

Managing Multiple Outsourcing: Service Quality and Volume Issues

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Abstract

Service operations like call center operations and business processes are being outsourced to offshore locations because of many operational reasons including cost savings. The main trade-off is the lack of control on service quality. When multiple suppliers are operating and service requirements are allocated to one or another, differential quality level is especially being an irritant to the firm's customers. In our model, the firm uses the allocation of quantity to motivate the multiple suppliers so that the equilibrium service quality in the market equalizes over all suppliers. The buyer's customers are assured of the same consistent quality level regardless of which of the multiple suppliers their calls are routed to. We obtain optimal policies of both the buyer and the suppliers in a two-supplier market and also study an asymmetric information case.

Keywords

Offshoring, Outsourcing, Service quality, Multiple suppliers

1. Introduction

Outsourcing of manufacturing processes has been used as an operations strategy for a number of decades now, the main motivations being manufacturing activities to reduce labor cost, free up capital, and improve worker productivity. Recently, service operations like call center operations and business processes are also being outsourced basically because of the same reasons. BPO (business process outsourcing) is another popular outsourcing vehicle for companies, where a non-core business process is outsourced. Examples include processes in investment companies, law firms, and pharmaceutical companies. Service operations like data entry, digitization, data preparation and validation are also popular outsourcing activities. The trade-off companies face while outsourcing a process is the lack of control on the service quality. Many companies have experienced that the call center service quality is not up to the mark for many of the offshore companies including Dell, Lehmann Brothers and Delta. Thus the quality of service is another important issue in outsourcing. One can see that any problem with the quality in the service will reflect directly to the buyer company and thus the buyer is deeply concerned with the service quality and needs tools to motivate the supplier to enhance his quality.

In a call center operation, there is always a risk in allocating all of its calls to one single call center in case the offshore company turns out to have a less than desirable quality level. One way the firms can hedge the risk is by selecting more than one company and at the same time retaining the option of keeping some calls inshore. This is a widespread practice and is called co-sharing (Aksin et al. (2008)). A research question in this context is how to allocate the total call volume among the supplier companies. In our paper, we advocate that quality of service can be the basis of this allocation of volume. The revenue received by the call center is, therefore, dependent on the volume contracted, and in turn on the service quality. The call center, therefore, has an incentive to invest into improving its service quality. Our paper is concerned with finding the optimum investment policy in service quality improvement. A game theoretic model is used to design optimal contracts between the buyer and the service supplier.

2. Brief Literature Survey

There are several research related to strategic outsourcing. Arya et al. (2008) study a firm's make-or-buy strategic outsourcing decision. Novak and Stern (2008) study how outsourcing affects performance dynamics. An empirical analysis of offshore software development is reported in Gopal et al. (2003). Kalnins (2004) studies the information technology setting and finds the role of relationships in choice of contracts types. IT outsourcing is reviewed in recent studies (Kenion (2005) and King and Torkzadeh (2008)). The topic of call center has been studied extensively in literature. One way to deal with the call center problem is treating it as a queue. A review can be found in Gans and Zhou (2007) and Gans et al. (2003). In the quality area, price and

quality competition in differentiated demand environments are studied in Kouvelis and Mukhopadhyay (1999). These papers use game theory (Yue et al. (2006)) to discuss quality under full information. Information asymmetry cases like moral hazard (one side or the other may deliberately shirk or degrade quality to reduce cost) and adverse selection (quality may be an innate characteristic or its level of one party is unknown to the other) are dealt with in Ho and Zheng ((2004) and Das et al. (2000)). A comprehensive analysis of the concept and models related to outsourcing is given in Dolgui and Proth (2010).

Our research focuses on outsourcing at the strategic level. We will discuss the business model, where a buyer outsources from two suppliers (Multiple Outsourcing, MO). Our model contributes to existing literature in the following respects. First, we assume that the suppliers can have different levels of efficiency of their investment to improve service quality. Most supplier selection literatures ignore this fact of differential cost efficiency. Second, our model captures the interplay of major factors like outsourcing contract, quality and quantity. Third, our model can be applied to many business outsourcing scenarios, from manufacturing, to services, to BPO.

3. The Model Scenario

The outsourcing model we consider in this paper consists of a buyer and two suppliers to whom the buyer can outsource all or part of her service work like call center jobs. The volume allocated to a supplier is dependent on his quality level. The higher the service quality level of the supplier more is the volume allocated to him. This paper is concerned with developing a supply contract which can be used for motivating a supplier to improve his service quality.

3.1 Cost of Quality

The supplier incurs a certain amount of cost to bring up his quality of service to a competitive level. This cost, which is fixed with respect to the quantity supplied, includes the cost of training of employees, provision of bandwidth, installation of computer systems and databases and number and quality of phone lines. As is extensively used in literature (see Mukhopadhyay et al. (2008)), we assume that this fixed cost is increasing (and convex) in the level of quality offered. We assume the functional form as $\frac{\eta_i}{2} Q_i^2$. $\square_i$, a fixed cost parameter, is a measure of the supplier i 's quality cost efficiency, in the sense that if $\square_i$ is low for a supplier, he will need less fund to attain a given level of quality. Thus $\square_i$ plays an important role on determining the level of service quality.

3.2 Service Volume Function

The buyer outsources q_i units of service per period to supplier i . Supplier i offers service of these q_i units at a quality level Q_i which is a decision variable for the supplier. The quantity allocated to supplier i then is a decreasing function of price p and an increasing function of Q . The quantity allocated to supplier i is given as $q_i = \tilde{a} - \tilde{b} p + \tilde{e} Q_i$, $i = 1, 2$. Adding the two quantities for the two suppliers, the total outsourced quantity will be $\sum_{i=1,2} q_i = \tilde{a} - \tilde{b} p + \tilde{e} \sum_{i=1,2} Q_i$. After inverting this volume function, the inverse volume function can be written as:

$$p = a - b \sum_{i=1,2} q_i + e \sum_{i=1,2} Q_i \quad (1)$$

Here, b and e are the sensitivity parameters of sales volume and quality respectively.

From (1), note that the synergistic effect of quality improvement carried out by all suppliers in a market helps the market as a whole, by improving the price position from the supplies' point of view. We mention here that we assume no capacity constraints at the suppliers' facilities.

3.3 Profit Functions

The supplier i 's objective function is:

$$\text{Max}_{\{Q_i\}} \pi_{Si} = (p - c)q_i - \frac{\eta_i}{2} Q_i^2, \quad (2)$$

where c is the variable cost of servicing one call. Here we make the mathematically convenient assumption that the variable cost is same for both suppliers. The buyer's objective function is:

$$\underset{\{q_1, q_2\}}{\text{Max}} \pi_B = (B - p) \sum_{i=1,2} q_i \quad (3)$$

The buyer's objective function is essentially her savings by outsourcing. The cost for the buyer to finish the work by herself is B per unit. Then $B-p$ is the dollar amount saved for each unit outsourced. The buyer decides the quantities q_1 and q_2 to be outsourced to each of the two suppliers. A supplier decides the optimal level of his own quality, each maximizing his own profit function. Then $B-p$ is the dollar amount saved for each unit outsourced. The buyer decides the quantities q_1 and q_2 to be outsourced to each of the two suppliers. A supplier decides the optimal level of his own quality, each maximizing his own profit function. Note that the price is automatically determined via (1), once the quantity and quality levels are known.

4. The Game

This outsourcing problem essentially turns into a game, which is typically played in two stages. It is realistic to assume that the buyer would act as a leader and the suppliers as followers. This gives rise to a Stackelberg game. In stage one of the two-stage game, the buyer makes the decision on the outsourcing quantity q_i allocated to supplier i with the knowledge of each supplier's best response function. In stage two, the supplier will decide the quality level. The two suppliers compete against each other by individually choosing quality level Q_i . For this Stackelberg game, we generate the sub-game perfect Nash equilibrium using backward induction. A subgame perfect Nash equilibrium is an equilibrium such that the players' strategies constitute a Nash equilibrium in every subgame of the original game. It may be found by backward induction, an iterative process for solving finite sequential games.

Supplier i 's best response functions can be given by, $Q_i = \frac{eq_i}{\eta_i}$, $i=1,2$. Note from above that the supplier i 's decision variable, Q_i , is a function of only q_i , η_i and e . Next, the buyer substitutes these best response functions into her profit function and obtain the following optimization problem:

$$\begin{aligned} \underset{\{q_1, q_2\}}{\text{Max}} \pi_B &= (B - p) \sum_{i=1,2} q_i \\ \text{s.t. } Q_i &= \frac{eq_i}{\eta_i} \\ \pi_{Si} &\geq 0, \quad i = 1, 2 \end{aligned}$$

4.1 Multiple outsourcing suppliers (MO) scenario

The solution procedure for this formulation starts by substituting p from (1) making it a function of q_i alone. Taking Lagrangian functions. The optimal values of the decision variables q_1 and q_2 , and thereby of Q_1 and Q_2 are given in the next two propositions. All proofs are given in the Appendix.

Proposition 1.

For the MO case:

When $\frac{e^2}{2b} < \eta_2 < \frac{e^2}{b}$, the equilibrium quantity, quality, and price are given by:

$$\left\{ \begin{array}{l} q_1^{A*} = 0 \\ q_2^{A*} = \frac{(B-a)}{2\left(\frac{e^2}{\eta_2} - b\right)} \end{array} \right\}, \left\{ \begin{array}{l} Q_1^{A*} = 0 \\ Q_2^{A*} = \frac{(B-a)e}{2(e^2 - \eta_2 b)} \end{array} \right\}, p^{A*} = \frac{a+B}{2}.$$

The corresponding profits of buyer, supplier 1 and supplier 2 at equilibrium are given as:

$$\pi_B^{A*} = \frac{(B-a)^2}{4\left(\frac{e^2}{\eta_2} - b\right)}, \left\{ \begin{array}{l} \pi_{S1}^{A*} = 0 \\ \pi_{S2}^{A*} = \left(\frac{a+B}{2} - c\right)q_2^{A*} - \frac{(eq_2^{A*})^2}{2\eta_2} \end{array} \right\}.$$

Proposition 2.

(a) When $\eta_2 > \frac{e^2}{b}$, the equilibrium quantity, quality, and price are given by:

$$\begin{cases} q_1^{B*} = \frac{2\eta_1 a}{A} \\ q_2^{B*} = \frac{2\eta_2 a}{A} \end{cases}, Q_1^{B*} = Q_2^{B*} = \frac{2ea}{A}, p^{B*} = \frac{e^2 a}{A},$$

where $A = 2b\eta_1 + 2b\eta_2 - 3e^2 > 0$.

(b) The corresponding profits of buyer, supplier 1 and supplier 2 at equilibrium are given as:

$$\pi_B^{B*} = \frac{2a}{A^2} (2Bb\eta_1 + 2Bb\eta_2 - 3Be^2 - ae^2)(\eta_1 + \eta_2),$$

$$\begin{cases} \pi_{S1}^{B*} = 0 \\ \pi_{S2}^{B*} = 0 \end{cases}$$

Discussions.

Propositions 1 and 2 give the optimal policies and profit for all three parties under two different cases. For the purpose of discussion, let us assume that Supplier 1 is more efficient than Supplier 2 in attaining a certain quality level i.e., $\eta_1 < \eta_2$. The optimum results divide the domain of $\square_2$ into two cases: Case A is where

$\frac{e^2}{2b} < \eta_2 < \frac{e^2}{b}$ and Case B is where $\eta_2 > \frac{e^2}{b}$. Proposition 1 gives the results of Case A. Here we see that the efficient Supplier 1 cannot get any outsourcing work from the buyer and inefficient supplier 2 gets all of it. Supplier 1 is basically driven out of the market. As revealed by our result, the efficient supplier is unable to get a high enough price for the outsourcing work to warrant his high quality. Therefore, the efficient supplier will exit the market without doing any outsourcing work. This is a paradoxical case of “the bad driving out the good” in

the outsourcing market. Case B, for the condition $\eta_2 > \frac{e^2}{b}$, gives an interesting scenario. In this case, the optimal outsourcing quality is exactly the same for both suppliers. That means that no matter what their efficiencies are, they have to bring their quality to the same level. In other words, there is a single quality level in the market. This shows the power of competition in this market. The outsourcing volume is allocated according to their efficiencies. In Part (b), we obtain closed form expression for the buyer’s optimal profit. Both the suppliers’ profits came out as zero. This means that both suppliers will receive the minimum reservation level of profit. This reveals that, due to competitive pressure, the suppliers would increase their quality to such a high level that the investment cost is as high as their respective revenue. The buyer takes the whole system profit as her profit. We can write the buyer’s profit as:

$$\pi_B^{B*} = (B - c)(q_1^{B*} + q_2^{B*}) - \frac{\eta_1}{2}(Q_1^{B*})^2 - \frac{\eta_2}{2}(Q_2^{B*})^2.$$

The buyer’s saving from outsourcing is determined by the difference of her costs and the Supplier 1 and Supplier 2’s costs, including both variable costs and fixed costs. A very important guideline emerges from our analysis. From the buyer’s point of view, the only criterion of importance is the level of service quality received from the two suppliers. The buyer is not concerned about the suppliers’ internal cost of investment for improving quality. From this point of view, she can play the competition game in a very profitable (to her) way. She pays the same price to both suppliers and allocates higher volume to the less cost efficient supplier. As a result, the optimal quality level for the less efficient supplier is as high as the other supplier. This way, the buyer is assured of the same level of quality no matter which supplier the call goes to. This is one of the main contributions of our paper.

Proposition 3.

In the two supplier case, under Case B,

(i) The optimal quality level offered by the supplier Q_i is increasing in a and e , and decreasing in η_i , and b ;

- (ii) The quantity outsourced q_i is increasing in a and e , and decreasing in b and the other supplier's cost efficiency.

The insight here is that the buyer can be sure of receiving a higher service quality when she carefully selects the suppliers with higher cost efficiency. We also see that if the buyer can use marketing strategy variables to increase the total market potential (a), or the quality sensitivity of the market (e), then she will enjoy a higher optimal quality. From the supplier's point of view, these increased a and e would benefit him too by increasing his service volume. One counter-intuitive finding is that a supplier's outsourcing quantity will increase if the other supplier (or the competitor) becomes more cost efficient.

4.2 Multiple suppliers (MO) scenario under Asymmetric Information

Here the buyer does not know the exact value of the suppliers' cost efficiency parameter η_i . She only knows the prior density function $f(\eta)$ and cumulative distribution $F(\eta)$ defined on the range $[\underline{\eta}, \bar{\eta}]$. We found that closed form solution for the general case does not exist. Thus we obtained closed form solution for a special case using the assumption of $N^A = \underline{\eta}$ (where is N^A the cut-off value of η for a contract to be signed by the supplier) denoting that the supplier will always sign at least one contract. The results are shown in the next four propositions.

Proposition 4.

When $\frac{e^2}{2b} < \eta_2 < \frac{e^2}{b}$, the equilibrium quantity, quality, and price are given by:

$$\left\{ \begin{array}{l} q_{S1_AS}^{A*} = 0 \\ q_{S2_AS}^{A*} = \frac{(B-a)}{2 \left(e^2 \frac{\int_N^{\bar{\eta}} \frac{f(\eta_2)}{\eta_2} d\eta_2}{F(\eta_2)|_N^{\bar{\eta}}} - b \right)} \end{array} \right\}, \left\{ \begin{array}{l} Q_{S1_AS}^{A*} = 0 \\ Q_{S2_AS}^{A*} = \frac{e}{\eta_2} q_{S2_AS}^{A*} \end{array} \right\},$$

$$p_{AS}^{A*} = a + \left(\frac{e^2}{\int_N^{\bar{\eta}} \eta_2 d(\eta_2)} - b \right) q_{S2_AS}^{A*}.$$

The corresponding profits of buyer, supplier 1 and supplier 2 at equilibrium are given as:

$$\pi_{B_AS}^{A*} = (B - p_{AS}^{A*}) q_{S2_AS}^{A*},$$

$$\left\{ \begin{array}{l} \pi_{S1_AS}^{A*} = 0 \\ \pi_{S2_AS}^{A*} = \left(\frac{a+B}{2} - c \right) q_{S2_AS}^{A*} - \frac{(eq_{S2_AS}^{A*})^2}{2\eta_2} \end{array} \right\}.$$

When Supplier 1, a cost efficient supplier, did not get any outsourcing contract, Supplier 2's cost efficiency level η_2 will be the only piece of asymmetric information in the game to affect the optimal values of outsourcing quality, quantity, price and profits.

Proposition 5.

- (i) The optimal quality level offered by Supplier 2 is $q_{S2_AS}^{A*}$ is increasing in B and b , and decreasing in a and e ;
- (ii) The optimal quantity level offered by Supplier 2 is $Q_{S2_AS}^{A*}$ is increasing in B and b , and decreasing in a .

Note that the price under the asymmetric information depend on not only a and B , (as in the full information case) but also on other parameters, like e , b and the distribution of η_2 . This is an important insight for information asymmetry.

Proposition 6.

When $\eta_2 > \frac{e^2}{b}$, the equilibrium quantity, quality, and price are given by:

$$\begin{cases} q_{S1_AS}^{B*} = \frac{\eta_1}{e} Q_{S1_AS}^{B*} \\ q_{S2_AS}^{B*} = \frac{\eta_2}{e} Q_{S2_AS}^{B*} \end{cases},$$

$$Q_{S1_AS}^{B*} = Q_{S2_AS}^{B*} = \frac{(B-a)F(\eta_1)|\bar{\eta}_N}{\left(3e - \frac{2b\eta_2}{e}\right)F(\eta_1)|\bar{\eta}_N - \frac{2b}{e} \int_N \bar{\eta}_1 f(\eta_1) d\eta_1 + \left(\frac{e}{\eta_1} - \frac{b}{e}\right) \int_N \bar{\eta}_2 f(\eta_2) d\eta_2}$$

$$p_{AS}^{B*} = a + \left(\frac{e^2}{\eta_1} - b\right) q_{S1_AS}^{B*} + \left(\frac{e^2}{\eta_2} - b\right) q_{S2_AS}^{B*}.$$

The corresponding profits of buyer, supplier 1 and supplier 2 at equilibrium are given as:

$$\pi_{B_AS}^{B*} = (B - p_{AS}^{B*})(q_{S1_AS}^{B*} + q_{S2_AS}^{B*}),$$

$$\begin{cases} \pi_{S1_AS}^{B*} = (p_{AS}^{B*} - c)q_{S1_AS}^{B*} - \frac{\eta_1}{2} (Q_{S1_AS}^{B*})^2 \\ \pi_{S2_AS}^{A*} = (p_{AS}^{B*} - c)q_{S2_AS}^{B*} - \frac{\eta_2}{2} (Q_{S2_AS}^{B*})^2 \end{cases}.$$

In this case, Supplier 1 and Supplier 2 will offer the same level of quality. The asymmetric information of cost efficiency of Supplier 1 and Supplier 2, both η_1 and η_2 will decide the quantity level, quality, price and profits.

Proposition 7

- (i) The optimal quality level offered by the supplier Q_i is increasing in B and b , and decreasing in a ;
- (ii) The quantity outsourced q_i is increasing in B , b and η_i , and decreasing in a ;
- (iii) Supplier 1 gets less contract quantity than Supplier 2, $q_{S1_AS}^{B*} \leq q_{S2_AS}^{B*}$.

Comparing with the multiple outsourcing suppliers scenario under the full information Case B, we see that under the asymmetric information of the suppliers cost efficiency η_i , the buyer's benchmark cost B will affect the outsourcing quality, quantity (from Proposition 7, parts *i* and *ii*), price and profits. Our explanation is that because the buyer has no accurate information about the suppliers cost efficiency level, one of the thoughts when she decides the outsourcing quantity would be about her own cost level. So she includes her cost level B as the benchmark to set up the quantity level.

We also carried out extensive numerical analysis to gain more insights into the problem. The results are omitted from this paper.

5. Conclusions

The main insight from this paper which is also an important contribution to literature is as follows. The buyer is concerned about how to influence the service quality. Our analysis came up with the non-intuitive strategy of using the volume allocation strategy to manage quality. The buyer can use differential volume allocation to force the non-efficient supplier to improve his quality so that both the suppliers have the same equilibrium quality level in the market. Our study showed that the actual values of the decision variable in the

optimum policy like relative quantity allocated and the market quality, depend on the market parameters like the base demand, the sensitiveness of price and quality. We analyzed the effect on the optimal policies of any change in the values of these parameters, by conducting extensive sensitivity analysis, both analytically and numerically, and produced guidelines for the managers. As an illustrative example, we can suggest that a company like Dell can select two call centers in two countries, say in India and in Philippines, and still be assured of the same quality level due to the implementation of the policies derived in this paper.

Our model can be applied to different outsourcing scenarios like manufacturing, call center and BPO scenarios. The only change would be the way the variables or parameters are interpreted. For example, price p can be the amount charged by supplier for each product, each call and so on.

Our model can be extended in many different directions. We could include bonus and penalty for the high and low quality in the contract, using non-linear pricing schemes. We can extend our model by including a reservation quality level which is the minimum requirement for the quality of outsourcing. In some scenarios, the buying company insists on a minimum quality level. Another avenue for future research will come from allowing each supplier to set his own price, either in the first stage of the game or later. It will be interesting to study the buyer's response. The duration of the outsourcing agreement is not included in the current study which can be another line of further research, along with a sensitivity analysis of the decision parameters. A cooperative learning effect in the outsourcing process can be used to determine the optimal duration of contract for supplier and buyer.

References

- Akşin, O., de Véricourt, F., and F. Karaesmen (2008). "Call center outsourcing contract analysis and choice", *Management Science*, vol. 54, no. 2, pp. 354-368.
- Arya, A., Mittendorf, B., and D. Sappington (2008). "The make-or-buy decision in the presence of a rival: strategic outsourcing to a common supplier", *Management Science*, vol. 54, no. 10, pp. 1747-1758.
- Das, A., Handfield, R., Calantone, R., and S. Ghosh, (2000). "A contingent view of quality management - the impact of international competition on quality", *Decision Sciences*, vol. 31, pp. 649-690.
- Dolgui, A. and J.M. Proth (2010). *Supply Chain Engineering: Useful methods and techniques*, Springer.
- Gans, N. and Y. Zhou (2007). "Call-routing schemes for call-center outsourcing", *Manufacturing & Service Operations Management*, vol. 9, pp. 33-50.
- Gans, N., Koole, G. and A. Mandelbaum (2003). "Telephone call centers: tutorial, review, and research prospects", *Manufacturing & Service Operations Management*, vol. 5, pp. 79-141.
- Gopal, A., Sivaramakrishnan, K., Krishnan, M., and T. Mukhopadhyay (2003). "Contracts in offshore software development: an empirical analysis", *Management Science*, vol. 49, pp. 1671-1683.
- Ho, T., and Y. Zheng (2004). "Setting customer expectation in service delivery: an integrated marketing-operations perspective", *Management Science*, vol. 50, pp. 479-489.
- Kalnins, A. (2004). "Relationships and hybrid contracts: an analysis of contract choice in information technology", *Journal of Law Economics and Organization*, vol. 20, pp. 207-229.
- Kenion, C. (2005). "Optimal price design for variable capacity outsourcing contract", *Journal of Revenue and Pricing Management*, vol. 4, pp. 124-155.
- King, W. and G. Torkzadeh (2008). "Information systems offshoring: research status and issues", *Management Information Systems Quarterly*, Vol. 32, no. 2, pp. 205-225.
- Kouvelis, P. and S. Mukhopadhyay (1999). "Modeling the design quality competition for durable products", *IIE Transactions on Operations Engineering*, vol. 31, pp. 865-880.
- Mukhopadhyay, S., Zhu, X., and X. Yue (2008). "Optimal contract design for mixed channels under information asymmetry", *Production and Operations Management*, vol. 17, pp. 641-650.
- Novak, S. and S. Stern (2008). "How does outsourcing affect performance dynamics? Evidence from the automobile industry", *Management Science*, vol. 54, no. 12, pp. 1963-1979.
- Yue, X., Mukhopadhyay, S., and X. Zhu (2006). "A Bertrand model of pricing of complementary goods under information asymmetry", *Journal of Business Research*, vol. 59, pp. 1182-1192.

Biography

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Appendix

Proof of $Q_i = eq_i / \eta_i$.

Substituting Equation (1) into Equation (2) generates: $\pi_{Si} = (a - b \sum_{i=1,2} q_i + e \sum_{i=1,2} Q_i - c)q_i - \frac{\eta_i}{2} Q_i^2$. The first

order condition (FOC) of each supplier's profit maximization with respect to Q_i requires: $\frac{\partial \pi_{Si}}{\partial Q_i} = eq_i - \eta_i Q_i = 0$

. This implies that $Q_i = \frac{eq_i}{\eta_i}$.

Proof of Propositions 1 and 2.

Set up Lagrange Function:

$$L(q_1, q_2, \lambda_1, \lambda_2) = [B - a + b(q_1 + q_2) - \frac{e^2 q_1}{\eta_1} - \frac{e^2 q_2}{\eta_2}](q_1 + q_2) + \lambda_1 \{ [a - b(q_1 + q_2) + \frac{e^2 q_1}{\eta_1} + \frac{e^2 q_2}{\eta_2} - c]q_1 - \frac{e^2 q_1^2}{2\eta_1} \} + \lambda_2 \{ [a - b(q_1 + q_2) + \frac{e^2 q_1}{\eta_1} + \frac{e^2 q_2}{\eta_2} - c]q_2 - \frac{e^2 q_2^2}{2\eta_2} \}.$$

Kuhn-Tucker conditions require:

$$\max_{L, w.r.t. q_1} \Rightarrow \begin{cases} L_{q_1} = B - a + 2b(q_1^* + q_2^*) - \frac{2}{\eta_1} e^2 q_1^* - e^2 q_2^* (\frac{1}{\eta_1} + \frac{1}{\eta_2}) + \lambda_1^* (a - 2bq_1^* - bq_2^* + \frac{1}{\eta_1} e^2 q_1^* \\ + \frac{1}{\eta_2} e^2 q_2^* - c) + \lambda_2^* (-bq_2^* + \frac{1}{\eta_1} e^2 q_2^*) \leq 0 \\ q_1^* [B - a + 2b(q_1^* + q_2^*) - \frac{2}{\eta_1} e^2 q_1^* - e^2 q_2^* (\frac{1}{\eta_1} + \frac{1}{\eta_2}) + \lambda_1^* (a - 2bq_1^* - bq_2^* + \frac{1}{\eta_1} e^2 q_1^* \\ + \frac{1}{\eta_2} e^2 q_2^* - c) + \lambda_2^* (-bq_2^* + \frac{1}{\eta_1} e^2 q_2^*)] = 0 \\ q_1^* \geq 0 \end{cases}$$

$$\begin{aligned}
\max_{L.w.r.t. q_2} \Rightarrow & \begin{cases} L_{q_2} = B - a + 2b(q_1^* + q_2^*) - \frac{2}{\eta_2} e^2 q_2^* - e^2 q_1^* \left(\frac{1}{\eta_1} + \frac{1}{\eta_2} \right) + \lambda_1^* (-bq_1^* + \frac{1}{\eta_2} e^2 q_1^*) \\ + \lambda_2^* (a - bq_1^* - 2bq_2^* + \frac{1}{\eta_1} e^2 q_1^* + \frac{1}{\eta_2} e^2 q_2^* - c) \leq 0 \\ q_2^* [B - a + 2b(q_1^* + q_2^*) - \frac{2}{\eta_2} e^2 q_2^* - e^2 q_1^* \left(\frac{1}{\eta_1} + \frac{1}{\eta_2} \right) + \lambda_1^* (-bq_1^* + \frac{1}{\eta_2} e^2 q_1^*) \\ + \lambda_2^* (a - bq_1^* - 2bq_2^* + \frac{1}{\eta_1} e^2 q_1^* + \frac{1}{\eta_2} e^2 q_2^* - c)] = 0 \\ q_2^* \geq 0 \end{cases} \\
\min_{L.w.r.t. \lambda_1} \Rightarrow & \begin{cases} L_{\lambda_1} = [a - b(q_1^* + q_2^*) + \frac{e^2 q_1^*}{\eta_1} + \frac{e^2 q_2^*}{\eta_2} - c] q_1^* - \frac{e^2 q_1^2}{2\eta_1} \leq 0 \\ \lambda_1^* \{ [a - b(q_1^* + q_2^*) + \frac{e^2 q_1^*}{\eta_1} + \frac{e^2 q_2^*}{\eta_2} - c] q_1^* - \frac{e^2 q_1^2}{2\eta_1} \} = 0 \\ \lambda_1^* \geq 0 \end{cases} \\
\min_{L.w.r.t. \lambda_2} \Rightarrow & \begin{cases} L_{\lambda_2} = [a - b(q_1^* + q_2^*) + \frac{e^2 q_1^*}{\eta_1} + \frac{e^2 q_2^*}{\eta_2} - c] q_2^* - \frac{e^2 q_2^2}{2\eta_2} \leq 0 \\ \lambda_2^* \{ [a - b(q_1^* + q_2^*) + \frac{e^2 q_1^*}{\eta_1} + \frac{e^2 q_2^*}{\eta_2} - c] q_2^* - \frac{e^2 q_2^2}{2\eta_2} \} = 0 \\ \lambda_2^* \geq 0 \end{cases}
\end{aligned}$$

We get the results in Propositions 3 and 4 by solving the above twelve inequalities and equations.

Proof of Propositions 3

From Proposition 2, we have $Q_i^{B*} = \frac{2ea}{2b \sum_{i=1}^2 \eta_i - 3e^2}$.

It is easy to see that

$$\frac{dQ_i^{B*}}{da} > 0, \frac{dQ_i^{B*}}{db} < 0, \frac{dQ_i^{B*}}{d\eta_i} < 0$$

$$\text{and } \frac{dQ_i^{B*}}{de} = \frac{2a(2b \sum_{i=1}^2 \eta_i + 3e^2)}{(2b \sum_{i=1}^2 \eta_i - 3e^2)^2} > 0.$$

Similarly, we have $q_i^{B*} = \frac{2\eta_i a}{2b \sum_{i=1}^2 \eta_i - 3e^2}$.

It is easy to see that

$$\frac{dq_i^{B*}}{da} > 0, \frac{dq_i^{B*}}{de} > 0, \frac{dq_i^{B*}}{db} < 0, \text{ and } \frac{dq_i^{B*}}{d\eta_j} < 0.$$