenges that occur due to limitation of refueling infrastructure and vehicle driving range and to help them to plan for refueling and incorporate stops at Alternative Fuel Stations (AFS) so as to eliminate the risk of running out of fuel while sustaining low cost routes. The solution of the model shows that the problem could be extended for further adoptions and techniques as discussed.

Keywords
Vehicle routing and scheduling problem, optimization, Green vehicle routing problem, G-VRP, exact solution, Alternative fuel stations.

Introduction

In recent years, the greenhouse effects plays an important role worldwide with many laws and regulations to reduce greenhouse gas pollution has already been passed that have strong effects on the logistics industry. Many logistics companies have already started to establish "Green Logistics" projects to reduce CO2 emissions Transportation is one of the most important parts of logistics and a fundamental infrastructure for economic growth and one of the hugest petroleum consumers and accounts for a large part of the overall pollutants (Salimifard et al., 2012).

Logistic companies may use optimization methods to improve route planning, which helps to decrease the traveled distance of their vehicles and hence emissions. Researchers tend to pay close attention to the role that transportation will play in achieving positive environmental effects. Exploring the relationship between environmental effect and transportation through route planning will be able to provide practical and valuable suggestions regarding this green logistics campaign

In this paper, a particular consideration is given to routing and scheduling model that relate to environmental issues, it was previously presented as G-VRP by Erdogan and Miller (2012), a class of the capacitated vehicle routing problem. The contribution of this paper is to present a mathematical model that solve a class of the Green Vehicle routing Problem (G-VRP) and highlight the need for more future research directions for the GVRP and solutions approaches.

The remaining of this paper is organized as follows: Section 2 concerns the review of literature of the evolution of G-VRP with the existing research literature and its variants. Section 3 presents the mathematical model for solving the problem, and a brief introduction to main benchmark test instances in Section 4. In Section 5, computational results and a conclusion is drawn.

Review of Literature

During recent years, green logistics has received a growing attention from organizations and governments. Green Logistics deals with the activities of measuring the environmental effects of different distribution strategies, reducing the energy consumption, recycling refuse and managing waste disposal (Sbihi & Eglese 2007a). There is a wide variety of problems concerning Green logistics, such as the promotion of alternative fuels, next-generation electronic vehicles, green intelligent transportation systems, and other eco-friendly infrastructures. Also, a better deployment of vehicles and a cost effective vehicle routing solution would more directly accomplish sustainable transportation patterns

Green transportation was initiated with the studies of Sbihi and Eglese (2007) and the PhD dissertation of (Palmer 2007). Sbihi and Eglese (2007a, 2007b) employed the Time-dependent VRP as an approach to deal with the minimization of emissions during routing. The existing literature argue that reduction in total distance will in itself provide environmental benefits due to the reduction in fuel consumed and the consequent pollutants.

Bektas and Laporte (2011) introduced the "Pollution Routing Problem (PRP)" where they developed the PRP as an extension of the classical VRP with a more comprehensive objective function that deals with the travel distance, the amount of green-house emissions, fuel, travel times and their costs. They also presented the various tradeoffs between various parameters such as vehicle load, speed and total cost. Xiao et al. 2012 considered Fuel Consumption Rate (FCR) as a load dependent function, and add it to the classical CVRP to extend traditional studies on CVRP with the objective of minimizing fuel consumption and called it FCVRP and develop a simulated annealing algorithm with a hybrid exchange rule to solve it. According to Canhong Lin, et al. (2013), there are three major categories of GVRP, including Green-VRP, Pollution Routing Problem and VRP in Reverse Logistics.

In their paper Erdogan and Miller-Hooks (2012) introduced Green Vehicle Routing Problem (G-VRP) that in which AFV are allowed to refuel on the tour to extend the distance it could travel. To the best of our knowledge they were the first to consider the possibility of recharging or refueling a vehicle on the route in VRP. Their model seeks to eliminate the risk of running out of fuel as well as considering the service time of each customer and the maximum duration restriction was posed on each route. Also, Schneider, Stenger, and Goeke D. (2012), extended the G-VRP with time windows and denoted it Electric Vehicle Routing Problem with Time Windows and Recharging Stations (E-VRPTW), which incorporates the possibility of recharging at any of the available stations using an appropriate recharging scheme. They considered limited vehicle freight capacities as well as customer time windows and presented hybrid heuristic that combines a Variable Neighborhood Search (VNS) algorithm with a Tabu Search as a solution method.

The G-VRP mathematical model presented here differs from the one introduced by Erdogan and Miller-Hooks, 2012 in many substantial ways. First, it is MILP and all the constraints are linear; it could be used as multi depot and it is not too restrictive as in their paper; meaning it permits return paths that visit more than one AFS without being concerned of being stranded.

Mathematical Formulation

As discussed above the G-VRP problem seeks to find at most m tours, one for each vehicle that starts and ends at the depot visiting a subset of vertices using alternative fuel stations AFSs when needed such that the total distance traveled is minimized. In the mathematical model presented below it seeks to minimize the distance traveled using one type of fuel – the Alternative fuel. An exact solution of minimum distance traveled is expected as an outcome for the model.

The G-VRP is defined on an undirected, complete graph G = (V, E), where vertex set V is a combination of:

- v= $\{v_1, v_2, \; , v_n\}$where $n= nd+nf+nc$, is the number of nodes of depot, AFS and customers.
- Depot nodes D= $\{d_1, d_2,\ldots,d_{nd}\}= \{v_1, v_2, \; , v_{nd}\}$,
- Fuel resources set S = $\{s_1,s_{2,..,}s_{nf}\} = \{v_{nd+1,} v_{nd+2,} \; , v_{nd+nf}\}$,
- *Customer set* C = $\{c_1, c_2, \ldots, c_n\} = \{v_{nd+\,nf+1,} v_{nd+\,nf+2,} v_{nd+\,nf+2,} \;.., .v_{nd+\,nf+nc}\}$,

Thus, V the set of all nodes is $\{v_1, v_{nd+nf+1}, v_{nd+nf+2},\ldots, v_{nd+nf+nc}\}$

The set E = $\{(v_i,v_j)$: v_i, v_j € V, i <j$\}$ corresponds to the edges connecting vertices of V. Each edge (v_i, v_j) is associated with a non-negative travel time t_{ij}, and distance d_{ij}.
The Vehicle driving range constraints that are dictated by fuel tank capacity limitations and tour duration constraints meant to restrict tour durations to a pre-specified time limit T_{max} and tank capacity F_*max.*

Assumptions:

1. The depot could be used as a refueling station and all refueling stations have unlimited capacities;
2. Travel speeds are assumed to be constant over a link;
3. No limit is set on the number of stops that could be made for refueling;

4. When refueling is undertaken, it is assumed that the tank is filled to full capacity;
5. Customers could be served by a vehicle that begins its tour at the depot and returns to the depot after visiting the customer once and directly within T_{max};
6. It is assumed that all customers could be visited directly by a vehicle beginning and returning to the depot with at most one visit to an AFS.

Decision Variables

x_{ij}	Binary variable equal to 1 if a vehicle travels from vertex *I* to *j* and 0 otherwise
y_j	Fuel level variable specifying the remaining tank fuel level upon arrival to vertex j. It is reset to *F_max* at each refueling station and at the depot
τ_j	Time variable specifying the time of arrival of a vehicle at vertex j, initialized to zero upon departure from the depot

MATHEMATICAL MODEL FOR THE GVRP

The following notations will help in the description of G-VRP

Notation:

$$min \quad \sum_{i,j\in V} d_{ij}x_{ij} \tag{1}$$

$$s.t.$$

$$x_{ii} = 0 \qquad \forall i \in V$$

$$\sum_{j\in V} x_{ij} = \sum_{j\in V} x_{ji} \qquad \forall i \in V \tag{2}$$

$$\sum_{i\in V} x_{ij} = 1 \qquad \forall j \in C \tag{3}$$

$$\sum_{i\in V} x_{ij} \le 1 \qquad \forall j \in S \tag{4}$$

$$\sum_{i\in V, i\in D} x_{ij} \le m \tag{5}$$

$$T_{max} - \left(p_i + t_{ij}\right)x_{ij} - \tau_j + T_{max}\left(1 - x_{ij}\right) \ge 0 \qquad \forall i \in D, j \in V \tag{6}$$

$$\tau_i - \left(p_i + t_{ij}\right)x_{ij} - \tau_j + T_{max}\left(1 - x_{ij}\right) \ge 0 \qquad \forall i \in V\backslash D, j \in V \tag{7}$$

$$F\_max - f_{ij}x_{ij} - y_j + F\_max\left(1 - x_{ij}\right) \ge 0 \qquad \forall i \in V\backslash C, j \in V \tag{8}$$

$$y_i - f_{ij}x_{ij} - y_j + F\_max\left(1 - x_{ij}\right) \ge 0 \qquad \forall i \in C, j \in V \tag{9}$$

$$x_{ij} \in \{0,1\} \qquad \forall i, j \in V \tag{10}$$

$$0 \le y_j \le F\_max \qquad \forall j \in V \tag{11}$$

$$0 \le \tau_j \le T_{max} \qquad \forall j \in V \tag{12}$$

x_{ij}	Binary variable indicating visit from node i to node j
y_j	Real variable indicating remaining fuel upon arrival at node j
τ_j	Real variable indicating remaining time upon arrival at node j
d_{ij}	Distance from node i to node j
t_{ij}	Travel time from node i to node j
f_{ij}	Fuel consumption from node i to node j
p_i	Service time at node i

The objective function seeks to minimize total distance d_{ij} traveled from vertex i to vertex j by the AFV fleet in a given day. Constraint (2) controls the flow as it forces the vehicles entering the node to be equal to those out of that node; thus eliminating sub tours occurrence. Constraint (3) ensures that each customer vertex has exactly one visit so it forces the trip. Constraint (4) purpose is to ensure that the fuel station is visited at most once so that there would be no overlapping or interference. Constraint (5) defines that at most m vehicles routes from the depot, it will guarantee that the number of tours will remain less than the number of vehicles so that there would be no sub-tours occurrence. Time constraints 6 and 7, guarantee that the time of arrival at each vertex by each vehicle should be less than the T_{max} whether the trip started from the depot or not while the time of arrival at each vertex is being tracked. Fuel constraints 8 and 9 are to track the vehicle's fuel level based on vertex sequence and type. Constraint (10) is a binary integrality is guaranteed as x travels to j then x_{ij}=1 otherwise no trips then x_{ij}=0, finally constraint 11, 12 guarantee that the vehicles will be routed through a given day without being stranded or violating the time, or fuel constraints.

The experiments run using sets of instances for the GVRP are used for testing the model proposed by Erdogan and Miller-Hooks (2012). There are ten sets of 5, and 10 customer nodes each The customer distribution are random and clustered and the number of AFS are 2, and 3.

Experimental Environment & Parameter Settings

Exact solutions were obtained by implementing the model using MATLAB 2012B to solve the G-VRP mathematical model by using the IBM.ILOG.CPLEX Optimizer (version 12.5) and solver Yalmip *toolbox* for modeling and optimization. When running the program to solve set of small instances~~,~~ the output are graphed tours for each instance showing the nodes visited, time elapsed, and the distance travelled in each tour and the total distance as well. All tests are performed on a CPU configurations are Intel® Core™ i7 CPU 860 @ 2.80 GHz 2.79 GHz, and 16.0 GB of RAM operating Windows 7 Professional with 32 bits.

The fuel tank capacity is set to 60 liter and fuel consumption rate of 0.2 liter per mile and the average vehicle speed is assumed to be 40 miles per hour (mph) and the total tour duration limitation is assumed to be 11 h unless not stated. The service times are assumed to be 30 min at customer locations and 15 min at AFS locations

Computational Results

Table 1: Results obtained from after running the test instances

Instance Number	Dimensions		No. of Tours	Visited Nodes	Elapsed Time (seconds)	Problem Complexity
	No. of Customers	AFS				
1	10	3	3	1: [1 7 11 4 8 15 1] 2: [1 13 9 10 1] 3: [1 14 6 12 1]	12.15971	94848
	5	3	2	1: [1 5 9 10 7 8 1] 2: [1 6 1]	0.697867	17748
	10	2	3	1: [1 12 8 9 1] 2: [1 13 5 11 1] 3: [1 14 7 4 10 6 1]	1.9853	73033
	5	2	2	1: [1 5 3 7 1] 2: [1 6 9 8 1]	0.624147	11952
2	10	3	3	1: [1 6 7 8 1] 2: [1 10 4 15 9 12 1] 3: [1 13 11 5 14 1]	1.161604	96976
	5	3	2	1: [1 9 1] 2: [1 10 7 6 8 1]	0.639838	17136
	10	2	2	1: [1 9 6 5 7 1] 2: [1 13 12 10 8 14 11 1]	0.956932	73892
	5	2	2	1: [1 8 1] 2: [1 9 6 5 7 1]	1.493225	11454
3	10	3	3	1: [1 8 13 7 1] 2: [1 10 6 9 1] 3: [1 15 14 11 3 12 1]	4.960813	95910
	10	2	3	1: [1 7 12 6 1] 2: [1 9 5 8 1] 3: [1 11 3 10 13 14 1]	1.922406	72834

Instance Number	Dimensions		No. of Tours	Visited Nodes	Elapsed Time (seconds)	Problem Complexity
	No. of Customers	AFS				
3	5	3	2	1: [1 7 8 1] 2: [1 9 6 10 1]	0.655420	17850
	5	2	2	1: [1 6 7 1] 2: [1 8 5 9 1]	0.704203	12035
4	10	3	3	1: [1 10 1] 2: [1 11 13 12 7 1] 3: [1 15 8 4 9 6 14 1]	13.64972	96976
	5	3	2	1: [1 7 1] 2: [1 8 4 9 6 10 1]	5.14434	16932
	10	2	3	1: [1 6 11 12 10 1] 2: [1 9 1] 3: [1 13 5 8 4 7 14 1]	7.822659	74704
	5	2	2	1: [1 6 1] 2: [1 7 4 8 5 9 1]	0.663142	10956
5	10	3	3	1: [1 9 12 10 1] 2: [1 11 15 1] 3: [1 14 6 3 13 7 8 1]	5.036555	96512
	5	3	2	1: [1 6 7 8 1] 2: [1 9 10 1]	0.707039	16524
	5	2	2	1: [1 7 6 5 1] 2: [1 8 9 1]	0.731499	10956
6	10	3	3	1: [1 7 9 1] 2: [1 8 3 11 12 6 1] 3: [1 14 4 10 15 13 1]	5.70742	98368
	5	3	3	1: [1 6 1] 2: [1 9 7 1] 3: [1 10 8 1]	0.60066	16700
	10	2	3	1: [1 7 14 11 9 1] 2: [1 12 5 1] 3: [1 13 10 6 8 1]	4.00266	76125
	5	2	2	1: [1 8 6 1] 2: [1 9 7 2 5 1]	0.65855	11016
7	10	2	3	1: [1 6 8 1] 2: [1 7 3 10 11 5 1] 3: [1 12 14 9 4 13 1]	3.37543	72360
	5	2	2	1: [1 7 8 6 1] 2: [1 9 5 1]	0.73690	15189
	5	3	2	1: [1 8 9 7 1] 2: [1 10 6 1]	0.64855	16524
	10	3	3	1: [1 7 9 1] 2: [1 8 3 11 12 6 1] 3: [1 14 4 10 15 13 1]	5.54446	95220
8	10	3	2	1: [1 9 11 10 8 6 7 1] 2: [1 15 13 12 14 1]	9.09060	98832
	5	3	1	1: [1 7 6 8 10 9 1]	0.68480	17952
	10	2	2	1: [1 8 10 9 7 5 6 1] 2: [1 13 11 12 14 1]	4.10480	76531
	5	2	1	1: [1 6 5 7 9 8 1]	0.66605	12284

Instance Number	Dimensions		No. of Tours	Visited Nodes	Elapsed Time (seconds)	Problem Complexity
	No. of Customers	AFS				
9	5	3	2	1: [1 8 9 7 6 1] 2: [1 10 1]	0.59465	180
	10	2	3	1: [1 7 3 13 11 12 1] 2: [1 8 14 1] 3: [1 9 10 6 5 1]	2.69369	93708
	5	2	2	1: [1 7 8 6 5 1] 2: [1 9 1]	0.70026	4050
	10	3	3	1: [1 6 10 1] 2: [1 9 15 11 5 7 1] 3: [1 13 12 14 3 8 1]	3.02226	71441
10	10	3		Infeasible problem		
	5	3	2	1: [1 8 1] 2: [1 10 6 9 7 1]	0.73142	15040
	10	2		Infeasible problem		
	5	2	2	1: [1 7 1] 2: [1 9 5 8 6 1]	0.58525	49320

Results obtained from running CPLEX solver are displayed in the above table - Note that some customers contained in the small instances are infeasible, i.e., they cannot be served under the given restriction (tank and time constraints) that each customer has to be reached in time. The computing times in seconds and the problem complexity are provided in table 1.

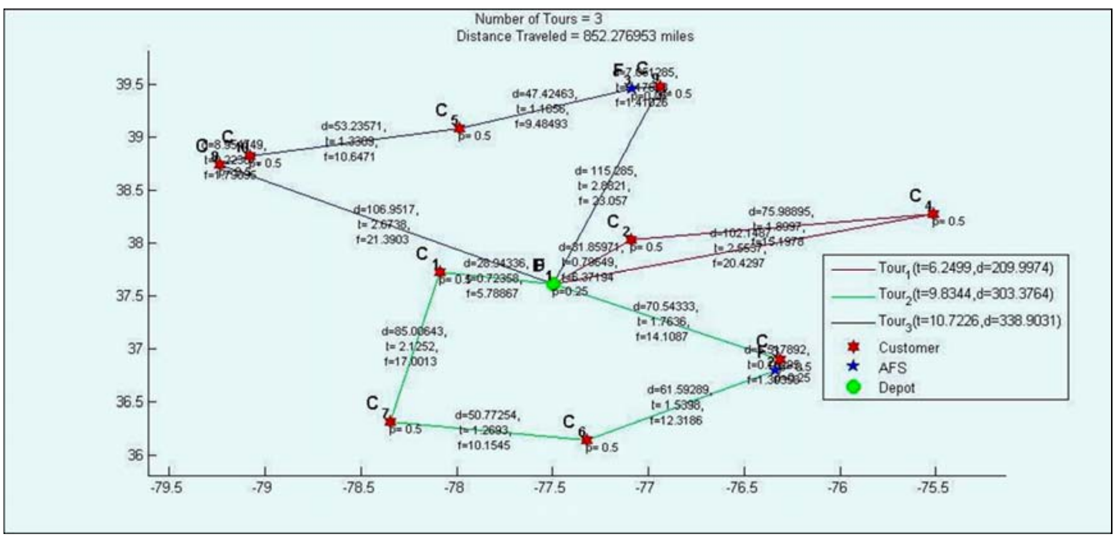

Figure 1: Graphed Solution for a Test Instance

Figure 1 explicit the routing for instance 7 that serve 10 nodes with 3 AFS. The tours are presented as closed connected nodes. There are two tours with total miles travelled 852.276 mile. Each tour started and ended at the depot whiles the visited nodes as shown in the graph. The Computational time in seconds is 94 seconds and the problem complexity is 99296.

Results obtained from running CPLEX solver are displayed in the above table. It is worth state that the infeasible solutions could be the result of the restricted constraints as tank capacity and time. The results clearly show the ability of the model to solve small instances G-VRP for one type of fuel and small sized instances.

Limitations

The vehicle routing problem is NP-hard combinatorial problem that the use of exact optimization methods may be hard to solve these problems in acceptable CPU times, when the problem involves real-world data sets that are very large. So, Exact solutions are used only for small problems and solvers are high specialized and inflexible and it needs a lot of effort to adapt them to modified problems.

Conclusion

Vehicle routing problem forms an integral part of supply chain management, which plays a significant role for productivity improvement in organizations through efficient and effective delivery of goods/services to customers. In this paper, an attempt has been made to solve the GVRP with the recent developments in the vehicle routing problem (VRP) and its variants. While the initial findings are promising as the topic is at its very beginning and is still too attractive and demanding, further research is necessary. The results of this study suggest a number of new avenues for research as considering a heterogeneous fleet of vehicles run by many different types of fuel. A practical purpose, it is hoped that these idealized models could help governments, nonprofit organizations, and companies to evaluate the possible economic and environmental significance of real-world transportation problems and to take action at different levels to contribute to Green Logistics.

References

Bektas, T and Laporte, G., The Pollution-Routing Problem, *Transportation Research Part B,* Methodological, vol. 45, no.8, pp. 1232-1250, 2011.

Baldacci, R., Mingozzi, A., and Roberti, R., Recent exact algorithms for solving the vehicle routing problem under capacity and time window constraints, *European Journal of Operational Research*, vol. 218, no. 1, pp. 1–6, 2012.

Erdogan, S., and Miller-Hooks, E., A Green Vehicle Routing Problem, *Transportation Research Part E,* vol. 48, no. 1, pp. 100–114, 2012.

Lin, Canhong, Choy, K.L., Ho, G.T.S., Chung, S.H. and Lam, H.Y., Survey of Green Vehicle Routing Problem: Past and future trends. *Expert Systems with Applications*, vol. 41, no. 4, Part 1, pp. 1118–1138, March 2014.

Palmer, A., The Development of an integrated routing and carbon dioxide emissions model for goods vehicles, *Ph. D. Thesis*, *Cranfield University*, School of Management, 2007.

Salimifard, K., Shahbandarzadeh, H., and Raees Ramin, Green transportation and the role of operation research, *International Proceedings of Computer Science & Information Tech*, vol. 26, pp. 74-79, 2012.

Sbihi, A., and Eglese, R. W., Combinatorial optimization and green logistics, *4OR: A Quarterly Journal of Operations Research*, vol. 5, no. 2, pp. 99–116, 2007a.

Sbihi, A., and Eglese, R. W., The relationship between vehicle routing and scheduling and green logistics – a literature survey, *Working paper, Department of Management Science, Lancaster University Management School*, LA14YX, UK, 2007b.

Schneider, M., Stenger, A., and Goeke D., The electric vehicle routing problem with time windows and recharging stations, Technical Report, University of Kaiserslautern, Kaiserslautern, Germany, 2012.

Xiao, Y., Zhao, Q, Kaku, I. and Y. Xu, Development of a Fuel Consumption Optimization Model for the Capacitated Vehicle Routing Problem, *Computers and Operations Research,* vol. 39, no. 7, pp. 1419–1431, July 2012.

Biography

Mandy Taha, is a PhD student of Industrial Engineering, Production Engineering Department, Alexandria University, Egypt. Taha, obtained her master in 2007 in Supply Chain management, Industrial Engineering.

M. Nashat Fors, is Professor Emeritus of Industrial Engineering, Production Engineering Department, Alexandria University, Egypt. He earned his PhD from Production Engineering Department, Faculty of Engineering, Alexandria University. He has written or co-authored research papers and articles on Industrial Applications of Mathematical Programming, Supply Chain Management, Maintenance planning, Water Management, Scheduling & Distribution. Dr. Fors has many joint projects, consultation, and training programs with industry in the area of Operations planning & scheduling, Maintenance planning, and Project management.

Amin A. Shoukry was born in Alexandria, Egypt. He obtained his B.Sc. (1973) and M.Sc. (1978) from the Computer Science and Automatic Control Department, Faculty of Engineering, Alexandria University. In 1979 he obtained a diploma in Computer Science from Pierre and Marie Curie University, Paris, France. Then he joined the AI group at the Informatics Research Center, Nancy, France, where he obtained his Doctor of Engineering degree in 1983.