Location Routing Inventory Problem with Transshipment (LRIP-T)

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Abstract

Location Routing Inventory Problem (LRIP) is an integration of three key logistics decision problems — location-allocation, vehicle routing and inventory management with the objective of minimizing total cost. In this study, the LRIP with transshipment, or LRIP-T is proposed. The LRIP is modified by introducing one of the customer points as a transshipment point in the logistics system. Selection of the transshipment point was done using *p*-median. Computational results for four cases were presented. The results indicated that the LRIP with transshipment (LRIP-T) gives satisfactory results compared with a LRIP.

Keywords
Location Routing Inventory Problem with Transshipment (LRIP-T), Location Routing Inventory Problem (LRIP), *p*-median

1. Introduction
Location routing inventory problem (LRIP) involves three decisions in the logistic optimization system, which are facility location, vehicle routing and inventory control. In distribution systems, LRIP deals with the allocation of depots from several potential locations, to schedule vehicle routes to meet customer demand and to determine the inventory policy based on the information of customers' demands in order to minimize the total system cost (Liu and Lin (2005). LRIP has not been widely studied due to the interrelated decision areas. However all the decisions are important as in reality, every process in logistics system contributes to total operation costs. Granada and Silva (2012) and Xuefeng (2010) studies explained three hierarchical levels of decision that make effectiveness of logistic management: (1) Strategic decision, involving plans for a long time period and important capital investment, (2) tactical decisions, involving plans for the short term (a year or a semester) and moderate investment, and (3) operational decisions, involving day to day operations and low capital investments. These three levels of decision are highly correlated to the distribution network. Hence, using the LRIP in optimizing the total operation cost can lead to getting the optimum solution for the total supply chain problem,

Among the unavoidable cost in total supply chain is the stockout cost. A stockout is a situation where an excess demand exists and causes inventories to be exhausted. It is the opposite of surplus. Surplus is a situation where it has an excess inventory and it is too much to retain. However, shortage and surplus may cause the various losses to the supplier and retailers. The supplier requires understanding of the causes for stockout and improves their availability through internal measures. Stock out will frustrate retailers and force them to take the correction actions. Stock out will cause lost sales, dissatisfy retailers, diminish store loyalty, jeopardize marketing efforts, and obstruct sales planning. When retailers are unable to find the inventory that they are intended to purchase, they might switch to the other suppliers who can give better services or not to buy the inventory at all. This will cause the supplier incur losses. Surplus is another one of the largest costs that organization faces. It involves the cost of inventory, the loss of working cash and the ongoing holding costs. These are few ways that it can make the organization reduce the profits. Surplus takes up space, tie up capital and negatively affect the organization bottom line. Both suppliers and retailers need to find ways and initiatives to prevent stock out and surplus happen. In order to consider solving this problem, we would like to incorporate transshipment process into the LRIP.

Transshipment is the process of mutual agreement and inventory sharing among the supply chain partners, suppliers, end-customers and retailers. Transshipment can occur if all the partners are allowed to share their inventory and their contributions will be return in profit sharing sales and revenue. In order to transship the products, the vehicle management needs to be considered as well as the amount of inventory. Several studies have been done and proved the effectiveness of transshipment in Inventory Routing problem (IRP) and Vehicle Routing Problem (VRP). Some researchers incorporated transshipment into IRP and transformed it into IRPT involving single supplier to serve many retailers (Bertazzi et. al.,2002; Kleywegt et. al. (2002). In IRPT, the supplier delivers the goods to the retailer when the inventory hold by the supplier is meeting demand requirement. However, the task will be subcontracted to the retailers who are collaborating with the supplier in the situation where inventory hold by suppliers is unable to meet demand. In this case, the retailer who is collaborating with the supplier to take on the task of sending goods to the other retailers is known as a transshipment point. Before handing over the task to a transshipment point, the supplier ensures that the inventory at the transshipment points is enough to accommodate demand. Other than that, transshipment is allowed when the location of transshipment point with retailer are nearer compared to the location of supplier. This can shorten the travel distance and at the same time it can save the transportation cost.

Similarly transshipment has been incorporated into VRP and becomes VRPT. Yang and Xiao (2007) in their study believed the transshipment center plays an important role in connecting suppliers and customers. The proper arrangement must be made at the transshipment center to avoid the increase in logistics cost and complicated of distribution task. They considered a multi-period single-product logistics system with transshipment centers. They assumed the customer own multiple vehicles with limited capacity and limited time during a period. If the total demand of customer exceeds its vehicles' ability, the rental vehicles are always available in a period. In their study, mathematical models has applied and developed the corresponding algorithms on a basis of dynamic programming while combining two stages method and the brand and bound technique. In this study, we consider the IRPT and VRPT and propose LRIP-T or Location Routing Inventory Problem with Transshipment. In order to validate the performance of the new proposed model, we propose a modified data to be used for LRIP-T as well as cost saving method in selecting the transshipment center.

2. Methodology

In this study, two objectives are considered, which are to include the transshipment process in the LRIP model and to evaluate the effect of transshipment into the performance of LRIP.

2.1 Model

We adapt the LRIP model from Chanchan et al. (2008) study, the following are the notations and decisions variables used to describe the LRIP model.

Notations:

N : number of customer points, which are distribution points are collection points.
M : number of logistics centers.
g : index of customer point or logistics center ($1 \leq g \leq N+M$).
h : index of customer point or logistics center ($1 \leq h \leq N+M$).
i : index of customer point ($1 \leq i \leq N$).
j : index of logistics point ($N+1 \leq j \leq N+M$).
F_j : establishing cost of logistics center j during the planning horizon.
dis_{gh} : distance between point g and point h.
T : length of the planning horizon, which have l period.
CK : capacity of vehicles.
k : index of vehicles or routes ($1 \leq k \leq K$).
t : index of time periods of the planning horizon ($1 \leq t \leq l$).
c : unit cost of vehicles.
u_j : cost of transport the new product from factory to logistics center j.
μ_j : probability of being reused after a circulation.
I_j : holding cost of unit useable product in one day in logistics center j.
A_j : ordering cost of every time in logistics center j.
L_j : lead time in logistics center $j, L_j < T/l$
$s_j(t)$: inventory level of logistics center j on the beginning of period t, where $s_j(t=0)=0$ means that the inventory is zero in beginning of the planning horizon.

Decision variables:

X_{ghkt} : 1, if point g immediately proceeds point h on route k in period t; 0, otherwise.
O_j : 1, if logistics center j is opened; 0, otherwise.
Y_{ij} : 1, if customer i is served by logistics center j; 0, otherwise.
Z_{jkt} : 1, if route k is served by logistics center j in period t; 0, otherwise.
$H_j(t)$: 1, if there is a ordering for new product in logistics center j; 0, otherwise.
$\gamma_i(t)$: actual collection volume from customer i in period t.

- The cost of establishing logistics center; $F_j.o_j$
- The transportation cost during the distribution-collection process in the whole planning horizon is;
$$c.\sum_{t=1}^{l}\sum_{k=1}^{K}\sum_{h=1}^{N+M}\sum_{g=1}^{N+M} dis_{gh}X_{ghkt}$$
- At the beginning of each period t, the volume that vehicle k prepare to start in the logistics center is $V_k^t(0) = \sum_{i\in I} E\big(d_i(t)\big).Y_{ij}.Z_{jkt}$, and $V_k^t(0) \le CK$
- When customer i is a distribution point, $V_k^t(i) = V_k^t(i-1) - E(d_i(t))$ or a collection point $V_k^t(i) = V_k^t(i-1) + \gamma_i(t)$ Where $V_k^t(i) \le CK$, $\gamma_i(t) \le E\big(p_i(t)\big)$

- The total demand of distribution in period t, $D_j(t+1) = \sum_{i=1}^{N} E\big(d_i(t+1)\big).Y_{ij}$
- The volume of collection in period t, $P_j(t) = \sum_{i=1}^{N} \gamma_i(t).Y_{ij}$

- When $s_j(t) + \mu_j.P_j(t) < D_j(t+1)$, it is need to ordering new, and the ordering volume is
$$D_j(t+1) - s_j(t) - \mu_j.P_j(t)$$
- The expression of holding cost is; $IV_j = \sum_{t=1}^{l}\big(u_j + A_j\big)H_j(t) + I_j.\frac{T}{l}.\sum_{i=1}^{l}(s_j(t) + \mu_j P_j(t))$

The model is written as:

Minimize $SC = \sum_{j=N+1}^{N+M}\big(F_j + IV_j + u_j\big).O_j + c.\sum_{t=1}^{l}\sum_{k=1}^{K}\sum_{h=1}^{N+M}\sum_{g=1}^{N+M} dis_{gh}.X_{ghkt}$
Subject to

$\sum_{k=1}^{K}\sum_{g=1}^{N+M} X_{gikt} = 1$, for all i, t (1)
$\sum_{g=1}^{N+M} X_{ghki} - \sum_{g=1}^{N+M} X_{hgki} = 0$, for all h, k, i (2)
$\sum_{h=1}^{N+M}\sum_{g=1}^{N+M} X_{ghki} \le 1$, for all k, t (3)
$V_k^t(i) \le CK$, where $i \in \{0, 1, 2, \dots, N\}$ (4)
$\gamma_i(t) \le E\big(p_i(t)\big)$, for all i (5)
$\sum_{h=1}^{N+M} X_{ihkt} + \sum_{h=1}^{N+M} X_{jhkt} - Y_{ij} \le 1$, for all i, j, k, t (6)
$X_{ghkt} \in \{0, 1\}$, for all g, h, k, t (7)
$O_j \in \{0, 1\}$, for all j (8)
$Y_{ij} \in \{0, 1\}$, for all i, j (9)
$Z_{ikt} \in \{0, 1\}$, for all i, k, t (10)
$Y_j \in \{0, 1\}$, for all j (11)

The objective function of the above formulation is to minimize the total system cost, which is the sum of establishing cost of logistics centers, transportation cost and inventory cost. Constraint (1) insures that every customer point appears in only one route. Constraint (2) states that every point entered by the vehicle should be the same point the vehicle should be the same point the vehicle leaves. Constraint (3) states that each route cannot be served by more than one logistics center. Constraint (4) and (5) insures that in any period, in any point the total load is less than the capacity of the vehicle and the actual collection volume is equal to or less than the expected volume. Constraint (6) states a customer can be allocated to a depot only if there is a route passing by that customer. Constraints (7)-(11) insure the integrality of decision variables. In order to incorporate the transshipment cost into the total system cost from LRIP model by Chanchan et al. (2008), the following two parameters are added to the formulation.

b_{ij} : the unit cost associated with transshipping product from i to j

w_{sit} : the quantity transshipped to the customers i in period t.

2.2 Assumptions and Data

Chanchan et al. (2008) listed out the assumptions and the parameters values of the candidate logistics centers and customers as follows:

- 20 customer points and 5 candidate logistics centers.
- The expected distribution-collection demand is created randomly which obey Poisson distribution with $\lambda_i = 2, \delta_i = 1$.
- $CK = 125$ units, $c = 2/$ unit distance, $l = 50$ weeks, $L_j = 2$ days, $A_j = 5$ for each time, $I_j = 10$/unit/day, $\mu_j = 0.9$.
- In any time period, each vehicle travels at most on one route, and customers are visited at most once (Hiassat and Diabat (2012).

Table 1: Parameter values of logistics centers

Logistics center	Location (coordinate)	Managing cost	μ_j
D1	(51, 30)	12000	25
D2	(61, 59)	10000	20
D3	(13, 77)	12000	30
D4	(16, 20)	13000	10
D5	(34, 45)	15000	15

Table 2: Parameter values of customer points

C1	C2	C3	C4	C5
(74, 20)	(60, 89)	(95, 75)	(76, 69)	(52, 18)
C6	C7	C8	C9	C10
(18, 54)	(22, 98)	(2, 58)	(69, 23)	(87, 14)
C11	C12	C13	C14	C15
(49, 8)	(55, 83)	(45, 53)	(55, 21)	(45, 21)
C16	C17	C18	C19	C20
(61, 32)	(3, 55)	(31, 96)	(33, 91)	(45, 65)

In order to observe the performance of the model, the relevant data will be adapted such that it will be suitable to be run using LRIP model in introduced in Section 2.1. The second step is to do the sensitivity analysis on various variables in LRIP model. Next the best output in LRIP will be used to choose the possible transshipment centers in order to extend LRIP into LRIP-T.

2.3 Data Manipulation

Since the benchmark data are not directly suitable to test the LRIP-T, we consider the assumptions and constraint of data being used in IRPT and VRPT and adapt it into LRIP to test LRIP-T. Several variations of the data will be considered and simulated. For the sensitivity analysis, we first apply the adapted data into LRIP by varying the values of the parameters: total supply, number of facility to open and number of vehicle used to fulfill the total customers demand. Next, we locate a facility to open such that it will minimize the total distribution cost (in terms of distance) using p-median problem (Shariff et al., 2012). The best located facility is designated as the distribution centre and we identify the next open facility among the customers to act as a transshipment center. This should minimize the total cost as set-up cost for the new logistics center is not needed due to the use of existing customer point as the transshipment point. The identified facilities are used to solve the LRIP-T.

3. Results and Analysis

In this section, we incorporate transshipment process into the LRIP formulation. The objective function of the LRIP is to minimize the total system cost, which is the sum of establishing cost of logistics centers, transportation cost and inventory cost. Hence, for LRIP-T, the objective function is modified to minimize the total system costs which include the following cost:

1. total cost of establishing the logistics centers. Since, the transshipment center will be chosen among the customers' points, no establishment cost is necessary.

2. transportation cost that covers the transportation from the distribution centers as well as the transportation cost from the transshipment centers.
3. Inventory cost at the distribution centers as well as that at the transshipment centers. However, the level of inventory at the transshipment center is assumed to be zero, or the center is assumed NOT to keep any stock.

3.1 Data

Data in this study are adopted from Chanchan et al. (2008) where 20 customer points and 5 logistics centers are considered. The logistics centers are the centers for distribution of inventory to customers. The inventory delivered to the customer is according to demand from customers. The customer demands are randomly selected in range of 30 to 70 inventories for each customer points and amount of total of 1009. We determine the demand of customer points do not exceed the range to ensure that the vehicle can distributes the inventories more than once from each logistics center. The parameters values of logistics centers used are similar to Chanchan et al. (2008) as indicated in Table 1. Table 3 describes the output of the parameter values and demand of customer points to be used in this study.

Table 3: Parameter values and demand of customer points

Customer Point	Coordinate	Demand	Customer Point	Coordinate	Demand
C1	(74,20)	42	C11	(49,8)	32
C2	(60,89)	56	C12	(55,83)	69
C3	(95,75)	53	C13	(45,53)	45
C4	(76,69)	34	C14	(55,21)	62
C5	(52,18)	48	C15	(45,12)	47
C6	(18,54)	61	C16	(61,23)	55
C7	(22,98)	37	C17	(3,55)	30
C8	(2,58)	45	C18	(31,96)	70
C9	(69,23)	48	C19	(33,91)	68
C10	(87,14)	68	C20	(45,65)	39
Total demand =1009					

3.2 Sensitivity analysis using LRIP

We explore the changes in total number of supply the number of open facility and total number of vehicle used in order to analyze the stability of the data. We consider four cases: (1) unlimited total supply and open five logistics centers, (2) limited total supply and open five logistics centers, (3) unlimited total supply and open one logistics centers, and (4) limited total supply and open one logistics centers.

Here, we describe the details of the four cases. In Case 1, we assume there is an unlimited supply at each logistics centers. Case 2 involves a logistics center with limited total supply of inventories. In Case 3, we assume only one logistics center with unlimited total supply is open. The final case involves only one logistics center with limited total supply being opened. For Case 3, total demand is 1009. We consider a balanced transportation problem where total demand equals to total supply. However, total supply is set to report to 1050 or four percent higher than 1009. The inventories are distributed by a homogeneous fleet of vehicles of the same capacity. Each vehicle is loaded with 125 units of inventory. Each customer only can be served by one vehicle.

For each cases, we provide ten solutions to determine the route for each logistics center and which customer to serve. Then, we find the total distance and total waste for each logistics center. Distance for each route is calculated beginning from the logistics center to the customers and back to the logistics center. The vehicle needs return to the logistic center when the demand for next customer point exceeds the inventory capacity in the vehicle. We define the leftover inventory in the vehicle when it returns to the logistic center is called waste. We take into account the lowest total distance and the total waste for each logistics center for all solution. For the unlimited of total supply, the total vehicles used are also been considered

Case 1:

According to the result, solution 1 shows the lowest average distance that is 61.6 km using 10 vehicles. The lowest average distance of 61.6 km, gets the moderate average waste of 24 units.

Table 4: Unlimited total supply and five open logistics centers (Case 1)

Solution	Route	Number of Load	Vehicle Used	Distance	Waste	Average Distance	Average Waste
1	**D1**	**402**	**4**	**193.2**	**98**	**61.6**	**24**
	D2	**212**	**2**	**138.1**	**38**		
	D3	**220**	**2**	**149.8**	**30**		
	D4	**91**	**1**	**86.4**	**34**		
	D5	**84**	**1**	**48.4**	**41**		
2	D1	337	3	160.1	38	68.9	21
	D2	322	3	245.4	53		
	D3	168	2	185.4	82		
	D4	107	1	81.0	18		
	D5	107	1	97.5	18		
3	D1	233	2	251.9	17	88.0	24
	D2	212	2	138.0	38		
	D3	230	2	234.5	20		
	D4	140	2	139.0	110		
	D5	194	2	116.9	55		
4	D1	234	2	140.8	16	81.5	24
	D2	301	3	308.6	74		
	D3	214	2	145.4	36		
	D4	75	1	81.0	50		
	D5	185	2	139.4	65		
5	D1	334	4	165.2	166	67.9	33
	D2	319	3	293.7	56		
	D3	152	2	102.1	98		
	D4	91	1	86.4	34		
	D5	113	1	99.4	12		
6	D1	323	3	183.7	52	63.7	33
	D2	212	2	173.7	38		
	D3	175	2	107.0	75		
	D4	154	2	151.9	96		
	D5	145	2	85.1	105		
7	D1	368	4	288.0	132	70.0	33
	D2	235	2	170.0	15		
	D3	236	3	154.1	139		
	D4	75	1	81.0	50		
	D5	95	1	76.5	30		
8	D1	365	4	267.3	135	78.2	24
	D2	333	3	218.6	42		
	D3	115	1	95.9	10		
	D4	91	1	86.4	34		
	D5	105	1	113.4	20		
9	D1	402	4	193.2	98	65.2	24
	D2	212	2	173.3	38		
	D3	214	2	145.4	36		
	D4	75	1	81.0	50		
	D5	106	1	59.0	19		
10	D1	213	2	131.4	37	73.2	24
	D2	212	2	176.8	38		
	D3	220	2	149.8	30		
	D4	170	2	157.3	80		
	D5	194	2	116.9	56		

Case 2: The total number of load is equal to the total demand of the customers' points. According to Table 5, solution 6 is the best compared to other solutions since it has the lowest value of average distance of 83 km and average waste of 8 units which is compatible with other.

Table 5: Limited total supply and five open logistics centers (Case 2)

Solution	Route	Number of Load	Total Distance (km)	Waste	Average Distance (km)	Average Waste
1	D1	206	127.9	4	90	8
	D2	198	193.1	12		
	D3	197	163.5	13		
	D4	202	202.5	8		
	D5	206	212.3	4		
2	D1	197	99.0	13	92	8
	D2	197	263.7	13		
	D3	205	153.6	5		
	D4	201	253.1	9		
	D5	209	150.7	1		
3	D1	197	87.9	13	92	8
	D2	197	182.9	13		
	D3	208	250.4	2		
	D4	201	166.4	9		
	D5	206	232.5	4		
4	D1	207	176.0	3	95	8
	D2	197	259.3	13		
	D3	205	160.3	5		
	D4	200	179.4	10		
	D5	200	176.2	10		
5	D1	190	150.2	20	99	8
	D2	198	190.3	12		
	D3	206	188.4	4		
	D4	210	243.8	0		
	D5	205	215.2	5		
6	**D1**	**206**	**127.9**	**4**	**84**	**8**
	D2	**204**	**147.1**	**6**		
	D3	**205**	**153.6**	**5**		
	D4	**185**	**161.9**	**25**		
	D5	**209**	**252.6**	**1**		
7	D1	198	175.5	12	96	8
	D2	203	258.7	7		
	D3	198	120.9	12		
	D4	208	234.2	2		
	D5	202	172.9	8		
8	D1	204	164.1	6	92	8
	D2	197	182.9	13		
	D3	206	201.5	4		
	D4	192	229.7	18		
	D5	210	138.8	0		
9	D1	204	161.8	6	95	8
	D2	203	199.8	7		
	D3	205	153.6	5		
	D4	192	251.7	18		
	D5	205	185.1	5		
10	D1	210	210.4	0	97	8
	D2	204	157.2	6		
	D3	200	197.2	10		
	D4	206	255.5	4		
	D5	189	148.9	21		

Case 3:
All the best solution for each logistics center is compared and determined for the lowest distance. The comparison is to find the best solution between the logistics center for this case. Table 6 shows the comparison all the best solution for all the logistics centers. From the table, D1 shows the lowest average distance (98.7 km) compared to the others.

Table 6: Comparison of the best solution for each logistics center (for unlimited supply)

Logistics Center	Vehicle Used	Total Distance (km)	Average Distance (km)	Average Waste (unit)
D1	**10**	**987.3**	**98.7**	**24**
D2	9	924.7	102.7	13
D3	9	1209.2	134.4	13
D4	10	1131.7	113.2	24
D5	10	1069.2	106.9	24

Case 4:
The average distance is calculated by dividing the total distance with the number of route. Meanwhile, the average distance per vehicle indicates the average distance for each of the vehicle. Table 4.6 (a) shows the average distance per vehicle for the best solution at D1 is 329.3 km. That is means the distance of each vehicle at the D1 logistics center to distribute the inventory to the customer is 329.3 km.

All the best solution for each logistics center is compared and the lowest distance is identified. The comparison is to find the best solution between the logistics center for this case. Table 7 shows the comparison all the best solution for all the logistics centers. From the table, D1 shows the lowest average distance (98.8 km) compared to the rest.

Table 7: Comparison of the best solution for each logistics center (for limited supply)

Solution	Total Distance (km)	Average Distance (km)	Average Distance per Vehicle (km)	Average Waste (unit)
1	**988.0**	**98.8**	**329.3**	**14**
2	1144.2	127.1	381.4	14
3	1068.2	106.9	356.2	14
4	1134.1	103.1	378.0	14
5	1097.7	122.0	365.9	14
6	1328.0	120.7	442.7	14
7	1398.6	139.9	466.2	14
8	1252.1	139.1	417.4	14
9	1001.5	111.3	333.8	14
10	1143.2	114.3	381.1	14

3.3 Assigning a Transshipment centre using *P*-Median
From analysis in Section 3.2, D1 is always giving the best solution. To analyse the effect of transshipment in the process, we open only one logistics center D1 and determine one of the customer point as the transshipment center. The selection process is solved using *p*-median (Shariff et al., 2012). In the *p*-median problem we are interested in finding the location of *p* facilities to serve demand nodes so that the transportation cost is minimized. The transportation cost is given by the product of the demand at the demand node and the distance between the demand node and the facility that serves the demand node. The result in this section is used to determine the logistics centers and its present's transshipment point among the customers points (C1 – C20).

Table 8 summarizes the results of selection process where the customer point C17 shows the lowest total values. Means that, logistics center D1 is selected as a main distribution point while customer point C17 is selected as a transshipment point. Upon being appointed a transshipment point, the customers' demand can be shipped from both the distribution center and the transshipment center which now is D1 and C17. The comparison is shown in the Table 10. The result shows a better performance compared to LRIP with 5 open distribution center (unlimited supply) when total travelled distance per vehicle is only 600 km (total travelled distance is equal to 18000.1 km with three vehicles used).

Table 8: Result for *p*-median

	Demand	42	56	53	34	48	61	37	45	48	68
	D1	**C1**	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**	**C9**	**C10**
D1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C1	25.1	0.0	1404.5	1329.2	852.7	1060.4	1529.9	928.0	1128.6	279.9	973.6
C2	59.7	2506.7	0.0	1997.9	870.8	2864.8	3335.0	1444.9	2685.7	2864.8	4058.4
C3	62.9	2472.7	2111.0	0.0	677.4	3021.0	3839.1	2328.6	2832.1	2790.6	4183.5
C4	46.3	1945.6	1434.3	1056.0	0.0	2223.6	2825.8	1714.0	2084.6	2223.6	3150.1
C5	12.0	505.7	674.3	638.2	409.4	0.0	734.5	445.5	541.9	578.0	818.8
C6	40.8	1713.8	2285.0	2162.6	1387.3	1958.6	0.0	1509.8	742.2	1958.6	2774.7
C7	73.9	3104.9	2186.9	3918.1	2084.0	3548.4	2695.1	0.0	2012.5	3548.4	5026.9
C8	56.4	2370.3	3160.4	2991.1	1918.8	2708.9	1006.0	1654.7	0.0	2708.9	3837.6
C9	19.3	244.9	1081.5	1023.6	656.6	850.6	1178.1	714.6	869.1	0.0	1313.3
C10	39.4	601.3	2206.1	2088.0	1339.4	1690.9	2403.1	1457.6	1772.8	966.0	0.0
C11	22.1	927.8	1237.1	1170.8	751.1	501.1	1347.5	817.4	994.1	1060.4	1502.2
C12	53.2	2232.3	437.4	2162.0	858.1	2551.2	2867.6	1341.2	2391.8	2551.2	3614.2
C13	23.8	998.3	1331.1	1259.8	808.2	1140.9	1450.0	879.5	1069.6	1140.9	1616.3
C14	9.8	413.7	551.5	522.0	334.9	203.6	600.8	364.4	443.2	472.7	669.7
C15	19.0	796.9	1062.5	1005.6	645.1	442.5	1157.4	702.0	853.8	910.7	1290.2
C16	10.2	428.3	571.1	540.5	346.7	489.5	622.1	377.3	458.9	489.5	693.5
C17	54.1	2273.0	3030.7	2868.4	1840.1	2597.8	917.0	1739.4	142.3	2597.8	3680.2
C18	69.0	2896.5	1670.6	3569.9	1784.3	3310.3	2681.9	341.1	2151.1	3310.3	4689.5
C19	63.6	2671.2	1516.1	3370.8	1642.2	3052.8	2435.4	482.4	2037.5	3052.8	4324.8
C20	35.5	1491.4	1584.9	1882.1	1062.7	1704.5	1778.4	1313.9	1598.0	1704.5	2414.7
TOTAL		30595.4	29537.2	35556.6	20270.1	35921.5	35404.9	20556.4	26809.7	35209.7	50632.5

	Demand	32	69	45	62	47	55	30	70	68	39
	D1	**C11**	**C12**	**C13**	**C14**	**C15**	**C16**	**C17**	**C18**	**C19**	**C20**
D1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	**0.0**	0.0	0.0	0.0
C1	25.1	802.6	1730.5	1128.6	1179.6	1178.8	973.0	**752.4**	1755.6	1705.4	978.1
C2	59.7	1909.8	538.9	1755.0	3700.3	2805.1	3135.5	**1790.5**	2088.3	1841.0	1103.8
C3	62.9	2014.0	2814.7	2458.2	3902.1	2958.0	3015.0	**1888.1**	4405.6	4279.7	1988.6
C4	46.3	1482.4	1741.5	1569.8	2872.1	2177.3	2195.9	**1389.7**	3242.7	3150.1	1219.0
C5	12.0	334.1	830.9	541.9	263.0	433.3	662.3	**361.2**	842.9	818.8	469.6
C6	40.8	1305.7	2815.5	1215.8	2529.9	1917.8	2244.2	**451.0**	2856.3	2714.9	1137.0
C7	73.9	2365.6	2501.2	2274.2	4583.4	3474.5	4065.9	**1410.3**	645.4	886.6	1568.8
C8	56.4	1805.9	3894.1	1948.0	3499.0	2652.5	3104.0	**94.9**	3346.1	3078.8	1699.1
C9	19.3	618.0	1332.6	869.1	876.8	907.7	662.3	**579.4**	1351.9	1313.3	753.2
C10	39.4	1231.1	2718.3	1772.8	2030.9	1851.6	1739.3	**1181.9**	2757.7	2678.9	1536.4
C11	22.1	0.0	1524.3	994.1	887.7	265.9	1215.0	**662.7**	1546.4	1502.2	861.5
C12	53.2	1700.8	0.0	1423.0	3295.3	2498.1	2824.3	**1594.5**	1910.6	1591.8	803.1
C13	23.8	760.6	1640.1	0.0	1473.7	1117.2	1307.3	**713.1**	1663.9	1616.3	468.0
C14	9.8	315.2	679.6	443.2	0.0	462.9	541.7	**295.5**	689.4	669.7	384.1
C15	19.0	181.0	1309.2	853.8	834.1	0.0	1043.6	**569.2**	1328.2	1290.2	740.0
C16	10.2	326.3	703.7	458.9	632.3	479.3	0.0	**305.9**	713.9	693.5	397.7
C17	54.1	1731.8	3734.3	1892.1	3355.5	2543.7	2976.6	**0.0**	3475.4	3186.6	1683.8
C18	69.0	2206.8	1883.3	2035.0	4275.8	3241.3	3793.0	**1489.5**	0.0	366.2	1326.6
C19	63.6	2035.2	1615.2	1793.2	3943.2	2989.2	3498.0	**1405.8**	377.0	0.0	1116.8
C20	35.5	1136.3	1420.8	540.0	2201.7	1669.0	1953.1	**1065.3**	2381.0	1947.2	0.0
TOTAL		24263.5	35428.6	25966.8	46336.5	35623.0	40950.0	**18001.0**	37378.2	35331.3	20235.2

4. Conclusions

Based on the solution for LRIP, we consider Case 1 and Case 2 where we open five logistics centers. When supply is limited, total distance travelled is higher at 832 km compared to only 649 km per vehicle when total supply is unlimited. In Case 3 and Case 4, we explore the same condition. When only one logistics center is open, total distance travelled for unlimited supply is quite close to that of limited supply. This might due to wrong route being considered. The logistics center that yields that lowest distance is D1 of 898.6 km. This is open to future research exploration.

Using *p*-median, C17 is chosen as the transshipment point with the shortest distance travelled. Upon being appointed a transshipment point, the customer demand can be shipped from both the distribution center and the transshipment center which now is D1 and C17. The comparison is shown in the Table 9. The result shows a better performance compared to LRIP with 5 open distribution center (unlimited supply) when total travelled distance per vehicle is only 600 km (total travelled distance is equal to 18000.1 km with three vehicles used).

Table 9: Comparison between LRIP and LRIP-T

LRIP	LRIP-T
615.9 km	600 km

Based on the findings, it is hoped that this study can be used by the researchers to do more research on the ideas and to the practitioners in logistics fields to consider the idea in their operations.

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