# The Effects of Salvage Value on the Age-Replacement Policy under Renewing Warranty

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## Abstract

This paper presents the effects of salvage value on the optimal age-replacement policy for non-repairable products sold with a renewing free-replacement warranty (RFRW). Cost models from the customer's perspective are developed for both warranted, and non-warranted products. The corresponding optimal replacement ages are derived such that the long-run expected cost rate is minimized. Under the increasing failure rate assumption, the existence and uniqueness of the optimal age for preventive replacement are shown, and the impacts of a RFRW on the optimal age-replacement policy are investigated analytically. Finally, a numerical example is provided for the optimal policy illustrated and verification.

## Keywords
Age-replacement, warranty, salvage value

## 1. Introduction

The primary role of a product warranty is to offer post-sale remedy for buyers when a product fails to fulfill its intended performance within a warranty period. A product warranty has also built-in incentives for sellers to do better than their commitments, such as reputation, market share, and potential profit. Accordingly, a product warranty has been an important part of merchandising, and became one type of additional value that is directly reflected on the product itself.

A detailed discussion and review of various issues related to product warranties can be found in Blischke and Murthy (1992a, 1992b, 1994, 1996, and 2004). It is worth noting for this discussion in particular that for non-repairable products, one of the most commonly used warranty policies is the renewing free-replacement warranty (Blischke and Murthy 1992a). Under this warranty, a product that fails during the warranty period is replaced with a new one with a full warranty at no cost to the buyer, thus, the buyer is assured of a working item for at least the length of the warranty period. Mathematical formulations and cost evaluations of the renewing free-replacement warranty policy were developed and investigated in the literatures.

The classical age-replacement policy is proposed by Barlow and Proschan (1965), where an operating system is replaced at time of failure (corrective replacement), or at age $T$ (preventive replacement), whichever comes first. Recently, Yeh *et al.* (2005) analysed the effects of the renewing free-replacement warranty (RFRW) on an age-replacement policy for non-repairable products. Chien (2008a) extended the models of Yeh *et al.* (2005) to a general repairable case. Chien (2008b) further proposed an imperfect RFRW, developed the model, and investigated its effects on the age-replacement policy. But, however, they did not consider the salvage value for a product that subject to replaced preventively.

In this paper, we focus on analyzing the impact of salvage on the optimal age replacement policy under a renewing free-replacement warranty. Taking product warranty into account, a mathematical model for a product under an age replacement policy is developed. For a product with an increasing failure rate function, we show that there exists a unique optimal replacement age such that the long-run expected cost rate is minimized. Furthermore, the optimal replacement ages for products with, and without warranty are compared analytically, and the result shows that the optimal replacement age should be adjusted toward the warranty period when a product is warranted.

## 2. Model Development

Under an age-replacement policy, a non-repairable product is replaced at a certain age $t$, or upon failure, whichever occurs first. More precisely, when the product fails at age $x \le t$, a corrective replacement is performed with a downtime cost $C_d > 0$, and a purchasing cost $C_p > 0$. If the age of the product reaches $t$, then a preventive replacement is carried out. Because a preventive replacement is a planned maintenance action, only the cost $C_p$ is incurred in this action. Under this policy, the design variable is the age for preventive replacement, $t$.

Without considering product warranty and salvage value, the age-replacement policy and its variation have been investigated by researchers, and numerous results have been obtained. In this paper, a renewing free-replacement warranty (RFRW) is considered in deriving the optimal age for preventive replacement. For a product purchased with the RFRW, if a failure occurs within the warranty period $w$, a new product with the same RFRW is offered, free of charge by the seller, to replace the failed one. However, any failure of the product incurs a downtime cost $C_d$ to the buyer. On the other hand, because a product that was preventive replaced at time $t$ is still operable, the salvage value of an un-failed product should be considered in the preventive replacement model. It seems reasonable that the salvage value of a used product which is still operable is proportional to the expected residual lifetime, that is $v_s \cdot (x-t) \big| \{x > t\}$. (Kaio and Osaki 1978 ).

Taking salvage value into account, the cost models for operating a product under the age-replacement policy can be established by a similar method to that of Yeh *et al*. (2005).

**2.1 Cost model without warranty**

We first develop the cost model for a product without any warranty. Without warranty, any two successive replacements of the product form a renewal cycle of the failure process. Hence, under a replacement age $t > 0$, the cycle time (denote by $T_0(t)$ ) is

$$T_0(t) = \begin{cases} x, & \text{if } x \le t, \\ t, & \text{if } x > t, \end{cases}$$

(1)

and the total cost incurred in a renewal cycle (denote by $C_0(t)$ ) is

$$C_0(t) = \begin{cases} C_d + C_p, & \text{if } x \le t, \\ C_p - v_s \cdot (x-t), & \text{if } x > t. \end{cases}$$

(2)

Thus, by (2) and (3), the long-run expected cost rate is

$$CR_0(t) = \frac{E[C_0(t)]}{E[T_0(t)]} = \frac{C_p + C_d F(t) - v_s \int_t^\infty \overline{F}(u)du}{\int_0^t \overline{F}(u)du}. \tag{3}$$

**2.2 Cost model with warranty**

For a product purchased with the RFRW, the failure process in a long run of the product between any two successive replacements again forms a renewal cycle because the warranty period is reset to $w$. However, the total cost incurred in a renewal cycle depends on whether a replacement is performed within the warranty period or not. Therefore, the cost model should be established for two cases: $t \ge w$, and $t < w$.

Case 1, $t \geq w$: As shown in Fig. 1, there are three possible replacements for a product when the replacement age $t$ is greater than or equal to the warranty period $w$. First, if the product fails within the warranty period (i.e., $x \leq w$), a downtime cost $C_d$ is incurred. Second, if a failure occurs after the warranty period, but before a preventive replacement (i.e., $w < x < t$), it incurs an additional purchasing cost $C_p$. Third, when the product reaches age $t$ (i.e., $x \geq t$), a preventive replacement is performed with cost $C_p$, and the salvage value $v_s \cdot (x-t)$ is also gained from that un-failed product. Thus, the elapsed time, and the total cost in a renewal cycle (denote by $T_1(t)$, and $C_1(t)$, respectively) become

$$T_1(t) = \begin{cases} x, & \text{if } x \leq w, \\ x, & \text{if } w < x < t, \\ t, & \text{if } x \geq t, \end{cases}$$

(4)

and

$$C_1(t) = \begin{cases} C_d, & \text{if } x \leq w, \\ C_d + C_p, & \text{if } w < x < t, \\ C_p - v_s \cdot (x-t), & \text{if } x \geq t. \end{cases}$$

(5)

Therefore, the long-run expected cost rate is

$$CR_1(t) = \frac{E[C_1(t)]}{E[T_1(t)]} = \frac{C_p \overline{F}(w) + C_d F(t) - v_s \int_t^{\infty} \overline{F}(u)du}{\int_0^t \overline{F}(u)du}. \tag{6}$$

Case 2, $t < w$: When the replacement age $t$ is less than the warranty period $w$, all the replacement is performed within the warranty period. In this case, either a corrective replacement with cost $C_d$, or a preventive replacement with cost $C_p$, will occur as shown in Fig. 2. That is, there are two possible states of replacement for a product under the case $t < w$. First, if a failure occurs before a preventive replacement (i.e., $x < t < w$), then a downtime cost $C_d$ is incurred. Second, when the product reaches age $t$ (i.e., $x \geq t$), a preventive replacement is performed with cost $C_p$, and the salvage value $v_s \cdot (x-t)$ is also gained.

According to the above descriptions, the elapsed time, and the total cost in a renewal cycle (denote by $T_2(t)$, and $C_2(t)$, respectively) are

$$T_2(t) = \begin{cases} x, & \text{if } x \leq t, \\ t, & \text{if } x > t, \end{cases}$$

(7)

and

$$C_0(t) = \begin{cases} C_d, & \text{if } x \leq t, \\ C_p - v_s \cdot (x-t), & \text{if } x > t. \end{cases}$$

(8)

Then the long-run expected cost rate becomes

$$CR_2(t) = \frac{E[C_2(t)]}{E[T_2(t)]} = \frac{C_p \overline{F}(t) + C_d F(t) - v_s \int_t^{\infty} \overline{F}(u)du}{\int_0^t \overline{F}(u)du}. \tag{9}$$

Note that

$$CR_1(w) = CR_2(w) = \frac{C_p + (C_d - C_p)F(w) - v_s \int_w^{\infty} \overline{F}(u)du}{\int_0^w \overline{F}(u)du} \tag{10}$$

from (6) and (9).

## 3. Optimization

To derive the optimal age for preventive replacement, we need to take the first derivative of the expected cost rate function with respect to $t$, and set it equal to 0.

**3.1 Optimal age for replacement without warranty**

For a product without warranty, the first derivative of (3) with respect to $t$ is

$$\frac{dCR_0(t)}{dt} = \overline{F}(t)\frac{C_d \cdot H(t) - (C_p - v_s \cdot \mu)}{\left[\int_0^t \overline{F}(u)du\right]^2},$$

(11)

where $\mu = \int_0^{\infty} \overline{F}(u)du$ is the expected lifetime for a product, and $H(t) = r(t) \times \int_0^t \overline{F}(u)du - F(t)$ is an intermediate function. Then, for a product with an increasing failure rate (IFR) function, the following Lemma from Yeh et al. (2005) and Chien (2010) show that $H(t)$ increases from 0 to $r(\infty)\mu - 1$ as $t$ increases.

**Lemma 1.** *If $r(t)$ is an increasing failure rate (IFR) function, and $F(0) = 0$, then $H(t)$ is an increasing function of $t$ with $\lim_{t \to 0} H(t) = H(0) = 0$, and $\lim_{t \to \infty} H(t) = H(\infty) = r(\infty)\mu - 1$.*

□

**Theorem 1.** *Given $r(t)$ is an IFR function for a product without warranty, the following results hold for $t \in [0, \infty)$.*

1) *When $C_p \le v_s \cdot \mu$, then the optimal age for preventive replacement is $t_0^* = 0$.*
2) *When $C_p > v_s \cdot \mu$, if $H(\infty) > (C_p - v_s \cdot \mu)/C_d$, or equivalently $(C_d + C_p - v_s \cdot \mu)/(C_d \cdot \mu)$ $< r(\infty)$, then there exists a finite, unique optimal replacement age $t_0^*$ for a product without warranty, where $t_0^*$ satisfies $C_d \cdot H(t_0^*) = C_p - v_s \cdot \mu$, and the resulting expected cost rate is $CR_0(t_0^*) =$ $C_d \cdot r(t_0^*) + v_s$. Otherwise, $t_0^* = \infty$, and $CR_0(\infty) = (C_p + C_d)/\mu$.*

□

As described in the Remark1 of Chien (2010), the condition $C_p \le v_s \cdot \mu$ means that the purchasing cost of a new product is lower than its expected salvage value over its lifetime. Thus, under this condition $C_p \le v_s \cdot \mu$, the optimal preventive replacement is always that the customer should preventively replace a new product when it is purchased. In fact, however, it seems more reasonable that the sale price of a new product should be larger than its expected salvage value over its lifetime. Therefore, the following discussion of the remainder of this paper on the optimal policies will focus on the condition $C_p > v_s \cdot \mu$.

**3.2 Optimal age for replacement with warranty**

To derive the optimal preventive replacement age for a product under a RFRW, two cases have to be investigated separately: $t \ge w$, and $t < w$. For $t \ge w$, the first derivative of (6) with respect to $t$ is

$$\frac{dCR_1(t)}{dt}=\frac{C_d\overline{F}(t)\left[H(t)-\frac{C_p\cdot\overline{F}(w)-v_s\cdot\mu}{C_d}\right]}{\left[\int_0^t\overline{F}(u)du\right]^2}.$$

(11)

Let $t_1^*$ be the optimal value minimizing (6) in the interval $[w,\infty)$. Then the following Lemma concerning $t_1^*$ can be obtained by (11).

**Lemma 2.** *Under a RFRW, given that $r(t)$ is an IFR function for a product with warranty period $w$, the following results hold for $t\in[w,\infty)$.*

1) *For $C_p\overline{F}(w)\le v_s\mu$, then $t_1^*=w$, and $CR_1(t_1^*)=CR_1(w)$ is as given in* (10).

2) *For $C_p\overline{F}(w)>v_s\mu$,*

   (a) *If $H(w)\ge[C_p\overline{F}(w)-v_s\mu]/C_d$, then $t_1^*=w$, and $CR_1(t_1^*)=CR_1(w)$ is as given in* (10).

   (b) *If $H(w)<[C_p\overline{F}(w)-v_s\mu]/C_d<H(\infty)$, there exists a finite, unique optimal replacement age $t_1^*>w$ satisfying $H(t_1^*)=[C_p\overline{F}(w)-v_s\mu]/C_d$ and $CR_1(t_1^*)=C_d r(t_1^*)+v_s$.*

   (c) *If $H(\infty)\le[C_p\overline{F}(w)-v_s\mu]/C_d$, or equivalently $[C_d+C_p\overline{F}(w)-v_s\mu]/(C_d\mu)\ge r(\infty)$, then $t_1^*=\infty$, and $CR_1(t_1^*)=CR_1(\infty)=(C_d+C_p\overline{F}(w))/\mu$.*

□

Now, for $t<w$, the first derivative of (9) with respect to $t$ becomes

$$\frac{dCR_2(t)}{dt}=\overline{F}(t)\frac{(C_d-C_p)H(t)-(C_p-v_s\cdot\mu)}{\left[\int_0^t\overline{F}(u)du\right]^2}.$$

(12)

Let $t_2^*$ be the optimal value minimizing (9) in the interval $[0,w)$. Then the following Lemma concerning about $t_2^*$ can be obtained by (12).

**Lemma 3.** Under a RFRW, given that $r(t)$ is an IFR function for a product with warranty period $w$, and $C_p>v_s\mu$ is true, then the following results hold for $t\in[0,w)$.

1) For $C_d\le C_p$, the optimal age for preventive replacement is $t_2^*=w$, and $CR_2(t_2^*)=CR_2(w)$ is as given in (10).

2) For $C_d>C_p$, if $H(w)>(C_p-v_s\mu)/(C_d-C_p)$, then there exists a finite and unique optimal replacement age $t_2^*<w$, where $t_2^*$ satisfying $H(t_2^*)=(C_p-v_s\mu)/(C_d-C_p)$, and $CR_2(t_2^*)=v_s+(C_d-C_p)r(t_2^*)$. Otherwise, $t_2^*=w$, and $CR_2(t_2^*)=CR_2(w)$ is as given in (10).

□

In the previous discussion (see Lemmas 2, and 3), we derived the local optimal replacement age for a warranted product under the constraint $t\ge w$ or $t<w$. However, in practice, the replacement age should not be pre-determined to be in a certain interval. Therefore, it is important to investigate the global optimal replacement age $t_w^*$ without any constraint. From Lemmas 2, and 3, it is obvious that whether $t_w^*\ge w$ or $t_w^*<w$ depends on the value of $H(w)$. Thus, combining Lemmas 2 and 3, the following Theorem concerning the global optimal replacement age $t_w^*$ can be obtained.

**Theorem 2.** *Under a RFRW, given that $r(t)$ is an IFR function for a product with warranty period $w$, and $C_p > v_s\mu$ is true, then the following results hold for $t \in [0, \infty)$.*

1) *For $C_d \le C_p$,*
   (a) *when $C_p\overline{F}(w) \le v_s\mu$, $t_w^* = w$;*
   (b) *when $C_p\overline{F}(w) > v_s\mu$, if $H(w) < [C_p\overline{F}(w) - v_s\mu]/C_d$, then $t_w^* = t_1^* > w$; otherwise, $t_w^* = w$.*
2) *For $C_d > C_p$,*
   (a) *when $C_p\overline{F}(w) \le v_s\mu$, if $H(w) > (C_p - v_s\mu)/(C_d - C_p)$, then $t_w^* = t_2^* < w$; otherwise, $t_w^* = w$.*
   (b) *when $C_p\overline{F}(w) > v_s\mu$,*
      (i) *if $H(w) < [C_p\overline{F}(w) - v_s\mu]/C_d$, then $t_w^* = t_1^* > w$,*
      (ii) *if $[C_p\overline{F}(w) - v_s\mu]/C_d \le H(w) \le (C_p - v_s\mu)/(C_d - C_p)$, then $t_w^* = w$,*
      *(iii) if $H(w) > (C_p - v_s\mu)/(C_d - C_p)$, then $t_w^* = t_2^* < w$.*

□

## 4. Further Discussion

In this section, the impact of a product RFRW on the optimal age-replacement policy is investigated by comparing the expected cost rates, as well as the optimal replacement ages. First, by the expected cost rates (3), (6), and (9), the following corollary results.

**Corollary 1.** *Given any replacement age $t$, the expected cost rate for a product without warranty is always greater than the expected cost rate for a product with* RFRW. □

Corollary 1 in turn implies that when the optimal policies ($t_0^*$, and $t_w^*$) are attained for both cases, the optimal expected cost rate for a warranted product results in a smaller value. Furthermore, the difference between the optimal expected cost rates provides a measure of the value of the RFRW.

Next, the difference between the optimal replacement ages $t_0^*$ and $t_w^*$ is compared to show the effect of a product warranty. Through Theorems 1 and 2, $H(w)$ plays an important role in the comparison of $t_0^*$ with $t_w^*$, and the following corollary results.

**Corollary 2.** *For a non-repairable product in which the period of a RFRW is $w$, and $C_p > v_s\mu$, the optimal replacement ages $t_0^*$, and $t_w^*$ which minimize the long-run expected cost rate have the following properties.*

1) *For $C_d \le C_p$,*
   (a) *when $C_p\overline{F}(w) \le v_s\mu$,*
      (i) *if $H(w) < (C_p - v_s\mu)/C_d$, then $t_0^* > t_w^* = w$;*
      (ii) *if $H(w) \ge (C_p - v_s\mu)/C_d$, then $t_0^* \le t_w^* = w$;*
   (b) *when $C_p\overline{F}(w) > v_s\mu$,*
      (i) *if $H(w) < [C_p\overline{F}(w) - v_s\mu]/C_d$, then $t_0^* > t_w^* > w$;*
      (ii) *if $[C_p\overline{F}(w) - v_s\mu]/C_d \le H(w) < (C_p - v_s\mu)/C_d$, then $t_0^* > t_w^* = w$;*
      (iii) *if $H(w) \ge (C_p - v_s\mu)/C_d$, then $t_w^* = w \ge t_0^*$;*
2) *For $C_d > C_p$,*
   (a) *when $C_p\overline{F}(w) \le v_s\mu$, if $H(w) > (C_p - v_s\mu)/(C_d - C_p)$, then $t_w^* = t_2^* < w$; otherwise, $t_w^* = w$.*

(i) *if* $H(w) \le (C_p - v_s\mu)/C_d$ , *then* $t_0^* \ge t_w^* = w$;

(ii) *if* $(C_p - v_s\mu)/C_d < H(w) \le (C_p - v_s\mu)/(C_d - C_p)$ , *then* $t_0^* < t_w^* = w$;

(iii) *if* $H(w) > (C_p - v_s\mu)/(C_d - C_p)$ , *then* $t_0^* < t_w^* < w$;

(b) *when* $C_p\overline{F}(w) > v_s\mu$ ,

(i) *if* $H(w) < [C_p\overline{F}(w) - v_s\mu]/C_d$ , *then* $t_0^* > t_w^* > w$;

(ii) *if* $[C_p\overline{F}(w) - v_s\mu]/C_d \le H(w) < (C_p - v_s\mu)/C_d$ , *then* $t_0^* > t_w^* = w$;

(iii) *if* $(C_p - v_s\mu)/C_d < H(w) \le (C_p - v_s\mu)/(C_d - C_p)$ , *then* $t_0^* \le t_w^* = w$,

(iv) *if* $H(w) > (C_p - v_s\mu)/(C_d - C_p)$ , *then* $t_0^* < t_w^* < w$.

□

Furthermore, to study the variation in the magnitude of savings in the expected cost rate by RFRW, we define

$$\Delta CR = \frac{CR_0\left(t_0^*\right) - CR\left(t_w^*\right)}{CR_0\left(t_0^*\right)} ,$$

(15)

where $CR(t_w^*) = CR_1(t_w^*)$ when $t_w^* \ge w$, and $CR(t_w^*) = CR_2(t_w^*)$ when $t_w^* < w$. That is, $\Delta CR$ provides a measure of the value of RFRW, and it will be evaluated through a numerical example in Section 5.

## 5. Numerical Example

In this section, numerical examples are given to illustrate the impact of a PRRW on the optimal age-replacement policy when the lifetime distribution of a product is Weibull with failure rate function $r(x) = (\alpha\lambda)(\lambda x)^{\alpha-1}$ for $x > 0$. Note that $r(x)$ is an increasing function of $x$, and $r(\infty) = \infty$ when $\alpha > 0$. Without loss of generality, $\lambda = 1$ is fixed in this example; and the purchasing cost $C_p = 200$ and the warranty period $w = 1$, are also assumed. Then the resulting optimal solutions (including $t_0^*$, $CR_0(t_0^*)$, $t_1^*$, $CR_1(t_1^*)$, $t_2^*$, $CR_2(t_2^*)$, $t_w^*$, $CR_w(t_w^*)$ ,are compared under $\alpha = 1.2$ and downtime costs $C_d (= 50,100,500)$, and the salvage value per unit time $v_s (= 0, 1, 10, 50)$ The numerical results are summarized in Table I.

Table I Numerical results

| $\alpha$ | $C_d$ | $v_s$ | $t_0^*$ | $CR_0(t_0^*)$ | $t_1^*$ | $CR_1(t_1^*)$ | $t_2^*$ | $CR_2(t_2^*)$ | $t_w^*$ | $CR(t_w^*)$ | $\Delta CR$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0 | 1705.3 | 265.8 | 50.3 | 131.4 | 1 | 158.8 | 50.3 | 131.4 | 50.6% |
| | 50 | 1 | 1673.4 | 265.8 | 48.4 | 131.4 | 1 | 158.4 | 48.4 | 131.4 | 50.6% |
| | | 10 | 1406.7 | 265.8 | 33.9 | 131.4 | 1 | 154.6 | 33.9 | 131.4 | 50.6% |
| | | 50 | 601.5 | 265.8 | 4.6 | 131.4 | 1 | 137.8 | 4.6 | 131.4 | 50.6% |
| | | 0 | 132.6 | 318.9 | 8.6 | 184.5 | 1 | 206.5 | 8.6 | 184.5 | 42.1% |
| 1.2 | 100 | 1 | 130.5 | 318.9 | 8.4 | 184.5 | 1 | 206.1 | 8.4 | 184.5 | 42.1% |
| | | 10 | 113.1 | 318.9 | 6.5 | 184.5 | 1 | 202.3 | 6.5 | 184.5 | 42.1% |
| | | 50 | 56.5 | 318.9 | 1.7 | 183.6 | 1 | 185.5 | 1.7 | 184.5 | 42.4% |
| | | 0 | 2.9 | 743.7 | 1 | 588.2 | 1 | 588.2 | 1 | 588.2 | 20.9% |
| | 500 | 1 | 2.9 | 743.6 | 1 | 587.8 | 1 | 587.8 | 1 | 587.8 | 21.0% |
| | | 10 | 2.7 | 743.4 | 1 | 584.0 | 1 | 584.0 | 1 | 584.0 | 21.5% |
| | | 50 | 2.0 | 741.6 | 1 | 567.2 | 1 | 567.2 | 1 | 567.2 | 23.5% |

## 6. Conclusion

This paper analyses the effect of salvage value on the optimal age-replacement policy for non-repairable products sold with a renewing free-replacement warranty. Cost models are developed for both a warranted, and a non-warranted product; and the corresponding optimal age for preventive replacement were derived. For products with an IFR function, there exists a unique optimal replacement age such that the long-run expected cost rate is minimized. Moreover, the optimal replacement ages, as well as the corresponding cost rates for products with and without RFRW, are compared analytically, and its structural properties are obtained.

We found that the timing for a preventive replacement will be affected strongly when a product is warranted with a RFRW. Most especially, when the product has a more rapidly increasing failure rate, adding the RFRW to it will move the optimal replacement age closer to the warranty period. Furthermore, the numerical examples also demonstrate that the adjustment of the optimal age-replacement policy is necessary because the cost rate reduction could be up to 50%.

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## References

Barlow, R.E., and Proschan, F., *Mathematical Theory of Reliability*, John Wiley & Sons, 1965.

Blischke, W.R., and Murthy, D.N.P., Product warranty management-I: A taxonomy for warranty policies, *European Journal of operational Research*, vol.62, pp. 127-148, 1992a.

Blischke, W.R., and Murthy, D.N.P., Product warranty management-III: A review of mathematical model, *European Journal of Operational Research*, vol. 63, pp. 1-34, 1992b.

Blischke, W.R., and Murthy, D.N.P., *Warranty Cost Analysis*, Marcel Dekker, 1994.

Blischke, W.R., and Murthy, D.N.P., Product Warranty Handbook, Marcel Dekker, New York, 1996.

Chien, Y.H., A general age-replacement model with minimal repair under renewing free-replacement warranty, *European Journal of Operational Research*, vol.186, pp. 1046-1058, 2008a.

Chien, Y.H., Optimal age-replacement policy under imperfect renewing free-replacement warranty, *IEEE Transactions on Reliability*, vol. 57, no. 1, pp. 125-133, 2008b.

Chien, Y.H., The effect of a pro-rata rebate warranty on the age-replacement policy with salvage value consideration, *IEEE Transactions on Reliability*, vol. 57, no. 1, pp. 125-133, 2008b.

Kaio, N., and Osaki, S., Optimum planned maintenance with salvage cost, *International Journal of Production Research*, vol.16, no. 3, pp. 249-257, 1978.

Murthy, D.N.P., Solem, O., and Roren, T., Product warranty logistics: Issues and challenges, *European Journal of Operational Research*, vol.156, pp. 110-126, 2004.

Yeh, R.H., Chen, G.C., and Chen, M.Y., Optimal age-replacement policy for nonrepairable products under renewing free-replacement warranty, *IEEE Transactions on Reliability*, vol. 54, no. 1, pp. 92-97, 2005.

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