Integrating a Modified Desirability Function Approach and Ant Colony Strategy to Solve Two-stage Multi-Response Optimization Problem

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Abstract

Evaluation of a product or process involves simultaneous study of several product quality characteristics or responses. The ultimate goal of manufacturing process optimization is to determine the settings of the process variables such that the best combination of the responses is obtained. In case of multiple processing stages, determining optimal process condition is a critical and difficult task. Interdependency between stages, multiple responses in more than one stage and correlation between responses add up to the complexity of the optimization problem. Individual optimal at each stage will not lead to overall best condition for a multistage and multiple response problem. In this paper, an integrated solution approach is recommended and used to solve a two-stage manufacturing process optimization problem. A modified desirability function and ant colony-based search strategy is used to derive the best solution. The case study selected is a two-stage machining of an automobile engine cylinder liner.

Keywords
Quality Characteristics, Multiple Response Optimization, Multistage, Desirability function, Ant Colony Optimization

1. Introduction

The primary goal of a process improvement initiative is to determine the best process operating conditions that simultaneously optimizes several process output characteristics. These output characteristics are also known as 'responses' in quality engineering terminology. Simultaneous optimization of multiple responses is generally referred to as *multiple response optimization* (MRO) problem (Khuri, 1996). Solution to a MRO problem generally implies determining the feasible set of point(s) or region(s) in the input space for which all the responses show the desired properties in terms of measured bias and variance. Bias is the measured deviation of predicted response from the specified target value. Variance is caused by model parameter uncertainty and presence of uncontrollable variables. Multiple responses generally have linear or nonlinear interactions. The presence of interaction between responses calls for compromise solution(s). The typical objective of a MRO problem is to determine the process settings so that response variables are close to their target values with minimum variability around their targets.

Various literatures have proposed different solution approaches for MRO problem. The primary strategy used by many researchers (Derringer and Suich, 1980; Khuri and Conlon, 1981; Pignatiello, 1993; Kim and Lin, 2000; Tong et al., 2005; Plante, 2001; Awad and Kovach, 2011; Bera and Mukherjee, 2013) is to solve a MRO problem by reducing the problem dimensionality. These strategies basically convert the MRO problem into a single objective optimization problem by suitable mathematical transformation function(s). However, there is solution approach (Chiao and Hamada, 2001; Peterson et al. 2009) which maximizes the probability of all responses conformance or reliability.

In multistage manufacturing process, each intermediate or final stage may consider several response characteristics of interest. Any intermediate stage input conditions and response characteristic(s) can significantly influence the final stage product quality characteristics (Williams and Peters, 1989; Tagaras and Lee, 1996). Thus, it can be assumed that process performance at intermediate stage(s) has an influence (direct or indirect) on the final product quality (Zantek et al., 2002). In addition, any deviation from the specified target value(s) of a response(s), in the intermediate stage, may directly or indirectly influence the response characteristic(s) of the subsequent stages. This leads to interdependency between stages in a multistage manufacturing situation. In this context, the response characteristics obtained from the final stage of a manufacturing process defines the product quality.

For a multistage process, sequential or isolated optimization of each stage response characteristics will not lead to final stage optimal solution. This is primarily due to interdependency that exists between various stages and also because of correlated nature of response characteristics. Secondly, the optimization approach must simultaneously optimize the product quality characteristics and determine the overall best setting conditions for all the stages. Controlling the response characteristics within the specified engineering tolerance and developing a solution approach for simultaneous optimization of correlated multiple responses is always a challenge for researchers. The research works focused on multistage MRO problem are presented below.

Jin and Shi (1999) developed a stage indexed state space model to incorporate the each stage error variance in multistage systems with multiple responses. The state space model has a linear structure and considers variance propagation due to deviations from the targets. The proposed model provides useful insight on sources of variation affecting the product quality. Zantek et al. (2002) proposed a systematic approach to measure the impact of previous stage variability on the subsequent stage(s) and also on final stage product quality in a correlated multistage manufacturing scenario. Their proposed approach identifies the sources of variation in product quality rather than process parameter optimization in different stages. Kwak et al. (2010) proposed 'multistage PRIM' procedure for optimizing multistage manufacturing processes. They proposed maximizing the performances at each stage sequentially starting from final stage and going backward to initial stage, considering the relationship between the stages. Their approach works to determine optimum input variable conditions without an explicit prediction model. Multistage PRIM has capability to handle high dimensional input space and is less sensitive to outliers. However, it relies on large amount of data observations. In addition, multistage PRIM is a black box-type approach, and does not provide deeper insight of manufacturing process variables.

It is observed from the literature review that a few research articles suggested a systematic and simplified solution approaches for multistage MRO problem. In this article, a solution approach for two-stage process is verified using case. The proposed approach mathematically defines an objective function and its associated constraints using modified desirability functions and an adaptive penalty-based 'maximin' desirability index. Ant colony-based metaheuristic search strategy is selected to determine best operating conditions.

This article is organized as follows. Section 2 provides detail on solution approach for a two-stage MRO problem. Section 3 discusses an industrial case to study the effectiveness of the integrated solution approach. Finally, section 4 concludes with some possible avenues of future research directions in this field of study.

2. An integrated solution approach for a two-stage process

In this section, the overall mathematical formulation and optimization search strategy are discussed.

2.1 Problem formulation

This subsection is divided into two sub-subsections. The first sub-subsection presents the schematic representation of inputs and responses of a two-stage manufacturing process. The second sub-subsection describes the problem formulation approach based on an adaptive penalty function.

2.1.1 Schematic representation of a two-stage process

A schematic diagram of various inputs and responses of a typical two-stage serial manufacturing process is shown in **Figure 1**. In such process, each stage may have single or multiple responses, which are measured in different measuring scale. The responses at any particular stage may be correlated, depends on the preceding stage response characteristic(s) and the input conditions of previous stage(s). Notations required for problem formulation and description are provided below.

Important Notations:

S : Maximum number of stages = 2

s : s^{th} stage of operation, $s = 1(1)S$

r_s : Numbers of response variables at s^{th} stage.

$\mathbf{X}_s^R$: Input variable at s^{th} stage

$\mathbf{X}_s^P$: Process control parameters (within stage variables) at s^{th} stage of operation.

$\mathbf{X}_{(s+1)}^R$: $(s+1)^{th}$ stage input variables which are actually response characteristics ($\mathbf{Y}_s$) of immediate preceding stage, s^{th} stage.

$\mathbf{Y}_s$: Response characteristics values at s^{th} stage of operation.

$\mathbf{X}_{(s+1)}^I$: $(s+1)^{th}$ stage input variables which are from previous stage input variables.

$y_{j(s)}$: Response of j^{th} characteristic at s^{th} stage of operation.

h_s : Prediction functional between responses $\mathbf{Y}_s$ and predictors $\mathbf{X}_s^P$, $\mathbf{X}_{(s-1)}^R$ at s^{th} stage.

$\hat{y}_{j(s)}$: Predicted j^{th} response characteristic using functional relationship h_s at s^{th} stage of operation for $j = 1, 2, . . , r_s$.

g : Functional relationship to determine individual desirability value from predicted response characteristic, $\hat{y}_{j(s)}$.

$t_{1j(s)}, t_{2j(s)}$: Desirability parameters or exponents for j^{th} response characteristic at s^{th} stage.

$d_{j(s)}$: Individual desirability measure of j^{th} response characteristic at s^{th} stage of operation, where $0 \le d_{j(s)} \le 1$ and $d_{j(s)} = g\left(\hat{y}_{j(s)}\right)$

q_s : Functional relationship to convert into stage desirability.

D_s : Desirability index at s^{th} stage of operation, where $D_s = q_s\left(d_{j(s)}\right)$.

λ_s : Minimum desirability value that should be achieved at s^{th} stage.

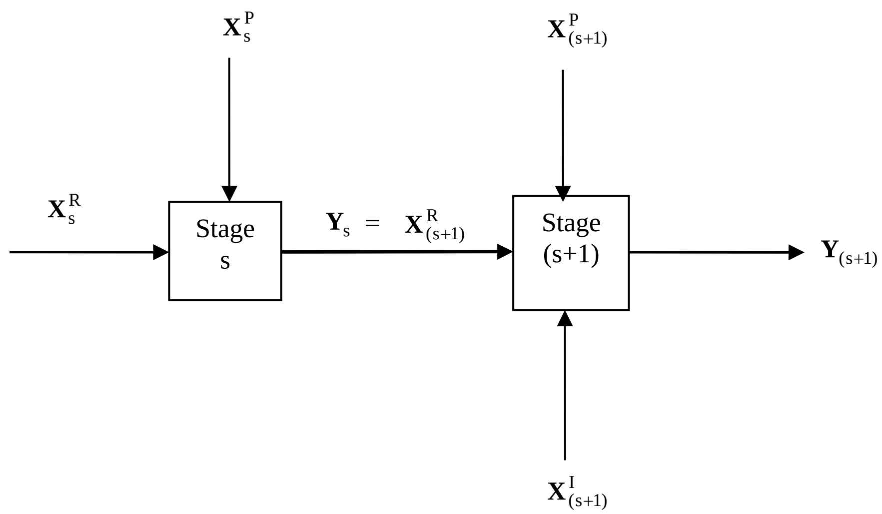

Figure 1: Schematic diagram for two-stage multiple response model

2.1.2 Adaptive penalty function based problem formulation

The proposed problem formulation and solution approach uses modified desirability functions, an adaptive penalty (Gen and Cheng, 1996) and 'maximin' desirability index function (Kim and Lin, 2000). The modified desirability function transforms all the predicted responses (derived from stage-wise prediction function) to scale-free desirability values at each processing stage. The minimum predicted desirability value among all the responses in a particular stage is considered as stage-desirability index. After computing the stage-wise desirability index, an adaptive penalty function-based maximin desirability index function is used to convert the stage-desirability indices to a process-desirability index. The expressions of a modified desirability function for STB and NTB type of response are provided in Eqn. 1 and 2.

$$d_{j(s)} = g(\hat{y}_{j(s)}(\mathbf{X})) = \left\{ \begin{array}{ll} 1 & \text{if } \hat{y}_{j(s)}(\mathbf{X}) \le y_{j(s)}^{min} \\ \left[\dfrac{y_{j(s)}^{max} - \hat{y}_{j(s)}(\mathbf{X})}{y_{j(s)}^{max} - y_{j(s)}^{min}} \right]^{t_{1\,j(s)}} & \text{if } y_{j(s)}^{min} < \hat{y}_{j(s)}(\mathbf{X}) < \tau_{j(s)} \\ \varepsilon & \text{if } \hat{y}_{j(s)}(\mathbf{X}) \ge y_{j(s)}^{max} \end{array} \right\} \quad (1)$$

$$d_{j(s)} = g(\hat{y}_{j(s)}(\mathbf{X})) = \begin{Bmatrix} \varepsilon & \text{if } \hat{y}_{j(s)}(\mathbf{X}) < y_{j(s)}^{\min} \text{ or } \hat{y}_{j(s)}(\mathbf{X}) > y_{j(s)}^{\max} \\ \left[\dfrac{\hat{y}_{j(s)}(\mathbf{X}) - y_{j(s)}^{\min}}{\tau_{j(s)} - y_{j(s)}^{\min}}\right]^{t_{1\,j(s)}} & \text{if } y_{j(s)}^{\min} \le \hat{y}_{j(s)}(\mathbf{X}) \le \tau_{j(s)} \\ \left[\dfrac{y_{j(s)}^{\max} - \hat{y}_{j(s)}(\mathbf{X})}{y_{j(s)}^{\max} - \tau_{j(s)}}\right]^{t_{2\,j(s)}} & \text{if } \tau_{j(s)} < \hat{y}_{j(s)}(\mathbf{X}) \le y_{j(s)}^{\max} \end{Bmatrix} \tag{2}$$

In the above equations, $d_{j(s)}$ is the desirability value for the j^{th} response at s^{th} stage. $d_{j(s)}$ always lies between ε and 1. $\hat{y}_{j(s)}(\mathbf{X})$, $y_{j(s)}^{\min}$ and $y_{j(s)}^{\max}$ are predicted response value, lower and upper tolerance bound of j^{th} response at s^{th} stage, respectively. $t_{1\,j(s)}$ and $t_{2\,j(s)}$ are the exponential desirability parameters for desirability conversion of j^{th} response at s^{th} stage. $\tau_{j(s)}$ is the target value of the j^{th} response at s^{th} stage. ε is a small positive constant (10^{-4}) selected for this study.

The objective function is shown in Eqn. (3). The proposed formulation approach sets the process-desirability index to a very small value for infeasible solution, but remains the same for a feasible solution. The proposed formulation approach is an extension of the APF MAXIMIN formulation approach (Bera and Mukherjee, 2012a). Thus, the overall multistage MRO problem can be formulated as

$$\text{Maximize } \left[\left(\underset{s}{\text{Minimum}}\,(D_s)\right) \times \left(1 - \frac{1}{S}\sum_{s=1}^{S}\left(\frac{v_s}{V^{\text{Max}}}\right)^{\kappa}\right)\right] \tag{3}$$

subject to

$$v_s = \text{Max}\{0,\ \lambda_s - D_s\} \qquad \forall\ s = 1(1)S \tag{4}$$

$$V^{\text{Max}} = \underset{s}{\text{Max}}\{\varepsilon_1,\ v_s\} \qquad \forall\ s = 1(1)S \tag{5}$$

$$D_s = \underset{j}{\text{Min}}\{d_{j(s)}\} \qquad \forall\ j = 1(1)r_s\ , \tag{6}$$

$$(\mathbf{X}_s^P)^{\min} \le \mathbf{X}_s^P \le (\mathbf{X}_s^P)^{\max} \qquad \forall\ s = 1(1)S\,, \tag{7}$$

$$(\mathbf{X}_s^I)^{\min} \le \mathbf{X}_s^I \le (\mathbf{X}_s^I)^{\max} \qquad \forall\ s = 1(1)S\,, \tag{8}$$

where D_s represents the stage desirability index at s^{th} stage. λ_s is the minimum stage-desirability value that should be achieved by s^{th} stage. ε_1 is a small positive constant (10^{-4}) used to avoid zero division situation. κ is a penalty co-efficient and its value is 1 or 2. Eqn. (4) works as a stage constraint and measures the magnitude of violation of stage-desirability with respect to minimum preset value (λ_s). High value of stage desirability index indicates that predicted responses values for that stage are more desirable towards their respective targets. The above formulation approach has the ability to handle correlation among the response characteristics at any processing stage due to adaptation of 'maximin' approach. The following subsection discusses on the optimization search strategy.

2.2 Search strategy

The mathematical formulations given in Eqn. (3) to Eqn. (8) depict a multiple response constraint optimization problem. There are several disjoint feasible operating regions with multimodal objective function. In such situation, suitable constraint handling metaheuristic technique is needed to determine approximate or best near-optimal solutions. Performance of simulated annealing and genetic algorithm is observed to be poor for constrained MRO problem (Mukherjee, 2007). Ant colony optimization (ACO) performs better than genetic algorithm (GA) for constraint nonlinear optimization problem (Kumar and Reddy, 2006). It was reported (Socha and Dorigo, 2008; Bera and Mukherjee, 2012b) that ant colony based optimization strategies have immense potential to determine near-optimal solution for nonlinear multimodal single and multiple response functions.

3. A Case Study to Demonstrate the Usefulness of the Solution Approach

Mukherjee (2007) reported a set of standardized manufacturing data collected from a two-stage engine cylinder liner bore honing process. The two-stage honing consists of a single-pass rough honing (say, stage 1 or first stage) and a two-pass finish honing (say, stage 2 or second stage). Total 198 sample observations on input conditions and corresponding response characteristics at both the stages are collected simultaneously at different time points for model development. The first stage has eleven in-process variables, and three response characteristics. In-process variables are specific machine controllable variables. The second stage has seven in-process variables, another three input variables those are responses of first stage, and five responses (quality characteristics) of engine cylinder liner bore. The five responses at the final stage are the average diameter, average surface finish, honing angle, maximum taper and maximum ovality. The objectives of the case study was to determine the optimal operating conditions for the two-stage honing process so as to get the desired surface texture on the engine cylinder liner bore.

3.1 Prediction models for each processing stage

In this case study, prediction models are developed based on two different techniques, viz. parametric multiple regression model and nonparametric back propagation neural network (BPNN). DUPLEX splitting algorithm (Snee, 1977) is separately used to split data set into 70:30 ratio (train vs. test) for each stage. The train data set is used to develop the two different types of models. Multiple linear regression model is developed in each stage separately and model adequacy is verified. A single hidden layer BPNN model is considered at each stage separately to develop nonparametric model. The activation function considered for each neuron is hyperbolic tangent sigmoidal function, as the normalized data lies between -1 to +1. The network is trained in batch mode using TRAINGDX network training function.

3.2 Determining near-optimal solutions

The optimal search strategy used to determine the near-optimal solutions is ant colony optimization in real space (ACO_R). The intrinsic parameters selected for the strategy is provided in Bera and Mukherjee (2012b). MATLAB 7.1 programming environment is used to perform *30* different computational run for the search strategy. The initial start point for any computational run is generated based on time dependent random number seed. **Table 1** provides summary statistics of the predicted process-desirability index. The process-desirability index is the minimum value among the stage-desirability indices. It is observed from table 1 that average process-desirability index is 0.9341 for linear regression model. In case of BPNN model, average process-desirability index is 0.8866. Thus, linear regression-based model provides higher average process-desirability than nonlinear BPNN model. This may be due to linear response surface which correctly map the input-output relationship. The detailed stage wise desirability indices using regression model and ACO_R optimization strategy is provided in **Table 2**.

Table 1: Summary statistics of predicted process-desirability index

Modeling Technique	Process-desirability index			
	Max	Min	Avg.	Stdev
Linear Regression	0.9376	0.9228	0.9341	0.0039
BPNN	0.8904	0.8788	0.8859	0.0016

Table 2: Desirability indices using regression model and ACO_R search strategy

Computational run no.	Stage 1 desirability (D_1)	Stage 2 desirability (D_2)	Process Desirability $(D_{Process})$
1	0.928	0.927	0.927
2	0.937	0.933	0.933
3	0.981	0.936	0.936
4	0.974	0.937	0.937
5	0.934	0.934	0.934
6	0.932	0.929	0.929
7	0.951	0.937	0.937
8	0.935	0.932	0.932
9	0.941	0.937	0.937
10	0.940	0.937	0.937
11	0.958	0.937	0.937
12	0.936	0.935	0.935
13	0.930	0.929	0.929
14	0.937	0.936	0.936
15	0.937	0.935	0.935
16	0.938	0.936	0.936
17	0.975	0.933	0.933
18	0.947	0.936	0.936
19	0.925	0.924	0.924
20	0.944	0.934	0.934
21	0.970	0.937	0.937
22	0.963	0.938	0.938
23	0.942	0.933	0.933
24	0.951	0.937	0.937
25	0.923	0.923	0.923
26	0.970	0.937	0.937
27	0.938	0.936	0.936
28	0.952	0.936	0.936
29	0.990	0.936	0.936
30	0.945	0.937	0.937

The results indicate the suitability of the integrated solution approach. ACO_R seems adequate to derive near optimal solutions for complex two-stage MRO problems. The search strategy is also quite consistent for both linear and nonlinear response functions.

4. Conclusions

In this article, an adaptive penalty-based integrated solution approach is used for a two-stage process optimization problem. The optimization problem formulation is based on simultaneous optimization of multiple response characteristics, considering interdependency between the stages and correlation among the responses. The response prediction models developed at each stage consider in-process variables (or within stage process variables) and relevant response variables of previous stage. Thus, interrelationships among the various stages are considered in the approach. The search strategy adopted is a variant of continuous ant colony optimization (ACO_R) that determines the near-optimal solutions. The near optimal solutions derived are consistent and accurate and seems adequate for implementation. However, there are certain limitations of the solution approach and this study. The proposed solution approach does not incorporate each stage error variance due to fluctuation of input variables in a multistage process. The problem formulation approach does not consider process model parameter uncertainty. This paper also does not report the sensitivity analysis and the number of stages considered is restricted to two. Studying different search strategies, using the same solution approach, for varied response functions and varied number of stages may be quite interesting to explore.

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