

Extend Optimal Replacement Model for a Deteriorating Systems with Inspections

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Abstract

In this study, we propose a generalized replacement model for a deteriorating system with failures that could only be detected through inspection work. The system is assumed to have two types of failures and is replaced at the N th type I failure (minor failure) or first type II failure (catastrophic failure) or at the working age T , depending on whichever occurs first. The probability of type I and II failure depends on the number of failures since the last replacement. Such systems can be repaired upon type I failure, but are stochastically deteriorating, that is, the lengths of the operating intervals are stochastically decreasing, whereas the durations of the repairs are stochastically increasing. Then, the expected net cost rate is obtained.

Keywords

Stochastic deterioration; Non-homogeneous Poisson process; Inspection.

Introduction

In most replacement models, it is typically assumed that the repaired item is returned to an “as good as new” state (perfect repair). However, for a repairable deteriorating system, it is frequently more realistic to assume that a failed item may return to an “as good as old” state (imperfect repair) upon repair. That is, if the original life distribution of the item when it was brand new was F , then the item upon repair may have survival function $\bar{F}_t$, where t is its age at failure and $\bar{F}_t(x) = \bar{F}(t+x)/\bar{F}(t)$. This is also known as minimal repair (Barlow and Proschan, 1965, 1975). Brown and Proschan (1983) examined a maintenance action, called imperfect repair, that with probability p is a perfect repair and with probability $1-p$ is a minimal repair. Their model has been generalized by Block et al. (1985) to the case in which the probability of perfect repair is age-dependent. These results have been generalized by Sheu and Griffith (1991) to the multi-item case.

If the items are stochastically deteriorating, the successive operating intervals after repairs become shorter and in view of the ageing and accumulative wear, the mean lengths of the repair durations become longer. Thus, an alternative approach is to use some kind of monotone process. Hence one of modeling the deteriorating systems is to use the Non-Homogeneous Poisson Process (NHPP). Baxter (1982), Gupta and Kirmani (1988), and Kochar (1996) have shown that if the failure rate function is non-decreasing (non-increasing) of the NHPP, the operating intervals are stochastically non-increasing (non-decreasing). Lam (1988) introduced the other one of modeling the deteriorating systems. He studied the geometric process replacement model in which the successive operating

interval $\{X_i, i = 1, 2, \dots\}$ of an item constitutes a non-increasing geometric process and the consecutive repair durations $\{Y_i, i = 1, 2, \dots\}$ constitute a non-decreasing geometric process.

In order to avoid the failure of an item during actual operation, maintenance and replacement policies are needed. There were many studies before. Makabe and Morimura (1963a, 1963b, 1965) proposed a replacement model where a system is replaced at the N th failure; they also discussed the optimal policy. This policy has been generalized by Morimura (1970), Nakagawa (1981), and Sheu and Griffith (1996). These authors have assumed that a failed system will function again upon repair, but without affecting its failure rate, that is, the failure rate will remain undisturbed after repair. They also assume that repair times are negligible. They used the NHPP for modeling these deteriorating systems.

In maintenance and replacement problems, it is usually assumed that failures are detected and repaired simultaneously. However, in some systems, failures are identified through inspection work. Cheng and Li (2012) studied a geometric process repair model with inspections from Lam (1988). Hence our research is motivated by theirs.

General model

We consider a generalized replacement model according to the follow scheme. We assume that the original system begins operating at time 0, that the system will have two types of failures, and that the failure will only be detected through inspection work. Type I failure (minor failure) is eliminated by repair, whereas when a type II failure (catastrophic failure) occurs, an entire unit has to be replaced. A system will be replaced at the N th type I failure (minor failure) or first type II failure (catastrophic failure) or at the working age T , depending on whichever occurs first. The probability of type II failure is now permitted to depend on the number of failures since the last replacement. Let M count the number of failures until the first type II failure occurs. Let $\bar{P}_k = P(M > k)$. That is, $\bar{P}_k$ is the probability that the first k are type I failures. We assume throughout that the domain of $\bar{P}_k$ is $\{0, 1, 2, \dots\}$ and that $1 = \bar{P}_0 \geq \bar{P}_1 \geq \dots$. We use the notation $\{\bar{P}_k\}$ as the abbreviation for a sequence of probabilities. The sequence $\{\bar{P}_k\}$ is supposed to be known. Let $p_k = P(M = k)$; then, we know that $p_k = P(M = k) = \bar{P}_{k-1} - \bar{P}_k = \bar{P}_{k-1} \left(1 - (\bar{P}_k / \bar{P}_{k-1})\right)$. Hence, if the k th failure occurs, it is categorized as either a type I failure with probability $q_k = \bar{P}_k / \bar{P}_{k-1}$ or a type II failure with probability $\theta_k = 1 - (\bar{P}_k / \bar{P}_{k-1})$.

Let X_i be the operating interval of the system after the $(i - 1)$ th repair. The cumulative distribution function of is $H_i(t)$. We assume that $\{X_i, i = 1, 2, \dots\}$ forms a stochastically decreasing sequence with decreasing means $E(X_i) = \lambda_i$. Let Y_i be the repair duration after the detection of the i th failure. Further, we assume that $\{Y_i, i = 1, 2, \dots\}$ forms a stochastically increasing sequence with increasing means $E(Y_i) = \mu_i$. We assume that $\{X_i, i = 1, 2, \dots\}$ and $\{Y_i, i = 1, 2, \dots\}$ are two independent sequences of independent, non-negative random variables. Let $\gamma^{i-1}h$ be the time period between two successive inspections in the i th repair cycle where h is a real number greater than zero. Let G_i be the idle period of the system when the i th failure occurs until it is detected. We can calculate $E(G_i)$ that is denoted as the mean idle period of the system when the i th failure occurs until it is detected by using the total probability decomposition method. We obtain

$$E(G_i) = \gamma^{i-1}h \sum_{g=0}^{\infty} \bar{H}_i(g\gamma^{i-1}h) - E(X_i) \quad (1)$$

where $\bar{H}_i(g\gamma^{i-1}h) = 1 - H_i(g\gamma^{i-1}h)$.

Let $Q_{(k,1)}$ be the number of inspections during the operating period of the system after the $(k-1)$ th repair when the system is finally replaced at the N th type I failure. We calculate the mean value of $Q_{(k,1)}$, that is, $E[Q_{(k,1)}]$ in the following manner:

$$E[Q_{(k,1)}] = \sum_{g=0}^{\infty} \bar{H}_k (g\gamma^{k-1}h) \quad (2)$$

Let $Q_{(k,2)}$ be the number of inspections during the operating period of the system after the $(k-1)$ th repair when the system is finally replaced at the first type II failure. We calculate the mean value of $Q_{(k,2)}$, that is $E[Q_{(k,2)}]$, in the following manner:

$$E[Q_{(k,2)}] = \sum_{g=0}^{\infty} \bar{H}_k (g\gamma^{k-1}h) \quad (3)$$

Let $Q_{(k,3)}$ be the total number of inspections when the system is finally replaced at the working age T for $1, 2, \dots, N$. We calculate the mean value of $Q_{(k,3)}$, that is $E[Q_{(k,3)}]$, in the following manner:

$$E[Q_{(k,3)}] = \sum_{i=1}^{k-1} \sum_{g=0}^{\infty} \bar{H}_i (g\gamma^{i-1}h) + \sum_{g=1}^{\infty} [F_{k-1}(T - g\gamma^{k-1}h) - P(\sum_{i=1}^{k-1} X_i = T - g\gamma^{k-1}h)] \quad (4)$$

The unit of repair cost rate is C_1 , the unit of reward rate whenever the system is operating is C_2 , the replacement cost is C_3 , the unit of penalty cost rate for the idle period is C_4 , and each inspection cost is C_5 .

Formulation

Let W_i be the length of the i th successive replacement cycle, and R_i be the operational cost over the renewal interval W_i for $i = 1, 2, 3, \dots$. Hence, $\{(W_i, R_i)\}$ constitutes a renewal reward process. If $D(t)$ denotes the expected cost of operating the unit over time interval $[0, t]$, then it is obvious that

$$\lim_{t \rightarrow \infty} \frac{D(t)}{t} = \frac{E(R_1)}{E(W_1)} \quad (5)$$

(see, e.g., Ross, 1970). We denote the right-hand side of Eq. (5) by $B(N, T, h)$. Let $U_n = \sum_{i=1}^n X_i$. Let $F_k(t)$ denote the distribution function of U_k . We shall use the convention that $F_0(t) = 1$ and $\bar{F}_0(t) = 1 - F_0(t) = 0$ for $t \geq 0$. For convenience, let $\sum_{i=1}^0 \mu_i = 0$. Thus, the expected length of a replacement cycle is given by

$$E[W_1] = \sum_{k=1}^N \bar{P}_{k-1} \int_0^T (F_{k-1}(t) - F_k(t)) dt + \sum_{k=1}^{N-1} \mu_k \bar{P}_k F_k(T) + \sum_{k=1}^N E[G_k] \bar{P}_{k-1} F_k(T) \quad (6)$$

Now, we can derive the expression for $E[R_1]$.

$$E[R_1] = C_3 + C_1 \sum_{k=1}^{N-1} \mu_k \bar{P}_k F_k(T) - C_2 \sum_{k=1}^N \bar{P}_{k-1} \int_0^T (F_{k-1}(t) - F_k(t)) dt + C_4 \sum_{k=1}^N E[G_k] \bar{P}_{k-1} F_k(T) +$$

$$C_5 \left[\sum_{k=1}^N \sum_{g=0}^{\infty} \bar{H}_k(g\gamma^{k-1}h) \bar{P}_{k-1} F_k(T) + \sum_{k=1}^N \sum_{g=1}^{\infty} [F_{k-1}(T - g\gamma^{k-1}h) - P(\sum_{i=1}^{k-1} X_i = T - g\gamma^{k-1}h)] \bar{P}_{k-1} (F_{k-1}(T) - F_k(T)) \right] \quad (7)$$

Hence, the expected net cost per unit time is given by

$$B(N, T, h) = \frac{\left\{ \begin{aligned} &C_3 + C_1 \sum_{k=1}^{N-1} \mu_k \bar{P}_k F_k(T) + C_4 \sum_{k=1}^N E[G_k] \bar{P}_{k-1} F_k(T) + \\ &C_5 \left[\sum_{k=1}^N \sum_{g=0}^{\infty} \bar{H}_k(g\gamma^{k-1}h) \bar{P}_{k-1} F_k(T) + \sum_{k=1}^N \sum_{g=1}^{\infty} [F_{k-1}(T - g\gamma^{k-1}h) - P(\sum_{i=1}^{k-1} X_i = T - g\gamma^{k-1}h)] \bar{P}_{k-1} (F_{k-1}(T) - F_k(T)) \right] \\ &- C_2 \sum_{k=1}^N \bar{P}_{k-1} \int_0^T (F_{k-1}(t) - F_k(t)) dt \end{aligned} \right\}}{\sum_{k=1}^N \bar{P}_{k-1} \int_0^T (F_{k-1}(t) - F_k(t)) dt + \sum_{k=1}^{N-1} \mu_k \bar{P}_k F_k(T) + \sum_{k=1}^N E[G_k] \bar{P}_{k-1} F_k(T)} \quad (8)$$

Conclusions

In this study, we propose a generalized replacement model for a deteriorating system with failures that could only be detected through inspection work. The system is assumed to have two types of failures and is replaced at the N th type I failure (minor failure) or first type II failure (catastrophic failure) or at the working age T , depending on whichever occurs first. The expected net cost rate was also obtained. The optimal N , the optimal T and the optimal time interval of inspections $\gamma^{i-1}h^*$ in the i th repair cycle that would minimize the cost rate were discussed. We hope that this generalized replacement model can be used for approaching the practical applications such as oil and gas industries, medicine and even nuclear power plants.

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Biography

Shey-Huei Sheu received the M.Sc. degree in applied mathematics from the National Tsing Hua University, Hsinchu City, Taiwan, in 1979, and the Ph.D. degree in statistics from the University of Kentucky, Lexington, USA, in 1987. He is currently a Chair Professor of the Department of Statistics and Informatics Science at the Providence University. He was a Chair Professor of the Industrial Management Department at the National Taiwan University of science and Technology. He has published over one hundred and thirty papers in several prestigious journals in the fields of statistics, reliability, operations research, and systems engineering.

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