# Optimal Economic Manufacturing Quantity and Process Target for Imperfect Systems

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## Abstract

Recently, Chen [1] presented a modified economic manufacturing quantity (EMQ) model under imperfect product quality and perfect measurement system. The model determines the EMQ and the process target simultaneously. However, the extend model did not consider the problem under imperfect rework where measurement error exists. In this paper, the authors extend Chen [1] model for obtaining the optimum process mean and economic manufacturing quantity under imperfect rework where measurement error exists. The proposed model is illustrated via an example from the literature. Comparison of the results of the proposed model with Chen model is provided. In addition sensitivity analysis to assess the impact of the measurement errors has been conducted.

## Keywords
Economic manufacturing quantity (EMQ), Process target, Taguchi's quadratic loss function, Measurement error.

## 1. Introduction
The problem of determining the economic manufacturing quantity and the process targeting are two important problems in production planning. Both problems attracted the attention of researchers for several decades. In this paper both problems are addressed in an integrated framework under imperfect measurement system.

Classical Economic order quantity (EOQ) models assume all received items in a lot Q meet specifications (perfect quality) and screening procedures for items are error free. However these types of assumptions rarely exist in real life. Therefore lots received may have imperfect items and the procedures used to screen the items have errors such as misclassifying a good item as rework, good item as scrap, rework item as good and so on. These lead researchers to address these issues and modify the classical models for obtaining the optimal (EOQ/EPQ). The researchers include Lee and Rosenblatt [2], Cheng [3, 4], Chiu [5], Eroglu and Ozdemir [6], Salameh and Jaber [7], Goyal and Barron [8], Khan et al. [9].

On the other hand a closely related problem is the process targeting problem. It deals with the determination and selection of the optimal process parameters (mean and variance) to optimize a selected objective function. It is important stems from the fact that selecting the optimal parameters has impact on quality, cost and customer satisfaction. This problem is an active area of research and has attracted the interest of many researchers in the last sixty years since the first initial process targeting model of Springer [1951]. The literature survey has shown that the original model of Springer has been extended in many directions. The directions include optimizing the process parameters using single but different objective functions, optimizing the parameters of several processes in sequence, studying the impact of measurements errors and integrating process targeting with other closely related problems.

In 2006, Chen [1] presented a modified economic manufacturing quantity (EMQ) model under the imperfect product quality using Taguchi's quadratic quality loss function for measuring the product quality. The model integrates the determination of EMQ and the process target. However **Chen [1]** in his paper assumed perfect measurement system and did not address the impact of measurement errors on the optimal EMQ and process target.

In this paper, Chen [1] model is extended for obtaining simultaneously the optimum process target and economic manufacturing quantity under imperfect rework wherea measurement error exists. The motivation behind this work stems from the fact it has been shown that measurement errors have considerable impact on other quality control and production problem and need to be studied when both these problems are integrated in one model and accounted for in production planning. The rest of the paper is organized as follows: Section 2 presents the literature review followed by the statement of the problem in section 3. Section 4 presents the proposed model and Section 5 illustrates the model with a numerical example and provides sensitivity analysis to assess the impact of the measurement errors. Section 6 concludes the paper.

## 2. Literature Review

**Harris [10]** proposed the classical famous EOQ model to the world; it has been broadly applied in many places. However, there are some drawbacks in the assumption of the original EOQ model and many researchers have tried to improve it with different viewpoints, and the absence of the inventory quality is one of these shortcomings. In the classical EOQ model, there is no defect in the quality of inventory or production line. However, this assumption does not exist in the real world. The relationship between quality and EOQ model has been extensively studied over the last decade and the work by Porteus was believed to be the starting point [11]. Rosenblatt and Lee [12] concluded that the presence of defective products motivates smaller lot sizes. In a subsequent paper, Lee and Rosenblatt[2] considered using process inspection during the production run so that the shift to out-of-control state can be detected and restored earlier. Furthermore, Lee and Rosenblatt [2, 12]. Lee and Park [13] introduced some inspection and maintenance mechanisms in order to monitor the production process. They assumed that the shift of the production process follows an exponential distribution and extended it to type I inspection error. Liou et al. [14] and Makis [15] extended the Lee and Rosenblatt's [2, 12] work. They considered the shift of the production process following a general distribution, the inspection interval being arbitrary, and type I and type II inspection errors existing in the EMQ model. Tseng [16] and Makis and Fung [17] further incorporated a preventive maintenance policy into Lee and Rosenblatt's [2, 12] deteriorating production system.

Tapiero [18] links optimal quality inspection policies and the resulting improvements in the manufacturing costs. Fine [19] uses a stochastic dynamic programming model to characterize optimal inspection policies. Fine and Porteus [20] refine the original work of Porteus to allow smaller investments over time with potential process improvement of random magnitude. Chand [21] brought the learning effect into the model. In a series of papers, Cheng [22, 23] has involved the production process reliability into a classic economic order quantity model. Hong et al. [24] have established the relationship between process quality and investment. Salameh and Jaber [7] considered a special inventory situation where items, received or produced, are not of perfect quality. Taguchi's [25] redefined the product quality as the loss of society and proposed the quadratic quality loss function for measuring the quality cost. His quality loss function has been successfully applied in the on-line and off-line quality control problem.

Pulak and Al-Sultan [26] extended the application of rectifying inspection plan in determining the optimum process mean setting. In rectifying inspection plan, the 100% inspection will be executed when the lot is rejected. All the non-conforming products during the inspection stage are usually replaced by conforming ones. Recently, there are some works addressing the economic models integrating production, maintenance and quality, e.g., Lin [27], Rahim and Ohta [28], Hariga and Al-Fawzan [29], Rahim and Al-Hajailan [30].Chen [31] presented a modified EMQ model by applying the modified Al-Sultan's [32] model with Taguchi's [25] symmetric quadratic quality loss function. However, the asymmetric quadratic quality loss function maybe occurs in the industrial application. In this paper, the author presents a modified EMQ model based on the modified Al-Sultan's[32] model with Taguchi's [25] asymmetric quadratic quality loss function for obtaining the maximum expected total profit per unit of time. The EMQ, maximum inventory level, and optimum process mean will be determined simultaneously. The advantage of this integrated model is to obtain a joint control of manufacturing quantity, inventory level, and production process.

Chen [1] proposes a modified EMQ model with the producer's loss and the customer's loss. The total inventory cost of his model includes the set-up cost, the holding cost, and the product cost. The 100% inspection, perfect rework, and imperfect rework of product are considered. However, his model does not consider the situation where measurement and inspection errors are present. Ignoring measurement or inspection errors in many situations is unrealistic. This paper is an attempt to address such situations when measurement errors are present.

## 3. Problem Statement

The problem modeled in this paper is the same one presented in Chen [1] except when measuring the quality characteristic of the product there is a measurement error. The inspector in this situation does not read the true value due to the measurement error.

Chen [1] considered a production process that is producing a product with a quality characteristic y that is normally distributed. The quality characteristic is inspected for determining if it meets specification (accepted), scrapped, or rework. The quality characteristic y has a lower specification $L_2$ (LSL) and an upper specification limit $L_1$ (USL). A product is classified as conforming if $L_2 < y < L_1$, scrapped if $y < L_2$, and need to be reworked if $y > L_1$. The producer will ship the conforming units to the customers. The rework process is assumed to be imperfect .Hence, the product may be reworked once more to meet a specification and the quality characteristic of rework is the same as that of production. The quality loss of conforming units will be measured by Taguchi's [25] quadratic quality loss function. Next Chen model is presented to make the paper self contained. Chen made the following assumptions to develop his model:

1. The manufacturing system consists of a single process or machine engaged in the production of a single item.
2. There is no shortage cost.
3. Demand of the produced item is continuous and constant and all demand s must be met (production rate > demand rate).
4. The price of per unit material in production is at a fixed cost.
5. The quality characteristic of product y is normally distributed with unknown mean μ and known standard deviation $\sigma$.

The cost function of product with imperfect rework is:

$$P_{II} = \begin{Bmatrix} R + E(P_{II}), If\ y > L_1 \\ A + k(y-t)^2, If\ L_2 \le\ y \le L_1 \\ A + S, If\ y < L_2 \end{Bmatrix}$$

Hence, the expected cost of a product is:

$$E(P_{II}) = \int_{L_1}^{\infty} (R + E(P_{II}))f(y)dy + \int_{L_2}^{L_1} (A + k(y-t)^2)f(y)dy + \int_{-\infty}^{L_2} (A+S)f(y)dy \qquad (1)$$

Where $P_{II}$ is the cost function of product under the imperfect rework; $A$ is the inspection cost per unit; $k$ is the quality loss coefficient ($=\frac{R}{\Delta^2}$); $\Delta$ is the tolerance ($=| L_1 - t | = | L_2 - t |$); $R$ is the rework cost; $t$ is the target value; $S$ is the scrap cost per unit.

$$E(P_{II}) = R\int_{L_1}^{\infty} f(y)dy + E(P_{II})\int_{L_1}^{\infty} f(y)dy + A\int_{L_2}^{L_1} f(y)dy + \int_{L_2}^{L_1} (k(y-t)^2)f(y)dy + (A + S)\int_{-\infty}^{L_2} f(y)dy \qquad (1)$$

Where:

$f(y) = \frac{1}{\sqrt{2\pi}} e^{\frac{1}{2\sigma}(y-\mu)^2}$ is the normal distribution density function with mean $\mu$ and standard deviation $\sigma$. Let $z = \frac{y-\mu}{\sigma}$ then, $\varphi(z) = \frac{1}{\sqrt{2\pi}} e^{z^2}$ is the standard normal distribution density function. Now consider the following:

$\int_{-\infty}^{y} f(y)dy = \int_{-\infty}^{\frac{y-\mu}{\sigma}} \varphi(z)dz = \Phi(z)$the standard normal cumulative distribution function.

Now let's define the following:

$\alpha = \frac{L_1 - \mu}{\sigma}$ , $\delta = \frac{L_2 - \mu}{\sigma}$

$\beta = \Phi\left(\frac{L_1-\mu}{\sigma}\right) = \Phi(\alpha)$, $\gamma = \Phi\left(\frac{L_2-\mu}{\sigma}\right) = \Phi(\delta)$

Standardizing the normal distribution function to standard normal using the transformation $z = \frac{y-\mu}{\sigma}$ and $\beta$,$\gamma$

$$\therefore E(P_{II}) = A + \frac{R[1-\beta] + \int_{L_2}^{L_1}(k(y-t)^2)f(y)dy + S.\gamma}{\beta} \qquad (2)$$

Where:

$$\int_{L_2}^{L_1} (k(y-t)^2)f(y)dy \qquad (3)$$

$$= k\{[(\mu-t)^2+\sigma^2]+[\beta-\gamma] - \sigma[(\mu-2t+L_1)\varphi(\alpha)-(\mu-2t+L_2)\varphi(\delta)]\}$$

The probability of the product scrapped is:

$$\Phi\left(\frac{L_2-\mu}{\sigma}\right) + \left[1-\Phi\left(\frac{L_1-\mu}{\sigma}\right)\right]\Phi\left(\frac{L_2-\mu}{\sigma}\right) + \left[1-\Phi\left(\frac{L_1-\mu}{\sigma}\right)\right]^2\Phi\left(\frac{L_2-\mu}{\sigma}\right) + \cdots$$

$$= \frac{\Phi\left(\frac{L_2-\mu}{\sigma}\right)}{\Phi\left(\frac{L_1-\mu}{\sigma}\right)} = \frac{\gamma}{\beta} \qquad (5)$$

The probability of the product is shipped to the customers is:

$$1 - \frac{\Phi\left(\frac{L_2-\mu}{\sigma}\right)}{\Phi\left(\frac{L_1-\mu}{\sigma}\right)} = \frac{\Phi\left(\frac{L_1-\mu}{\sigma}\right) - \Phi\left(\frac{L_2-\mu}{\sigma}\right)}{\Phi\left(\frac{L_1-\mu}{\sigma}\right)} = \frac{\beta-\gamma}{\beta} \qquad (6)$$

Hence, the modified **Chen [1]** EMQ model with imperfect rework including the set-up cost, the holding cost, and the production cost is:

$$\boldsymbol{TC_{(I)} = S_t \cdot \frac{D}{Q\left[\frac{\beta-\gamma}{\beta}\right]} + 0.5 \cdot Q.\left[\frac{\beta-\gamma}{\beta}\right]\left(1-\frac{O}{I}\right).h + D.E(P_{II})} \qquad (7)$$

where $TC_{(I)}$ is the total inventory cost per unit time; $D$ is the demand quantity in units per unit time; Q is the economic manufacturing quantity; $S_t$ is the set-up cost for each production run; $O$ is the demand rate inunits per day; $I$ is the production rate in units per day, $I > O$; $h$ is the holding cost per unit item per unit time

## 4. Proposed model with measurement error

Now consider the case where the inspection system is imperfect and has error when measuring the quality characteristic. Thus, inspectors classify product based on observed value x not the true value y due to the measurement error. Both quality characteristics (the observed $X$ and the actual $Y$ ) are normally distributed and the relation between them is as follows:

$$X = Y + \varepsilon \qquad (8)$$

Where $\varepsilon$ is a random variable which represents the inspection error. $\varepsilon$ is normally distributed with mean zero and known standard deviation $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ . The correlation coefficient between the actual and observed quality characteristics ρ is given by the formula:

$$\rho = 1 - \frac{\sigma_\varepsilon^2}{\sigma_x^2} = \frac{\sigma_y^2}{\sigma_x^2} \qquad (9)$$

Since, the actual and observed quality characteristics are both normally distributed; then, their joint distribution is bi-variate normal given as follows:

$$h(x,y) = \frac{1}{2\pi\sigma_y\sigma_x\sqrt{1-\rho^2}} e^{\frac{-1}{2(1-\rho^2)}*\left(\frac{(y-\mu)^2}{\sigma_y^2} + \frac{(x-\mu)^2}{\sigma_x^2} - \frac{2*\rho*(x-\mu)*(y-\mu)}{\sigma_y*\sigma_x}\right)} \qquad (10)$$

To reduce the effect of the inspection error, instead of using the original limits ($L_1$ and $L_2$) for inspection, we based the inspection on new limits (cut off points) and use these new limits as the classification criteria (figure 1).

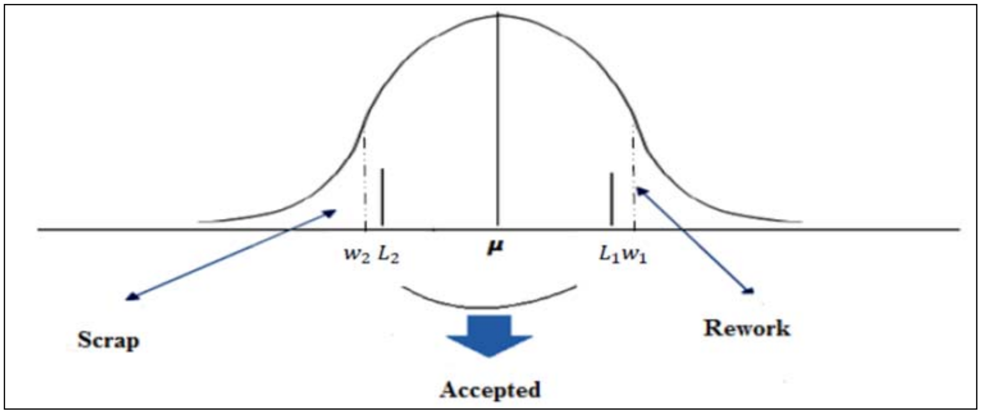


Figure 1: Cut off points for the inspection error

The location of these cut off points ($w_1$,$w_2$) depends on many factors, such as: the penalty associated with misclassifying a lower quality product with a higher quality, the value of the mean, the value of the standard deviation, etc.

Prior to model development, the types of losses and penalties associated with misclassification of the items will be described in Table 1.

Table 1: Penalties Associated with Misclassification

| Penalty | Due to | Penalty | Due to |
|---|---|---|---|
| $b_{RA}$ | Classify rework item as accepted item | $b_{AS}$ | Classify accepted item as scrap item |
| $b_{RS}$ | Classify rework item as scrap item | $b_{SA}$ | Classify scrap item as accepted item |
| $b_{AR}$ | Classify accepted item as rework item | $b_{SR}$ | Classify scrap item as rework item |

The cost function of product under perfect rework and measurement error is:

$$P_{IIe} = \begin{cases} A + R + E(P_{IIe}) & x \geq w_1, y > L_1 \\ A + b_{RA} & x \geq w_1,\ L_2 \leq y \leq L_1 \\ A + b_{RS} & x \geq w_1, y < L_2 \\ \\ A + b_{AR} & w_2 \leq x \leq w_1, y > L_1 \\ A + k(y-t)^2 & w_2 \leq x \leq w_1,\ L_2 \leq y \leq L_1 \\ A + b_{AS} & w_2 \leq x \leq w_1, y < L_2 \\ \\ A + S + b_{SR} & x < w_2, y > L_1 \\ A + S + b_{SA} & x < w_2,\ L_2 \leq y \leq L_1 \\ A + S & x < w_2, y < L_2 \end{cases}$$

Where $P_{IIe}$, is the cost of a product under-imperfect rework and measurement error. Hence, the expected cost function of product under-imperfect rework and measurement error is:

$$E(P_{Ie}) = \int_{\omega_1}^{\infty}\int_{L_1}^{\infty}[A + R + E(P_{IIe})]h(x,y)\,dyd + \int_{\omega_1}^{\infty}\int_{L_2}^{L_1}[R + b_{RA}]h(x,y)\,dyd + \int_{\omega_1}^{\infty}\int_{-\infty}^{L_2}[R + b_{RS}]h(x,y)\,dyd$$
$$+ \int_{\omega_2}^{\omega_1}\int_{L_1}^{\infty}[A + b_{AR}]h(x,y)\,dydx + \int_{\omega_2}^{\omega_1}\int_{L_2}^{L_1}[A + k(y-t)^2]h(x,y)\,dydx$$
$$+ \int_{\omega_2}^{\omega_1}\int_{-\infty}^{L_2}[A + b_{AS}]h(x,y)\,dydx + \int_{-\infty}^{\omega_2}\int_{L_1}^{\infty}[A + S + b_{SR}]h(x,y)\,dydx$$
$$+ \int_{-\infty}^{\omega_2}\int_{L_2}^{L_1}[A + S + b_{SA}]h(x,y)\,dydx$$
$$+ \int_{-\infty}^{\omega_2}\int_{-\infty}^{L_2}[A + S]h(x,y)\,dydx \quad (11)$$

Now, add the similar term together:

$$E(P_{Ie}) = \frac{A + R\int_{\omega_1}^{\infty}\int_{L_1}^{\infty}h(x,y)\,dydx + b_{RA}\int_{\omega_1}^{\infty}\int_{L_2}^{L_1}h(x,y)\,dydx + b_{RS}\int_{\omega_1}^{\infty}\int_{-\infty}^{L_2}h(x,y)\,dyd}{\left[1 - \int_{\omega_1}^{\infty}\int_{L_1}^{\infty}h(x,y)\,dyd\right]}$$
$$+ \frac{\int_{\omega_2}^{\omega_1}\int_{L_2}^{L_1}[k(y-t)^2]h(x,y)\,dydx + b_{AR}\int_{\omega_2}^{\omega_1}\int_{L_1}^{\infty}h(x,y)\,dydx + b_{AS}\int_{\omega_2}^{\omega_1}\int_{-\infty}^{L_2}h(x,y)\,dydx}{\left[1 - \int_{\omega_1}^{\infty}\int_{L_1}^{\infty}h(x,y)\,dyd\right]}$$
$$+ \frac{S\int_{-\infty}^{\omega_2}\int_{-\infty}^{\infty}h(x,y)\,dydx + b_{SA}\int_{-\infty}^{\omega_2}\int_{L_2}^{L_1}h(x,y)\,dydx + b_{SR}\int_{-\infty}^{\omega_2}\int_{L_1}^{\infty}h(x,y)\,dydx}{\left[1 - \int_{\omega_1}^{\infty}\int_{L_1}^{\infty}h(x,y)\,dyd\right]} \quad (12)$$

$$E(P_{Ie}) = \frac{A + S.\xi + R.\int_{\omega_1}^{\infty}\int_{L_1}^{\infty}h(x,y)\,dydx + b_{RA}\int_{\omega_1}^{\infty}\int_{L_2}^{L_1}h(x,y)\,dydx + b_{RS}\int_{\omega_1}^{\infty}\int_{-\infty}^{L_2}h(x,y)\,dyd}{\left[1 - \int_{\omega_1}^{\infty}\int_{L_1}^{\infty}h(x,y)\,dyd\right]}$$
$$+ \frac{\int_{\omega_2}^{\omega_1}\int_{L_2}^{L_1}\left[k(y-t)^2\right]h(x,y)\,dydx + b_{AR}\int_{\omega_2}^{\omega_1}\int_{L_1}^{\infty}h(x,y)\,dydx + b_{AS}\int_{\omega_2}^{\omega_1}\int_{-\infty}^{L_2}h(x,y)\,dydx}{\left[1 - \int_{w_1}^{\infty}\int_{L_1}^{\infty}h(x,y)\,dyd\right]}$$
$$+ \frac{b_{SA}\int_{-\infty}^{\omega_2}\int_{L_2}^{L_1}h(x,y)\,dydx + b_{SR}\int_{-\infty}^{\omega_2}\int_{L_1}^{\infty}h(x,y)\,dydx}{\left[1 - \int_{\omega_1}^{\infty}\int_{L_1}^{\infty}h(x,y)\,dyd\right]} \quad (13)$$

Where:

$\omega = \frac{w_1-\mu}{\sqrt{\sigma_y^2+\sigma_e^2}}$, $\eta = \frac{w_2-\mu}{\sqrt{\sigma_y^2+\sigma_e^2}}$

$\Omega = \Phi\left(\frac{w_1-\mu}{\sqrt{\sigma_y^2+\sigma_e^2}}\right) = \Phi(\omega)$, $\xi = \Phi\left(\frac{w_2-\mu}{\sqrt{\sigma_y^2+\sigma_e^2}}\right) = \Phi(\eta)$

The probability of the product is shipped to the customers is ($\frac{\Omega-\xi}{\xi}$). Hence, the expected total cost of modified EMQ model with imperfect production process quality and inspection process including the set-up cost, the holding cost, and the production cost is:

$$ETC2 = S_t \cdot \frac{D}{Q.\left(\frac{\Omega-\xi}{\xi}\right)} + 0.5\,.\,Q.\left(\frac{\Omega-\xi}{\xi}\right).\left(1-\frac{O}{I}\right).h + D.E(P_{IIe}) \quad (14)$$

## 5. Numerical Example and Sensitivity Analysis

Consider a production process, which produces a product that have a normally distributed quality characteristic y with unknown mean $\mu$. A product will be inspected for determining if it is accepted, scrap,

or rework. A product whose quality characteristic falls between the two limits (10<$y$ <15) is accepted, while a product with quality characteristic below lower specification limit ($y$ <10) is scrap with cost \$1. Finally, a product whose quality characteristic fall above the upper specification limit ($y$ >15) need to be reworked with cost \$2. The process standard deviation is 1.3.The inspection cost per item A=0.2.The error in the measurement system is represented by the correlation coefficient having the value $\rho$=0.85, i.e.$\sigma_\varepsilon$=0.557 and $\sigma_x$=1.414. The uniform search over $w_2 \in$ [8, 12] and $w_1 \in$ [13, 17] is conducted. Knowing that:
I=100,O=80, $S_t$ =20, $h$=1, D=2000 $b_{RA} = 10, b_{RS} = 10, b_{AR} = 70, b_{AS} = 70, b_{SA} = 70, b_{SR} = 60$. Chosen of penalties take a lot of our consideration because we aim to prevent our product from customer loss cost, so the penalties associated with classifying accepted item as rework or scrap items is chosen more higher cost than other penalties. For the numerical analysis, 'NLPSolve' command of Maple 12 software programming is used. Table (2), summarize the results.

Table 2: Proposed model with error

| | Proposed model with error |
|---|---|
| $Q*$ | 695 |
| $\mu*$ | 12.41 |
| $w_1^*$ | 14.921 |
| $w_2^*$ | 10.120 |
| $ETC$ | 36,171.3 |

Table (3), summarize the results of Chen [1] model.

Table 3: Chen [1] model with error-free

| | Proposed model with error |
|---|---|
| $Q*$ | 653 |
| $\mu*$ | 12.43 |
| $ETC$ | 1576 |

It is clear that the expected total cost under measurement error is greater than when error-free, the reason is a penalty cost is added when the system under measurement error, which lead to increasing economic manufacturing quantity ,consequently, increasing the expected total cost.

The sensitivity analysis for the correlation coefficient ρ and the penalty costs is conducted, to study their effect on the model and the results. First, the effect of the correlation coefficient between actual quality characteristic y and the observed quality characteristic x is studied. Table (3); below show the effect of the correlation coefficient on the modified Chen [1] model.

Table 4: Sensitivity analysis of the correlation coefficient on the Chen [1] model with measurement error

| $\rho$ | Chen [1] model with perfect rework and measurement error. | | | | | |
|---|---|---|---|---|---|---|
| | $w_1$ | $w_2$ | $\mu$ | $Q$ | $ETC1$ | Change percentage |
| 0.95 | 14.921 | 10.120 | 12.41 | 689 | 28,890.0 | -20.13% |
| 0.9 | 14.921 | 10.120 | 12.41 | 692 | 32,800.1 | -9.32% |
| **0.85(original)** | **14.921** | **10.120** | **12.41** | **695** | **36,171.3** | **0%** |
| 0.8 | 14.921 | 10.120 | 10.67 | 697 | 39,622.0 | 9.54% |
| 0.75 | 14.921 | 10.120 | 10.67 | 699 | 43,456.2 | 20.14% |

It is clear that as the correlation coefficient ρ increases the error standard deviation decreases as well. Therefore, as the correlation coefficient value increased the deviation between the actual and observed quality characteristics is decreased and approaches zero. Hence, the model tends to be closer to the model in chapter three with no inspection error. The higher the value of the correlation coefficient, the lower value for the expected total cost, because, if the correlation coefficient value is high then, more produced items are classified correctly according to their quality characteristic values therefore, no more penalty cost is going to be paid. While the small value of the correlation coefficient means more produced items are misclassified due to the high deviation between the actual and observed quality characteristics. Hence, more penalties are going to be paid which resulting in more loss which increase the expected total cost and more variability between the produced items.

## 6. Conclusion

In this paper, **Chen [1]** model is extended for the systems with measurement error. The proposed model is illustrated via a numerical example from the literature. Sensitivity analysis for the correlation coefficient between the actual and observed quality characteristics has been conducted to assess the impact of the errors on the optimal economic manufacturing quantity, process target mean and the expected total cost values. Further work is needed to consider asymmetric quadratic quality loss function for measuring the product quality. Also the work in this paper can be extended to multi-stage production processes.

## 7. References


1. Chen, C., 2006,“ The optimum selection of imperfect quality economic manufacturing quantity and process mean by considering quadratic quality loss function,” Journal of the Chinese Institute of Industrial Engineers 23, 12–19.
2. Lee, H., and Rosenblatt, M., 1987, “Simultaneous determination of production cycles and inspection schedules in a production system,” Management. Science, 33, 1125–1137.
3. Cheng, T., 1989, “An economic production quantity model with flexibility and reliability considerations,” European Journal of Operation Research, 39,174–179.
4. Cheng, T., 1991, “Economic order quantity model with demand-dependent unit production cost and imperfect production processes,”IIE Transactions, 23, 23–28.
5. Chiu, Y., 2003, “Determining the optimal lot size for the finite production model with random defective rate, the rework process, and backlogging,” Engineering Optimization, 35 (4), 427–437.
6. Eroglu, A., and G., Ozdemir, 2007, “An economic order quantity model with defective items and shortages”, International Journal of Production Economics, 106(2), 544-549.
7. Salameh, M., and Jaber, M., 2000, “Economic production quantity model for items with imperfect quality”, International Journal of Production Economics, 64(3), 59-64.
8. Goyal, S., and L. E. Cardenas-Barron, L., 2000, “Note on: Economic production quantity model for items with imperfect quality –a practical approach”, International journal of production economics 77(1), 85-87.
9. Khan, M., Jaber and, M., and Wahab, M., 2010, “Economic order quantity model for items with imperfect quality with learning in inspection”, International journal of production economics, 124(1), 87-96.
10. Harris, F., 1913, “How many parts to make at once,”Factory Management, 10 (152), 135–136.
11. Porteus, E., 1986, “Optimal lot-sizing process quality improvement and setup cost reduction,” Operation Research, 34, 137–144.
12. Rosenblat, M., and Lee, H., 1986, “Economic production with imperfect production processes,” IIE Transactions, 18, 48–55.
13. Lee, J., and Park, K., 1991, “Joint determination of production cycle and inspection intervals in a deteriorating production system,” Journal of the operational Research Society,42, 775-783.
14. Liou, M., Tseng, S., and T. M. Lin, 1994, “The effects of inspection errors to the imperfect EMQ Model,” IIE Transactions, 26, 42-51.
15. Makis, V., 1998, “Optimal lot sizing and inspection policy for an EMQ model with imperfect inspections,” Naval Research Logistics, 45, 165-186.
16. Tseng, S., 1996, “Optimal preventive maintenance policy for deteriorating production systems,” IIETransactions, 28, 687-694.
17. Makis V., and Fung, J., 1998, “An EMQ model with inspections and random machine failures,” Journal of the Operational Research Society, 49, 66-76.
18. Tapiero, C., 1987, “Production learning and quality control,” IIE Transactions, 19, 362–370.
19. Fine, C., 1988, “A quality control model with learning effects,” Operation Research, 36, 437–444.
20. Fine, C., and Porteus, E., 1989, “Dynamic process improvement,” Operation Research, 37, 580–591.

21. Chand, S., 1989, “Lot sizes setup frequency with learning in setups and process quality,” European Journal of Operation Research, 42, 190–202.
22. Cheng, T., 1989, “An economic production quantity model with flexibility and reliability considerations”,European Journal of Operational Research,39(2), 174-179.
23. Cheng, T., 1991, “An economic order quantity model with demand-dependent unit production cost and imperfect production processes”,IIE transactions, 23(1), 23-28.
24. Hong, J., Xu, S., and Hayya, J., 1993, “Process quality improvement and setup reduction in dynamic lot-sizing,” Int. J. Prod. Res. 31, 2693–2708.
25. Taguchi, G., 1986, “Introduction to Quality Engineering, Asian Productivity Organization,” Tokyo, Japan.
26. Pulak, M., Al-Sultan, K., 1996, “The optimum targeting for a single filling operation with rectifying inspection,” Omega 24, 727–733.
27. Lin, C., 2004, “Optimization of maintenance, production and inspection strategies while considering preventative maintenance error,” Journal of Information & Optimization Sciences, 25, 543–555.
28. Rahim, M., Ohta, H., 2005, “An integrated economic model for inventory and quality control problems,” Engineering Optimization, 37, 65–81.
29. Hariga, M., Al-Fawzan, M., 2005, “Joint determination of target value and production run for a process with multiple markets,” International Journal of Production Economics, 96, 201–212.
30. Rahim, M., Al-Hajailan, W., 2006,“An optimal production run for an imperfect production process with allowable shortages and time varying fraction defective rate,” International Journal of Advanced Manufacturing Technology, 27, 1170–1177.
31. Chen, C., and Lai, M., 2007,“ Determining the optimum process mean based on quadratic quality loss function and rectifying inspection plan, ”European Journal of Operational Research, 182(2), 755-763.
32. Al-Sultan, K., 1994, “An algorithm for the determination of the optimum target values for two machines in series with quality sampling plans”, International Journal of Production Research, 32, 37-45.

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