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Formulation of Integer Programming Model for Balancing and Scheduling of Production Line Having Shared Resources

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Abstract

In a mass production environment where resources such machines are shared, a balanced and scheduled production flow line can play a pivotal role through the enhancement of line efficiency by minimizing production time. Balancing of production flow line is a classical OR problem which has been addressed extensively during the last few decades. But only few researches focused on the resource constrained line balancing problem (RCLBP) where the elementary tasks can be performed by a single and/or multiple resources. In reality utilization of the limited capacities of such special machines, equipment, or resources, due to the prohibitively high investment cost, is a very common shop-floor phenomenon. For such cases, an integrated model capable of balancing, scheduling and allocating the resources sequentially to the appropriate task(s) is necessary. In this paper instead of treating the RCLBP in a traditional manner, we introduced a new dimension to it by formulating a relevant mathematical model through application of integer programming method for production environment where several tasks are performed using a particular resource through incorporation of the positive zoning and the shared resource constraints. The model allows the paralleling of work-stations to cope with the tasks having processing times exceeding the local work-station cycle time.

Keywords
Production line balancing and scheduling, Resource allocations, Shared resources problem, Integer programming

1. Introduction
With globalization of manufacturing activities, competition, complexity and challenges have increased drastically in the manufacturing industries. High volume of customer demand for a good quality product at a competitive price made these phenomena more complicated for the industrial managers. As a result, taking a strategic or tactical decision for unexpected events is being emphasized more to face these challenges. Though the time based strategies are often adopted for shortening the production lead time, managers can bring the competitive advantages by implementing the advanced technologies or selecting appropriate techniques for production shop floors (Steveson 2007). The production line balancing and scheduling is a similar kind of technique that helps to enhance the production rate by minimizing the idle time or the completion time (Sawik 2002, Betts and Mahmoud 1989).

A production flow line can be an assembly or a fabrication line or even the combination of both, where the standardized products are assembled and produced in a high volume with low verities (Becker and Scholl 2006, Ho and Emrouznejad 2009, and Kumar 2013). Based on the types of the end product, the production flow lines are classified as 1) single model production lines, 2) multi model production lines and 3) mixed model production lines (Gen et al. 2008). In addition, it can also be noticed from the literature that the main concern of these production flow lines is to determine the optimal allocation of the tasks to different stages or workstations under restrictions or constraints leading to production line balancing problem (PLBP) (Scholl and Becker 2006). In other words, PLBP can be interpreted as a task allocation problem among several workstations in a manner that the technical constraints remain valid and the task times should be kept within the considered local cycle time (Amen 2000).

On the basis of the underlying assumptions to represent the industrial problems, the researchers classified the LBP in two types; simple and general assembly production line balancing problems (Baybars 1986, Becker and Scholl 2006). The simple production line balancing problems (SPLBP) are mostly addressed in the literature, though the general production line balancing problems (GPLBP) are found to be more relevant to the industrial practice. For both SPLBP and GPLBP, the researchers introduce four different types of problem i.e. SLBP-1 (for a given cycle time minimize the number of work stations), SLBP-2 (for a given workstation minimize the cycle time), SLBP-E (enhance the line efficiency by minimizing the number of workstations and the cycle time) and finally the SLBP-F (find a feasible solution for a predefined workstation and cycle time) (Tuncel and Topaloglu 2013, Boysen et al. 2007, Becker and Scholl 2006).

During the last 50 years, numerous researches have been conducted to solve the simple and general production line balancing problems. But most of them consider that all the required resources are used in the most effective manner with availability (Ağpak and Gökçen 2005). However, inside the real industrial shop floor, the phenomena such as the limited capacity of the special purpose machines, non-availability of the resources, resource sharing among different tasks is not uncommon. To gain the benefit of the mass production strategy having shared resources, it is necessary to balance and schedule the production flow line by optimally allocating the resources to their corresponding tasks at different stages, so that the best possible schedule with the minimum completion time is achieved.

In this paper, a real life integrated balancing and scheduling problem of a single model production flow line having shared resources is attempted. To minimize the batch production time of the flow line, an integer programming model is formulated, which is deemed to be able to simultaneously balance and schedule the tasks by the optimal allocation of resources to the corresponding tasks at different stages through incorporation of the concept of workstation paralleling.

2. Literature review

In the past few decades, several mathematical models have been proposed to address the PLBP from different perspectives. Salveson (1955) made his first attempt to formulate the mathematical model for assigning the tasks to the workstations. Another attempt was taken by Bowman (1960), where he proposed two different linear programming approaches to solve the line balancing problem. In his first model, the problem was viewed as a simple assignment problem with cycle time restriction, precedence relationship, whereas, in the second, he simply modified the first model by introducing the clock time. By considering the SLBLP-1, Held et al. (1963) proposed a dynamic programming model and an iterative approximation technique for solving the complexities arising in a small and large enterprise respectively. In their proposed model, the assembly line balancing problem was considered as a job sequencing phenomenon where the precedence constraints were used to restrict the direct sequential task assignments. Patterson and Albracht (1975) formulated a binary integer model to improve the adequacy of the model proposed by Bowman and White. In their proposed approach, they limit the number of variables by featuring the precedence relationship and defining the earliest (Ei) and latest (Li) workstations for the given task.

Despites these efforts, urges for simultaneous balancing and scheduling have appeared in the literature. As in most of the line balancing works, the researchers limited their objectives to the assignment of tasks to the corresponding workstations regardless of the sequence of tasks inside the workstations (Andres et al. 2008). One of the justifications for this separate balancing and scheduling was the time frame, as the line balancing is a tactical decision which may be taken to last for several months (Öztürk et al. 2013b, Akgündüz and Tunalı 2011). Moreover, due to the presence of the uncertain circumstances, it is often necessary to rebalance the line frequently (Boysen et al. 2009). Hence, in the presence of the sequence dependent setup times, shared resources and even for some other uncertainties i.e. demand variation, rapid technological changes, the simultaneous line balancing and scheduling bring more competitive advantages rather than only focusing on the line balancing.

In 2002, Miltenburg presented a model for simultaneous line balancing and scheduling for a mixed model U-shaped production line. In his work, the tasks are assigned to their corresponding workstations to minimize the production time by choosing the sequence of the different products (Miltenburg 2002). In the same year, Sawik (2002) presented two different mixed integer models for balancing and scheduling the tasks of a flexible assembly line where the objective was assigning the tasks to different workstations with minimum completion time. In his first model, the tasks were balanced and scheduled simultaneously whereas the second model proposed to schedule the tasks after the line is balanced. However, the first approach has shown some improvement over the second in terms of the computational efforts for large sized problem. By considering the sequence dependent setup time, Andres et al. (2008) conducted a research to develop the optimal work schedule for the workers in each workstation. In their proposed model of balancing and scheduling the simple production line, the concept of travelling salesman problem was used to schedule the tasks where the setup time is considered as the distance between the cities. Moreover, to enhance the reliability of the flexible mixed model production line and satisfy the customers' demand of a just-in-time production system, [b] Öztürk et al. (2013a) addressed a new model for simultaneous balancing and scheduling by allowing the concept of workstation paralleling. The concept of paralleling can be viewed from two different perspectives i.e. workstation paralleling and task paralleling. The workstation paralleling allows opening one or more duplicate stations at the same stage with an aim of reducing the local cycle time while the task paralleling allows a single task to be performed in more than one workstation. Usually, the tasks or the workstations paralleling are used to enhance the production rate or to reduce the labor requirements with a concern of capital cost. By featuring the workstation paralleling, Ege et.al (2009) proposed two branch and bound algorithms for balancing the assembly lines.

As a part of the realistic considerations of line balancing, Ağpak and Gökçen (2005) presented two different resource constrained line balancing problems. In first case, it was considered that a task is allowed to be performed by a particular resource i.e. a production line needs to perform 5 tasks by two different resources R_1 and R_2 where R_1 performs tasks 1, 3, 5 and R_2 performs 2 and 4. In the second case, performing a task by different resources was allowed i.e. the task 5 in the previous example can be performed either by resource R_1 or R_2. The authors also presented two different 0-1 integer programming models for both the cases with an objective of minimizing the number of workstations through the allocation of minimum resources. Later, in 2011, this resource constrained ALBP is extended to a more generalized form through the work of Corominas et al. (2011). In their research, instead of limiting to the previous two cases they consider that a single task can be performed by a single or multiple alternate resources and/or their combinations. i.e. a production line needs to perform 9 tasks by three different resources R_1, R_2 and R_3 where a task may need 3 units of R_1 or the concurrent manner of resources i.e. 3 units of R_1 and 2 units of R_2 $(3\ R_1 \wedge 2\ R_2)$ or the alternative and/or concurrent resources i.e. $(2R_1 \vee 3\ R_2 \vee 3\ R_3)$ or $(4R_1 \wedge 5\ R_2) \vee (5R_2 \wedge 6\ R_3)$ or $(5R_1 \vee 3\ R_2) \wedge (2R_2 \vee 4\ R_3)$ and etc. Finally, they present a mathematical model that is expected to be more relevant for solving the real life industrial problems.

In the present paper, instead of viewing the line balancing problem in a traditional manner, we introduce a shared resource line balancing problem where several tasks require a particular type of resources i.e. a production line needs to perform 10 different tasks by 5 different resources R_1, R_2, R_3, R_4 and R_5 where the Tasks 1, 2 and 3 require same resource R_1, Tasks 4, 5 and 6 require R_2, Tasks 7 and 8 require R_3 and all other tasks can be performed by the dedicated resources. Moreover, the problem appears to be more complicated as some of the tasks have processing times greater than the local cycle (or station time). For this real life line balancing problem, we propose an integer programming based model that is able to balance and schedule the production flow line by allocating the resources to their corresponding tasks and opening the parallel workstations. Apart from balancing the production flow line, the model also ensures an optimal sequence of the tasks within the stages.

3. Brief description of the production line

Mass production strategy is commonly adopted in auto industries for manufacturing part/sub-assemblies where different parts are procured, fabricated and assembled in a high volume. In this strategy, the concept of production flow line having flexibility to handle various sub-assemblies or modules of car bodies of different models is adopted with a target to reduce the setup time, work in process inventories, material handling costs, production cycle time, etc. For this study a bumper part production flow line in a local automotive industry has been considered where 14 different resources are engaged to perform 22 different tasks needed to complete a single unit of a batch. Some of the resources are dedicated for individual tasks whereas others are used as shared resources for multiple tasks. As shown in Figure 1, machine-5 (M-5) performs tasks 5, 12 and 16; machine-8 (M-8) executes tasks 8, 13 and 17; machine-9 carries out tasks 9, 14 and 18; and machine-10 is engaged to accomplish tasks 10, 15 and 19. The rest of the resources are dedicated to individual tasks. Time study was conducted to record the task times and it has been found that task-7 and task-22 have the processing times greater than the local cycle time. Currently the problem of allocation of the shared resources for different tasks is done through intuition and common sense which is unlikely to end up with an optimal solution. As the scenario is a NP-hard type line balancing and scheduling problem, the intuitive or the judgmental approach may not be appropriate to guarantee to minimize the batch production time. Hence, we formulate a mathematical model, the solution of which is to minimize production cycle time of a batch by simultaneous balancing and scheduling of the flow line having shared resources and tasks requiring processing times greater than the local cycle time.

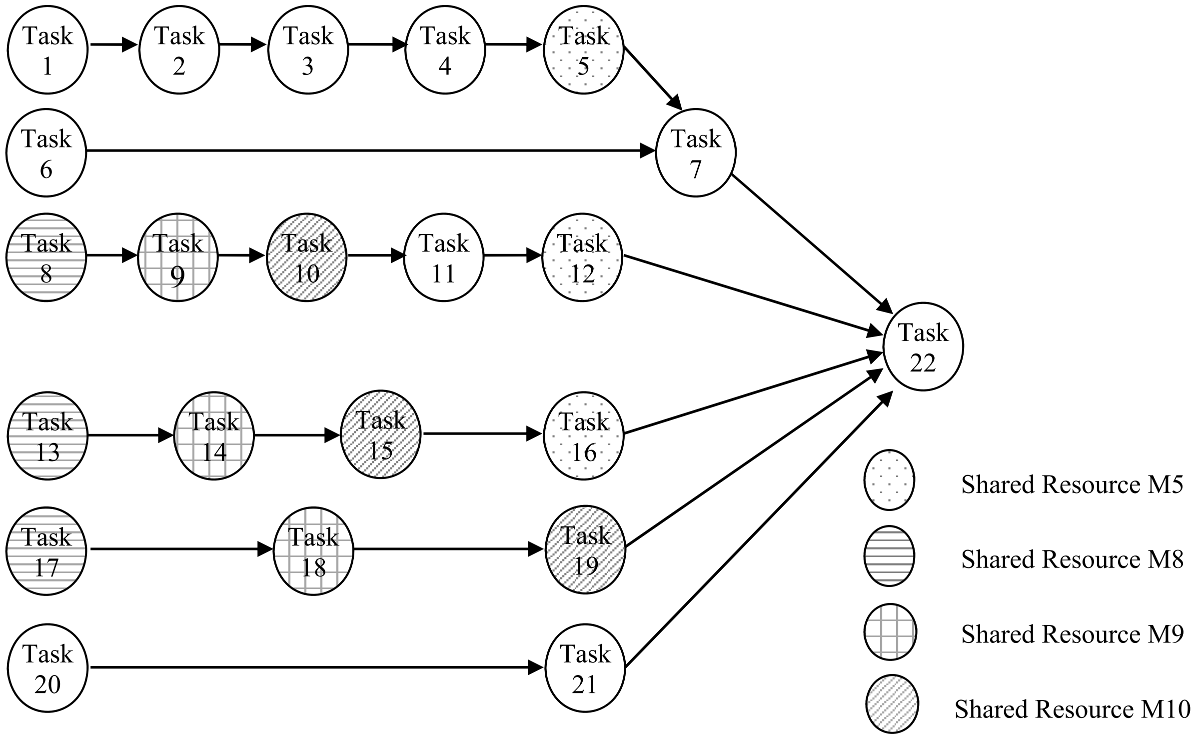

Figure 1: Network representation of production flow line

4. Formulation of the mathematical model

To derive an integer programming model, a simple task assignment problem is formulated initially to assign the tasks at different stages by maintaining the precedence relation and the cycle time constraints. Due to the requirement of resource sharing, positive zoning constraints are followed to group and assign the tasks at a particular stage. During the formulation of the assignment problem, it is also important to determine the sequence of the tasks inside each stage. As all the resources are not dedicated, the sequence of assignment of shared resources to their corresponding tasks affects greatly the completion time. For example, if a shared resource is assigned first to a non-critical task, the batch completion time is to increase. However, a sequence ensuring the assignment of a shared resource first to a critical task should minimize this time. Apart from this presence of tasks with processing times greater than the local cycle time is to linger the whole process. For such cases the model is framed to permit the workstation paralleling for the opened stages. The constraints in the developed model are integrated through an objective function derived to minimize the batch production time.

Notations, Indices and Sets

N: Total number of tasks in the problem
j: Number of stages that can be opened, j =1, 2, 3,…………………………………......m
i: Number of tasks, i =1, 2, 3,…………………………………………………………..n
k:Number of stations in parallel at each stage, k=1, 2, 3, ……………………………...K
j_{max}: Maximum number of stages that can be opened
E_i: Earliest station for assigning task i, given by the precedence relations
L_i: Latest station for assigning task i, given by the precedence relations
C: Local cycle time (takt time)
t_i: Task time of i where $i \epsilon$ SN
t_c: Task time of c where $c \epsilon SP_c$
t_d: Task time of c where $d \epsilon SP_d$
$T_{j:}$ Start time of each j stages.
$C_{ij:}$ Completion time of task i at stage j
C_{max}: Maximum completion time

K_p: Maximum number of parallel workstations
SN: Set of all tasks
SR: Set of shared resources
ST_c: Set of tasks which proceed task i
ST_d: Set of tasks which succeed task i
SR_r: Set of tasks that are using same resource r, where r ϵ SR and SRr is a subset of SN
ST: Set of tasks those have processing time less than the local cycle time
GT: Set of tasks those have processing time greater than local cycle time
ST_j: Set of tasks that can be assigned to stage j and ST_j is also a subset of SN
$SP_{a,b}$: Set of task pair (a,b) in immediate precedence relation where (a ,b)ϵSN and $SP_{a,b}$ is the subset of SN.
SP_i:Set of task i and all of its predecessors

Decision variables:
All the decision variables used in the proposed model are taken as binary variable, which are defined as follows:

$$x_{ij} = \begin{cases} 1 & \text{if task } i \text{ is assigned to stage } j \\ 0 & \text{otherwise} \end{cases} \qquad z_{jk} = \begin{cases} 1 & \text{if stage } j \text{ is opened with workstation } k \\ 0 & \text{otherwise} \end{cases}$$

$$R_{Dp} = \begin{cases} 1 & \text{if decision } n \text{ is taken} \\ 0 & \text{otherwise} \end{cases}$$

Where, R_{Dp} decides the best allocations of the shared resources and p=1, 2, 3…….

$$for\ n=1, R_{D1} = \begin{cases} 1 & \text{if task 8 get the resource before 13} \\ 0 & \text{otherwise} \end{cases} \qquad for\ n=2, R_{D2} = \begin{cases} 1 & \text{if task 8 get the resource before 17} \\ 0 & \text{otherwise} \end{cases}$$

$$for\ n=3, R_{D3} = \begin{cases} 1 & \text{if task 13 get the resource before 8} \\ 0 & \text{otherwise} \end{cases} \qquad for\ n=4, R_{D4} = \begin{cases} 1 & \text{if task 5 get the resource before 12} \\ 0 & \text{otherwise} \end{cases}$$

$$for\ n=5, R_{D5} = \begin{cases} 1 & \text{if task 5 get the resource before 16} \\ 0 & \text{otherwise} \end{cases} \qquad for\ n=6, R_{D6} = \begin{cases} 1 & \text{if task 12 get the resource before 16} \\ 0 & \text{otherwise} \end{cases}$$

4.1 Formulation of objectives function
The main objective of the proposed integer programming model is to minimize the batch production time or the completion time, which is directly interlinked with the task assignment at different stages as well as the workstations, allocation of the dedicated and shared resources and the optimal sequence of the tasks. Hence the objective function of this proposed model is framed as shown by Equation (1)

$$Minimize \sum_{j=1}^{m} C_{\max} \ ; \ \forall i,k \tag{1}$$

4.2 Formulation of Constraints
To formulate the intended mathematical model, the constraints are worked out as follows:

$$E_i = \left[\left(t_i + \sum_{c \in SP_c} t_c\right) \Big/ C\right]^+ \tag{2}$$

$$L_i = j_{\max} + 1 - \left[\left(t_i + \sum_{d \in SP_d} t_d\right) \Big/ C\right]^+ \tag{3}$$

$$\sum_{j=E_i}^{L_i} x_{ij} = 1; \forall i \tag{4}$$

$$\prod_{i \in SRr} (x_{ij}) \le 1; \forall j \tag{5}$$

$$\sum_{i \in ST_j} t_i * x_{ij} \le C * \left(\sum_{k=1}^{K} z_{jk}\right); \forall j \tag{6}$$

$$\sum_{i \in ST} t_i * x_{ij} \le C * z_{j1}; \forall j \tag{7}$$

$$\sum_{i \in GT} t_i * x_{ij} \le C * \left(\sum_{k=1}^{K} z_{jk}\right); \forall j \tag{8}$$

$$\sum_{j=1}^{m} z_{j1} \le [t_i / C]^+; \forall j \text{ and } i \in ST \tag{9}$$

$$j_{\max} = \left[\sum_{i=1}^{n} t_i \Big/ C\right]^+; i \in ST \tag{10}$$

$$\sum_{k=1}^{K} z_{jk} \le k_p; \forall j \tag{11}$$

$$k_p = \left[\sum_{i=1}^{n} t_i \Big/ C \right]^{+}; i \in GT \quad (12)$$

$$\sum_{i=1}^{n} x_{ij} - \left\| ST_j \right\| * z_{jk} \le 0; \forall j \quad (13)$$

$$\sum_{i \in ST} x_{ij} - \left\| ST_j \right\| * z_{j1} \le 0; \forall j \quad (14)$$

$$\sum_{i \in GT} x_{ij} - \left\| ST_j \right\| * \left(z_{jk} * z_{j(k+1)} \right) \le 0; \forall j \text{ and } k = 1,2,.......K \quad (15)$$

By considering the complexity, computational time and the size of the model, we use Equations (2) and (3) proposed by Patterson and Albracht (1975) to define the earliest (Ei) and latest (Li) stages for assigning a particular task. The constraint (4) ensures the assignment of each task exactly at one stage whereas the positive joining constraint shown by Equation (5) ensures that the task using shared resources will be assigned together to a particular stage. Constraints (6) limit the assignment of tasks to the stages by declaring that the stage time or the work load should be kept below or equal to the local cycle time. Equations (7) and (8) explain two different perspectives of constraint (6). The first one restricts the opening of the parallel workstation (k=1) as they only consider tasks that are elements of set ST while the second equation allow workstation paralleling (k=2) for the tasks that are elements of set GT. Equations (9) and (10) give the maximum theoretical number of stages that can be opened for assigning the tasks. However, depending on the problem type, this number is often determined subjectively (Bowman, 1960). Meanwhile, Equations (11) and (12) define the maximum number of parallel workstations that can be opened at each stage. Constraints (13), is formulated to limit the search area by addressing the probable set of tasks that can be assigned to a particular stage. Moreover, this constraint also ensures that a stage or parallel workstation should be opened if any of the tasks of set ST or GT is selected to be assigned. The Equations (14) and (15) are two different cases of constraint (13) which may apply for stages without parallel or with parallel work stations respectively. In addition, the following constraints are also formulated to ensure the optimal resources allocation to their corresponding tasks.

$$\sum_{i \in SR_r} x_{ij} * R_{Dn} \le 1; \forall j \quad (16)$$

$$x_{8j} * R_{D1} + x_{13j} * \left(1 - R_{D1}\right) = 1 \quad (17)$$

$$x_{8j} * R_{D2} + x_{17j} * (1 - R_{D2}) = 1 \quad (18)$$

$$x_{13j} * \left\{ R_{D1} R_{D2} R_{D3} + (1 - R_{D1})(1 - R_{D2}) R_{D3} \right\} \ge 0 \quad (19)$$

$$x_{17j} * \left\{ R_{D1} R_{D2} (1 - R_{D3}) + (1 - R_{D1})(1 - R_{D2})(1 - R_{D3}) \right\} \ge 0 \quad (20)$$

$$x_{5j} * R_{D4} + x_{12j} * (1 - R_{D4}) = 1 \quad (21)$$

$$x_{5j} * R_{D5} + x_{16j} * (1 - R_{D5}) = 1 \quad (22)$$

$$x_{12j} * \left\{ R_{D4} R_{D5} R_{D6} \right\} + (1 - R_{D4})(1 - R_{D5}) R_{D6} \} \ge 0 \quad (23)$$

$$x_{16j} * \left\{ R_{D4} R_{D5} (1 - R_{D6}) + (1 - R_{D4})(1 - R_{D5})(1 - R_{D6}) \right\} \ge 0 \quad (24)$$

The constraint (16) is used to determine the sequence for assigning the shared resources to their corresponding tasks. From Figure-1 it can be seen that due to the presence of three shared resources M-8, M-9 and M-10, the set of tasks (T-8,T-9,T-10,T-13, T-14, T-15, T-17, T-18, T-19) behaves like a permutation shop i.e. the sequence of task T-8, T-13, T-17 will be followed by both of the trio (T-9, T-14, T-18) and (T-10, T-15, T-19). Hence, we introduce the resource constraints (17) to (20) for identifying the optimal sequence of tasks T-8, T-13, T-17 that is to ensure the minimum production or the completion time. However, another sequence of tasks T-5, T-12, T-16 using shared resource, M-5 is also formulated as the constraints (21) to (24) because they are not included inside the permutation floor. As all other tasks, except those included in the set of SR_r, have their dedicated resources, the optimal sequence of tasks using shared resources should ensure the best schedule for the whole production line. The precedence relations among the tasks are maintained by constraints (25) and (26).

$$\sum_{j=E_a}^{L_a} j(x_{aj}) - \sum_{j=E_b}^{L_b} j(x_{bj}) \le 0; \; (a,b) \in SP_{a,b} \quad (25)$$

$$\left(C_{bj} x_{bj} - C_{aj} x_{aj} \right) \ge t_{bj}; \; (a,b) \in SP_{a,b} \quad (26)$$

$$C_{bj} \ge C_{aj} \quad (27)$$

$$C_{ij} \geq 0 \;\; ; \;\; \forall i, j \tag{28}$$

$$C_{\max} \geq C_{ij} \tag{29}$$

The Equations (27) to (29) are framed to define the completion time of each task i and their relationship with the maximum completion time. However, to consider the effect of the presence of shared resources on completion time and the precedence relation, the following constraints (30) to (53) are developed.

$$\left(C_{13j}x_{13j} - C_{8j}x_{8j}\right) * R_{D1} * (1 - R_{D2}) \geq t_{13j} \tag{30}$$

$$(C_{13j}x_{13j} - C_{17j}x_{17j}) * R_{D1} * (1 - R_{D2}) \geq t_{13j} + t_{8j} \tag{31}$$

$$\left(C_{17j}x_{17j} - C_{8j}x_{8j}\right) * R_{D2} * (1 - R_{D1}) \geq t_{17j} \tag{32}$$

$$(C_{17j}x_{17j} - C_{13j}x_{13j}) * R_{D2} * (1 - R_{D1}) \geq t_{17j} + t_{8j} \tag{33}$$

$$\left(C_{17j}x_{17j} - C_{13j}x_{13j}\right) * R_{D1} * R_{D2} * R_{D3} \geq t_{17j} \tag{34}$$

$$\left(C_{17j}x_{17j} - C_{8j}x_{8j}\right) * R_{D1} * R_{D2} * R_{D3} \geq t_{17j} + t_{13j} \tag{35}$$

$$\left(C_{13j}x_{13j} - C_{17j}x_{17j}\right) * R_{D1} * R_{D2} * (1 - R_{D3}) \geq t_{13j} \tag{36}$$

$$\left(C_{13j}x_{13j} - C_{8j}x_{8j}\right) * R_{D1} * R_{D2} * (1 - R_{D3}) \geq t_{13j} + t_{17j} \tag{37}$$

$$\left(C_{8j}x_{8j} - C_{17j}x_{17j}\right) * (1 - R_{D1}) * (1 - R_{D2}) * R_{D3} \geq t_{8j} \tag{38}$$

$$\left(C_{8j}x_{8j} - C_{13j}x_{13j}\right) * (1 - R_{D1}) * (1 - R_{D2}) * R_{D3} \geq t_{8j} + t_{13j} \tag{39}$$

$$\left(C_{8j}x_{8j} - C_{13j}x_{13j}\right) * (1 - R_{D1}) * (1 - R_{D2}) * (1 - R_{D3}) \geq t_{8j} + t_{17j} \tag{40}$$

$$\left(C_{8j}x_{8j} - C_{17j}x_{17j}\right) * (1 - R_{D1}) * (1 - R_{D2}) * (1 - R_{D3}) \geq t_{8j} \tag{41}$$

$$\left(C_{16j}x_{16j} - C_{5j}x_{5j}\right) * (1 - R_{D4}) * R_{D5} \geq t_{16j} \tag{42}$$

$$\left(C_{16j}x_{16j} - C_{12j}x_{12j}\right) * (1 - R_{D4}) * R_{D5} \geq t_{16j} + t_{5j} \tag{43}$$

$$\left(C_{12j}x_{12j} - C_{5j}x_{5j}\right) * (1 - R_{D5}) * R_{D4} \geq t_{12j} \tag{44}$$

$$\left(C_{12j}x_{12j} - C_{16j}x_{16j}\right) * (1 - R_{D5}) * R_{D4} \geq t_{12j} + t_{5j} \tag{45}$$

$$\left(C_{16j}x_{16j} - C_{12j}x_{12j}\right) * R_{D4} * R_{D5} * R_{D6} \geq t_{16j} \tag{46}$$

$$\left(C_{16j}x_{16j} - C_{5j}x_{5j}\right) * R_{D4} * R_{D5} * R_{D6} \geq t_{16j} + t_{12j} \tag{47}$$

$$\left(C_{12j}x_{12j} - C_{16j}x_{16j}\right) * R_{D4} * R_{D5} * (1 - R_{D6}) \geq t_{12j} \tag{48}$$

$$\left(C_{12j}x_{12j} - C_{5j}x_{5j}\right) * R_{D4} * R_{D5} * (1 - R_{D6}) \geq t_{12j} + t_{16j} \tag{49}$$

$$\left(C_{5j}x_{5j} - C_{12j}x_{12j}\right) * (1 - R_{D4}) * (1 - R_{D5}) * R_{D6} \geq t_{16j} + t_{5j} \tag{50}$$

$$\left(C_{5j}x_{5j} - C_{16j}x_{16j}\right) * (1 - R_{D4}) * (1 - R_{D5}) * R_{D6} \geq t_{5j} \tag{51}$$

$$\left(C_{5j}x_{5j} - C_{16j}x_{16j}\right) * (1 - R_{D4}) * (1 - R_{D5}) * (1 - R_{D6}) \geq t_{12j} + t_{5j} \tag{52}$$

$$\left(C_{5j}x_{5j} - C_{12j}x_{12j}\right) * (1 - R_{D4}) * (1 - R_{D5}) * (1 - R_{D6}) \geq t_{5j} \tag{53}$$

The constraints (30) to (41) enable the model to take into account of the effect of all the possible sequences of tasks using shared resources M-8, M-9, M10 i.e. T17-T8-T13, T13-T8-T17, T8-T13-T17, T8-T17-T13, T13-T17-T8,T13-T17-T8 respectively on the batch completion time. In the same manner, the constraints (42) to (51) are formulated to take into account of all possible sequences of T-5, T-12, T-16 on the completion time.

$$C_{ij} = T_j + \sum_{i \in SP_i} t_i x_{ij} \, ; \;\; \forall j \tag{54}$$

$$C_{8j} = [R_{D1}R_{D2}(1-R_{D3})]*(T_j + t_8)*x_{8j} + (R_{D1}R_{D2}R_{D3})*(T_j + t_8)*x_{8j}$$
$$+ [R_{D1}(1-R_{D2})]*(T_j + t_{17} + t_8)*x_{17j}*x_{8j} + [(1-R_{D1})R_{D2}]*(T_j + t_{13} + t_8)*x_{13j}*x_{8j}$$
$$+ [(1-R_{D1})(1-R_{D2})(1-R_{D3})]*(T_j + t_8 + t_{13} + t_{17})*x_{8j}*x_{13j}*x_{17j}$$
$$+ [(1-R_{D1})(1-R_{D2})R_{D3}]*(T_j + t_8 + t_{13} + t_{17})*x_{8j}*x_{13j}*x_{17j} \quad (55)$$

$$C_{13j} = [R_{D1}R_{D2}(1-R_{D3})]*(T_j + t_8 + t_{13} + t_{17})*x_{8j}*x_{13j}*x_{17j}$$
$$+ (R_{D1}R_{D2}R_{D3})*(T_j + t_8 + t_{13})*x_{8j}*x_{13j} + [R_{D1}(1-R_{D2})]*(T_j + t_8 + t_{13} + t_{17})*x_{8j}*x_{13j}*x_{17j}$$
$$+ [(1-R_{D1})R_{D2}]*(T_j + t_{13})*x_{13j} + [(1-R_{D1})(1-R_{D2})(1-R_{D3})]*(T_j + t_{13} + t_{17})*x_{13j}*x_{17j}$$
$$+ [(1-R_{D1})(1-R_{D2})R_{D3}]*(T_j + t_{13})*x_{13j} \quad (56)$$

$$C_{17j} = [R_{D1}R_{D2}(1-R_{D3})]*(T_j + t_8 + t_{17})*x_{8j}*x_{17j}$$
$$+ (R_{D1}R_{D2}R_{D3})*(T_j + t_8 + t_{13} + t_{17})*x_{8j}*x_{13j}*x_{17j}$$
$$+ [R_{D1}(1-R_{D2})]*(T_j + t_{17})*x_{17j} + [(1-R_{D1})R_{D2}]*(T_j + t_8 + t_{13} + t_{17})*x_{8j}*x_{13j}*x_{17j}$$
$$+ [(1-R_{D1})(1-R_{D2})(1-R_{D3})]*(T_j + t_{17})*x_{17j}$$
$$+ [(1-R_{D1})(1-R_{D2})R_{D3}]*(T_j + t_{13} + t_{17})*x_{13j}*x_{17j} \quad (57)$$

$$C_{5j} = [(1-R_{D4})R_{D5}]*(T_j + t_{12} + t_5)*x_{12j}*x_{5j}$$
$$+ [(1-R_{D4})(1-R_{D5})R_{D6})]*(T_j + t_{12} + t_{16} + t_5)*x_{12j}*x_{5j}*x_{16j}$$
$$+ (R_{D4}R_{D5}R_{D6})*(T_j + t_5)*x_{5j} + [R_{D4}R_{D5}(1-R_{D6})]*(T_j + t_5)*x_{5j}$$
$$+ [R_{D4}(1-R_{D5})]*(T_j + t_5 + t_{16})*x_{5j}*x_{16j}$$
$$+ [(1-R_{D4})(1-R_{D5})(1-R_{D6})]*(T_j + t_{12} + t_{16} + t_5)*x_{12j}*x_{16j}*x_{5j} \quad (58)$$

$$C_{12j} = [(1-R_{D4})R_{D5}]*(T_j + t_{12})*x_{12j} + [(1-R_{D4})(1-R_{D5})R_{D6})]*(T_j + t_{12})*x_{12j}$$
$$+ (R_{D4}R_{D5}R_{D6})*(T_j + t_5 + t_{12})*x_{5j}*x_{12j}$$
$$+ [R_{D4}R_{D5}(1-R_{D6})]*(T_j + t_5 + t_{12} + t_{16})*x_{5j}*x_{12j}*x_{16j}$$
$$+ [R_{D4}(1-R_{D5})]*(T_j + t_5 + t_{12} + t_{16})*x_{5j}*x_{12j}*x_{16j}$$
$$+ [(1-R_{D4})(1-R_{D5})(1-R_{D6})]*(T_j + t_{12} + t_{16})*x_{12j}*x_{16j} \quad (59)$$

$$C_{16j} = [(1-R_{D4})R_{D5}]*(T_j + t_5 + t_{12} + t_{16})*x_{5j}*x_{12j}*x_{16j}$$
$$+ [(1-R_{D4})(1-R_{D5})R_{D6}]*(T_j + t_{12} + t_{16})*x_{12j}*x_{16j}$$
$$+ (R_{D4}R_{D5}R_{D6})*(T_j + t_5 + t_{12} + t_{16})*x_{5j}*x_{12j}*x_{16j}$$
$$+ [R_{D4}R_{D5}(1-R_{D6})]*(T_j + t_5 + t_{16})*x_{5j}*x_{16j}$$
$$+ [R_{D4}(1-R_{D5})]*(T_j + t_{16})*x_{16j} + [(1-R_{D4})(1-R_{D5})(1-R_{D6})]*(T_j + t_{16})*x_{16j} \quad (60)$$

The completion time of each task performed by dedicated resources can be articulated by Equation (54) whereas the completion times of the tasks using shared resources possible to be expressed by Equations (55) to (60). Thus, the whole production environment having 22 tasks some of which performed by common resources in a sequential order, is modeled by integer programming approach. The solution of this problem is anticipated to provide an improved procedure to allocate the resources to the tasks in the optimal way leading to reduction of batch completion time with enhancement in productivity.

5. Conclusions and Future Works

It is evident that for a production environment using shared resources as addressed in this paper, allocation of resources in a random sequence of the tasks to different workstations through intuition and common sense is likely to end up with longer batch flow time, increased work-in process-inventory and reduced production rate. As a consequence overall cost for production of a batch is supposed to rise. In this context, the aim initiated for formulation of an integer programming model to contemplate the situation having simultaneous balancing and scheduling of operations through a production flow line embedded with shared resources is achieved. Main challenge in the formulation of the mathematical model has been encountered in defining the sequence of tasks and framing the constraints and objective function. The proposed model should be helpful in minimizing the batch completion time with enhanced productivity through possible assignment of extra-long tasks in parallel workstations and simultaneous scheduling within the stages. Securing a feasible and optimal solution of the problem and the successful implementation of the optimum values in practice are attributable to maximize resource utilization by reducing the batch completion time and work-in-process inventory with an end result of having a lean production environment. Additionally, the cost of incorporation of parallel work stations and mixed model production flow line may be taken into consideration from a wider perspective in the formulation of the problem and its solution. Alternatively a heuristic or a meta-heuristic approach may be developed to obtain easier solutions by avoiding the complexity and NP hard nature of the model.

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