A Relax-and-Fix Heuristic for a Production Planning Problem with Order Acceptance and Flexible Due Dates

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Abstract

We present a new production planning problem which integrates order acceptance decisions with due date flexibility while taking into consideration realistic production capacity constraints. The problem consists of choosing among a set of customer orders which ones to accept based on the profit of the orders and the available resource capacity. Realistic capacities are considered by linking the lead times to the utilization and explicitly considering the work-in-process inventory. Furthermore, due dates of customer orders are given as time intervals within which the order can be satisfied without penalty. A mixed integer linear programming formulation (MILP) is presented. The problem is solved using a two MILP heuristics: a reversals heuristic and a relax-and-fix heuristic. The performance of the heuristics is compared with that of a state-of-the-art commercial solver applied to the MILP formulation. The relax-and-fix heuristic outperforms the reversals heuristic and the solver solution as it yields the same integrality gaps for much less CPU effort. Furthermore, the performance of the heuristic can be adjusted using two parameters in such a way that one can sacrifice some CPU time to seek better solutions.

Keywords
Production Planning; Order Acceptance; Clearing Functions; Load-Dependent Lead Times, Flexible Lead Times;

1. Introduction
In traditional production planning models, two fundamental assumptions are made: *(i)* the production lead times do not dependent on the production workload, *(ii)* the cost of adding one unit (or order) to the production stage is zero as long as the capacity limit is not reached. As a consequence of these assumptions, production planning models try to satisfy as many customer orders with known due dates as production capacity permits. Because these assumptions do not hold, it might be more profitable if not all orders are accepted even if capacity is available. This can be achieved by integrating order acceptance and production planning decisions in a single model. In addition to that, when due date flexibility is allowed, i.e., the due date required by the customer is given as a set of possible dates rather than a fixed date, a win-win situation for the firm and customers can be achieved. In fact, this flexibility, when captured in production planning models results, in more accepted orders, higher profits, and more reliable due dates.

Production lead-times (the time required for material released into the production system to be transformed into finished goods that can be used to meet demand) depend on the production workload or the utilization level. Using queuing models, it was shown that lead-time increases non-linearly as the resource utilization approaches 100% (Buzacott and Shanthikumar, 1993 and Hopp and Spearman, 2001). This creates a circular, non-linear dependency between lead-time and utilization. Therefore, the more orders accepted the higher are the production lead times resulting in the possibility of missing customer due dates. This means that the planner can be faced with situations where production capacity is available but the next orders should be rejected in order not to delay some already accepted customer orders.

Furthermore, even if the unit price the customer pays exceeds the variable production cost and there is enough capacity to avoid shortage, it is not always clear that all orders should be accepted. There are two possible arguments for this fact. The first reason has to do with economies of scale. In fact, in the case of high fixed or set-up costs it might not be economical to satisfy a single order of a small quantity. The order must be aggregated with additional orders to justify the production setup (Geunes et al., 2006). The second reason has to do with the workload of the production stage. Kefeli et al. (2011) show that the marginal prices of capacitated resources are not necessarily equal to zero when the utilization is less than one. This means that even in the case where capacity is available, the revenue from an additional order should at least offset the variable production cost plus the dual of the capacity constraints that take into account workload. Therefore, models that integrate production planning decisions and order acceptance decisions have a great potential to improve the overall profitability of the firm.

The production planner in a capacitated production facility receives customer orders characterized by a reservation price (marginal revenue), quantity, and a specific due date. The planner needs to accept a set of orders and decide on the production plan to satisfy these accepted orders over a planning horizon of *T* periods. A realistic situation in this paper consists of considering a specific customer that takes as an input the promised due date. The promised due date in our situation is given to the customer as an output of the integrated model presented in this paper.

The dependency between resource utilization and lead times (or equivalently available capacity) has been addressed to some degree by some authors. Voss and Woodruff (2003) propose a nonlinear model where the function linking lead time to workload is approximated as a piecewise linear function. Ettl et al. (2000) take a similar approach, adding a convex term representing the cost of carrying work-in-process (WIP) as a function of workload to the objective function. Graves (1986), Karmarkar (1989), Missbauer (2002), and Asmundsson et al. (2006; 2009) use clearing functions (CFs) to model the dependency between workload and lead times. Several related models are proposed in the recent book by Hackman (2008). Pahl et al. (2005) and Missbauer and Uzsoy (2010) review production planning models with load-dependent lead times. Aouam and Uzsoy (2012) compare the performance of various production planning models with workload-dependent lead times under demand uncertainty. In this paper, a CF is used to model the capacity of the production stage in order to relate the production workload resulting from all accepted orders to the production lead-times.

Ivanescu et al. (2002) consider the order acceptance problem in the batch industries where the processing times are uncertain. The authors use regression based models in order to determine whether there is enough capacity to accept a customer order with the due date requested by the customer. Markov decision models are used by Defregger and Kuhn (2005) to decide about the orders to accept or to reject in a planning process over a number of periods. Geunes et al. (2002) consider a production planning problem with order acceptance and call it the "order selection" problem. This problem is a single item lot sizing problem with and without resource capacity constraints. The uncapacitated case is solved using a polynomial time algorithm and propose a Lagrangian relaxation approach for the capacitated case. For a more extensive review of order acceptance literature the reader is referred to Slotnick (2011). Aouam and Brahimi (2013) present a robust model that integrates production planning with load dependent lead-times and order acceptance decisions, which considers demand uncertainty and where a fraction of the order quantity can be accepted. They show that integrating the two decisions provides the planner with the flexibility to select the orders to be satisfied fully or partially. This flexibility enables the planner to maintain release quantities and utilization at desirable levels, which leads to high profits and high levels of customer satisfaction.

In this paper, the production planning problem with load dependent lead-times that integrates order acceptance decisions and flexibility in customer order due dates is formulated and solved. When an order is accepted it is scheduled over a planning horizon of *T* periods and incurs production costs and eventually inventory holding costs. The rejection of a customer order results however in a lost sale cost. By introducing due dates flexibility, the proposed model determines optimal periods to satisfy the customer order taking into account the workload resulting from all accepted customer orders. This flexibility in turn results in more customer orders satisfied. To quantify these benefits, the proposed model is compared to an integrated production planning model with order acceptance considering fixed due dates and lost sales (not due date flexibility).

The proposed integrated production planning and order decisions problem with flexible due dates is formulated as a mixed integer linear program (MILP). When the number of orders increases and/or for certain parameter settings it becomes difficult if not impossible to obtain good solutions in reasonable computation times. Therefore, an efficient relax-and-fix heuristic is proposed to solve the problem. Compared to other implementations of relax-and-fix heuristics for production planning problems (e.g. Stadtler, 2003; Federgruen et al., 2007), the decomposition in our heuristic is based on the orders instead of time periods.

The remainder of the paper is organized as follows. In section 2, CF basics are introduced and the production planning problem is formulated. In Section 3, heuristic solution approaches are presented. Numerical experiments and results are given in Section 4. The work is concluded in Section 5.

2. Production Planning with Load-Dependent Lead Times

2.1 Clearing function basics

Clearing functions (CFs) (Graves 1986; Srinivasan et al. 1988; Karmarkar 1989; Missbauer and Uzsoy 2010) express the expected throughput of a capacitated resource over a given period of time as a function of workload of the resource over that period, which, in turn, is defined by the amount of work available for the resource to process in that period. Figure 1, derived from Karmarkar (1989), depicts four examples of CFs considered in the literature to date, where X denotes the expected throughput in a planning period as a function of the WIP (W). The horizontal line X = C represents the regular capacity constraint that is implemented in most LP models but is supplemented with a fixed lead time. The linear CF of Graves(1986) is represented by the X=W/L line, which implies a lead time of L periods that is maintained independently of the WIP level. Given that the same proportion 1/L, for example 25%, of the WIP is always produced (cleared from the stage), the last unit to enter the stage and hence added to the WIP should wait L = 4 periods in the stage before it is produced. If a fixed lead time is maintained up to a certain maximum output, then X = min{W/L, C}. When queue delays are neglected, the lead time is equal to the average processing time p, then X = W/p. Assuming that lead time is equal to the average processing time up to a maximum output level gives the "Best Case" model X = min {W/p, C} of Hopp and Spearman (2001). The traditional LPs with fixed lead time differ from the linear CF of Graves in that the latter links the throughput to the WIP level leading to smoother production plans (Orcun et al. 2006). The CF always lies below the X = W/p and X =C lines. For most capacitated production resources subject to congestion, limited capacity leads to a saturating (concave) shape of the CF. Asmundsson et al. (2006; 2009) and Missbauer (2011) study the problem of estimating CFs from experimental data, obtained either from industry or from simulation models. Missbauer and Uzsoy (2010) review the state of the art in this area.

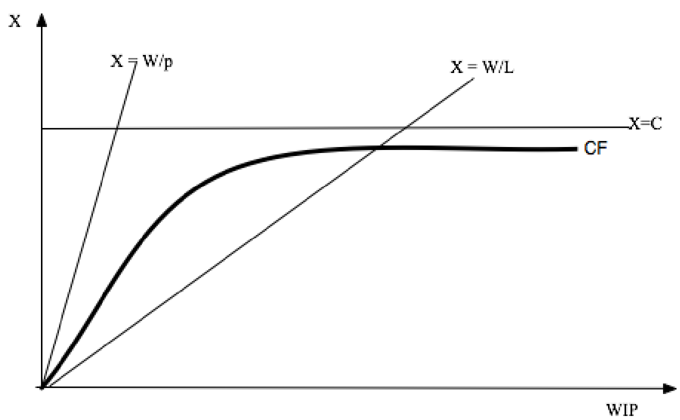

Figure 1: Examples of CFs (Karmarkar 1989).

2.2 Problem presentation and formulation

In production planning with order acceptance and flexible due dates (PP-OA-FDD), each customer order i is characterized by an order size q_i, reservation price or marginal revenue π_i. When due date flexibility is allowed, i.e., the due date required by the customer is given as a set of possible dates rather than a fixed date, a win-win situation for the firm and customers can be achieved. In fact, this flexibility when captured in production planning models results in more accepted orders, higher profits, and reliable due dates (lower delays). In this setting, a customer provides a time window with earliest delivery date e_i and a latest delivery date f_i which corresponds to a flexibility interval of size $\delta_i = f_i - e_i + 1$ periods. If an order is prepared before e_i, it will be kept in stock and a per unit inventory holding cost h_t is incurred. However, an order cannot be satisfied after f_i. Notice that this situation is similar to that presented by Lee et al. (2001) who considered aggregate models with and without backlogging. If an order is not satisfied, a lost sales penalty cost l_i is incurred. The PP-OA-FDD problem is also characterize by the quantity released R_t, the production level X_t, the Work-In-Process (WIP) level W_t, the inventory level I_t, as decision variables. Besides the inventory holding cost, the other marginal costs are: release cost r_t, processing cost c_t, and WIP cost w_t.

In order to model the high utilization mode, a CF, denoted by $f(.)$, that is increasing and concave with $f(0) = 0$ to relate the throughput to the WIP is defined as follows,

$$X_t = f(\bar{W}_t) \qquad \forall t$$

where $\bar{W}_t = W_{t-1} + R_t$ represents the resource load for period *t*, or the total amount of work that becomes available for processing during the period. Following Asmundsson et al. (2006), Missbauer (2002) and for tractability reasons, the CF is approximated using an outer linearization. In fact, $f(.)$ can be approximated by the convex hull of a set of affine functions of the form,

$$\hat{f}(W) = min_{j=1\ldots J}\{a_j W + b_j\}$$

where a_j is a strictly decreasing series and $b_1 = 0$.

Let the binary variable S_{it} such that $S_{it} = 1$ if order *i* is accepted and to be satisfied in period $t \in [e_i, f_i]$ and $S_{it} = 0$ otherwise. The integrated production planning and order acceptance model with flexible due dates can be formulated as follows:

PP-OA-FDD

$$Max\ Profit = \sum_i \pi_i q_i \sum_{t=e_i}^{f_i} S_{it} - \sum_t (r_t R_t + c_t X_t + w_t W_t + h_t I_t) - \sum_i l_i \left(1 - \sum_{t=e_i}^{f_i} S_{it}\right) \qquad (1)$$

Subject to:

$$W_t = W_{t-1} + R_t - X_t \qquad t = 1..T \qquad (2)$$

$$I_t = I_{t-1} + X_t - \sum_i q_i S_{it} \qquad t = 1..T \qquad (3)$$

$$X_t \leq a_k (W_{t-1} + R_t) + b_k \qquad t = 1..T,\ k = 1..K \qquad (4)$$

$$\sum_{t=e_i}^{f_i} S_{it} \leq 1 \qquad i = 1..N \qquad (5)$$

$$R_t, X_t, W_t, I_t \geq 0 \qquad t = 1..T \qquad (6)$$

$$S_{it}: binary \qquad i = 1..N, t = 1..T \qquad (7)$$

The objective function in equation (1) maximizes the total profit over the planning horizon. Constraints (2) and (3) define WIP and finished goods inventory balances, respectively for each period. Constraints (4) represent the capacity constraints defined by the CF. Constraints (5) ensure that an order is accepted only once and that it is only satisfied within its specified time window $[e_i, f_i]$.

3. Solving the PP-OA-FDD: A Heuristic Approach

3.1 General structure of the heuristics

This section presents two MILP heuristics: a reversals heuristic and a relax-and-fix heuristic with reversals. It the reversals idea consists is inspired by the sub-tour reversals heuristic for the traveling salesman problem (Taha, 2010). The pseudo-code of Algorithm 1 shows the common part of both heuristics.

The algorithm starts by sorting the orders based on some priority rules using a quick sort algorithm (line 10), which will generate a sequence $I = \{i_1, i_2, \ldots, i_N\}$. The initial sequence was obtained using two different priority rules: Most Profitable First (MPF) and Earliest Due Date (EDD).

1. *Most Profitable First (MPF) priority rule*

Initially, all orders are supposed to be released and satisfied in their earliest due date, which yields a unit profit of $\pi'_i = \pi_i - r(\tau_i)$ for each order i. The first sequence is obtained by sorting the orders in decreasing order of unit profit π'_i.

2. *Earliest Due Date (EDD) priority rule*

The first sequence is obtained by sorting the orders due date in their increasing value. This rule avoids missing orders due dates, and consequently, losing the orders.

Then, the sequence is used to build feasible solutions (line 180) in two different manners which results in the two heuristics that will be presented later. After updating the best solution (line 200), the first sequence is reversed two-by-two using subroutine *Reverse*, such that the resulting new sequences are: $I = \{i_2, i_1, \ldots, i_N\}$, $I = \{i_1, i_3, i_2, \ldots, i_N\}$, …, $I = \{i_1, i_2, \ldots, i_N, i_{N-1}\}$. The best reversal and solution value are saved. The best sequence in the two-by-two reversal is used as a starting point for a three-by-three reversal, and so on.

```
Algorithm 1: RerversalsHeuristic

10  BestSequence ← QuickSort(Orders);
20  BestProfit ← −∞
30  for Reversals ← 1 until N do
40     if Reversals = 1 then
50            MaxReversals ← 1
60     else
70            MaxReversals ← N-Reversals+1
80     end-if
90     ReversalBestProfit  ← −∞
100    for ReversePoint ← 1 until MaxReversals do
110           for i ← 1 until N do
120                S[i] ← 0;
130           flag ← true;
140           if (Reversals > 1) then
150                  Reverse(BestSequence, ReversePoint, ReversePoint+Reversals-1)
160           end-if
170           SequenceBestProfit ←  −∞
180           Find production plan based on current sequence
200           if (SequenceBestProfit ≥ ReversalBestProfit) then
210                  ReversalBestProfit ← SequenceBestProfit;
220                  UpdateBestSequence();
230           end-if
240    end-for
250    if (ReversalBestProfit ≥ BestProfit) then
260           BestProfit ← ReversalBestProfit;
270           UpdateBestSequence();
280    end-if
290  end-for
```

The two approaches used to find the production for a given sequence of a priori accepted orders (line 180) are presented below.

3.2 Finding production plans

1. Heuristic One-By-One with All Reversals (OBO-AR)

The orders are scheduled one by one, until no more increase in the profit is possible. The whole sequence and the last order accepted are saved and an initial solution is identified. The outer loop starting at line 30 in Algorithm 1 calculates this initial solution. In this loop, if Reversals=1 then this iteration corresponds to the calculation of the initial solution and the second loop starting at line 100 will be executed only once.

It is the subroutine OBO_AR(Sequence) which will accept orders as long as the profit increases. To schedule the first order i_1 using subroutine *OBO_AR(Sequence)*, the binary variables $S_{i_k t}$ associated with all its successors (i.e. $i_k \backslash k > 1$) must be set to zero. Then, the only binary variables left in the mathematical formulation are $S_{i_1 t}$. This simple problem was initially solved as a sequence of LPs in which only $S_{i_1 1}$ is set to 1 (i.e. order i_1 is satisfied in period 1), then only $S_{i_1 2} = 1$, and so on. Numerically, the commercial solver used with this problem solved the MILP (where $S_{i_1 t}$ are to be determined) much faster than the sequence of LPs. In the next iteration, the values of variables $S_{i_2 t}$ are to be determined in the same manner while fixing $S_{i_1 t}$ to the obtained solution in the first iteration and setting the binary variables $S_{i_k} \backslash k > 2$ to zero. In general, given that the last scheduled order is i_j, i.e. $S_{i_j t} = 1$ for a certain period t_{i_j} and $S_{i_j t} = 0$, for $t \neq t_{i_j}$, to schedule order i_{j+1}, all binary variables $S_{kt} \backslash k \neq j$ are fixed and the MILP with binary variables $S_{i_{j+1} t}$ is solved. Though it is easy to prove that the individual problems can be solved in polynomial time, we did not develop any specific algorithm for that purpose. The investment in the development of such an algorithm will not be justified given the quality provided by this heuristic compared to the relax-and-fix heuristic as it will be shown in the numerical results.

Algorithms 2: Subroutine OBO_AR(Sequence)

```
j ← 1;
while (j ≤ N) and (flag = true) do
begin
  i ← BestSequence[j];
  FindOptimalPeriod(i, Profit[i]);
  if Profit[i] ≥ SequenceBestProfit then
        SequenceBestProfit ← Profit[i];
  else
        flag ← false;
  end-if
  j ← j+1;
end;
```

2. Relax-and-Fix Heuristic with Reversals (RFR)

In relax-and-fix heuristics, the integer variables in a MILP are separated into subsets. The heuristic usually proceeds by fixing the most important variables and relaxing the integrality of the other variables. Then, it gradually fixes the relaxed variables (Wolsey, 1998). Relax-and-fix heuristics were applied to different production planning problems including the capacitated single level multi-item lot sizing problem (Federgruen et al. 2007), the multi-level lot sizing problem (Stadtler, 2003), and the lot sizing and scheduling problem with parallel machines (Beraldi et al. 2008). It was also used to solve problems in particular applications. Toso et al. (2009), for example, solve a lot sizing problem at an animal-feed plant using three different variants of a relax-and-fix heuristic. Ferreira et al. (2010) use the embedded relax-and-fix heuristic of commercial solver CPLEX to solve a production planning problem that arises in soft drink plants.

Some strategies used in the relax-and-fix heuristics consist of fixing different sub-categories of variables and relaxing the others (ex. Toso, 2009). However, most implementations in production planning consider partitioning the time horizon and forward or backward fixing integer variables (ex. Federgruen and Tzur, 1999; Stadtler, 2003; Federgruen et al. 2007; and Akartunali and Miller, 2009).

Our heuristic groups orders instead of time periods. It forces the integrality condition on binary variables of the first J orders with the highest profit π' (see Section 3.1) and relaxes the other binary variables. Then, it permanently fixes the solution for the first J' variables ($J' \leq J$), sets integrality constraints on variables indexed from $J'+1$ to $J'+J$ and relaxes integrality for orders after $J'+J+1$. The process is repeated until the last order in the sorted list is reached. Furthermore, compared to simple relax-and-fix heuristics, our heuristic applies the reversals function and explores more possible solutions.

A very detailed and practical presentation of a relax-and-fix heuristic for lot sizing problems can be found in Pochet and Wolsey 2007. The heuristic main inputs are the number of orders for which the integrality constraints are to be respected in each iteration (J), and the number of orders for which the binary variables are to be permanently fixed in each iteration (J'). The first step of the heuristic calculates the number of iterations based on these two parameters. Then, starting from the beginning of the sequence of the sorted orders, the sub-problems are solved until all binary decision variables are fixed.

Algorithms 3: Subroutine Relax-and-Fix(Sequence)

```
Input: J, J'
Caculate NumIter
for i ← 1 until NumIter
  Relax binary variables of orders after the last J interval
  Solve the sub-problem
  Permanently fix S_it variables for orders within J'
end-for;
```

4. Experimental Results

This section evaluates the added value from integrating production and order acceptance decisions and introducing due date flexibility. It also evaluates the efficiency of the proposed heuristics. The optimization model as well as the heuristics were implemented in FICO-Xpress version 1.24 (2013) and run on a PC with Intel CORE i7-2.4Ghz microprocessor and 16GB RAM.

4.1 Generated data sets

In the initial data sets, N =20 orders are received for a period of T =10 weeks. The release cost is equal to r_t =\$3, while c_t =0, w_t =\$35, and h_t =\$15, $\forall t$. The unit profit is equal to 100, 110 and 115 for small, medium, and large size orders, respectively. The earliest delivery date of each order is generated from a uniform distribution between 1 and T. The size of each order is generated from a uniform distribution between $\frac{1}{2}\bar{q}$ and $\frac{3}{2}\bar{q}$, where:

$$\bar{q} = \frac{T \times b_K \times DC}{N}$$

DC is the sum of all orders over the nominal capacity for the whole planning horizon, i.e. $DC = \frac{\sum_i q_i}{T \times b_K}$.

The lost sale cost is: $l_i = 1.2 \times \pi_i$. The intercepts and the slopes of the clearing function are defined as $(a_k, b_k) = (0.5, 0),\ (0.069, 136), (0.036, 154.8),\ (0.023, 161.8),\ (0, 180)$ for $k = 1,2,..,5$.

The analysis of the effectiveness of the model and the performance of the heuristics was based mainly on capacity tightness determined by coefficient $DC = \frac{\sum_i q_i}{T \times b_K}$ and order time window size δ_i. DC was varied between 0.6 (loose capacity) and 1.2 (demand exceeding capacity). To analyze the impact of due date flexibility, δ_i was varied between 1 and 6, where $\delta_i = 1$ corresponds to a no-flexibility situation.

4.2 Economic experiments

The effectiveness of integrating flexibility using the new model (PP-OA-FDD) is shown by comparing its total profits and other measures with those of a no-flexibility situation.

Figure 1 shows the total profit of the model with and without due date flexibility ($\delta_i = 3$ and $\delta_i = 1$, respectively). The effect of flexibility is more remarkable as DC increases. In the case of DC=1.2, the profit in the model with flexibility is almost double the profit when no flexibility is allowed.

The larger profit of PP-OA-FDD model is due to the increase in accepted orders as flexibility is introduced (See Figure 2).

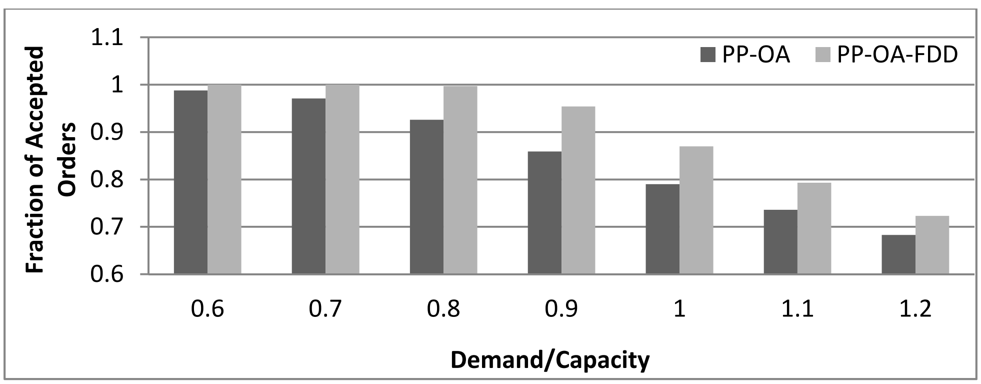

Figure 1. Accepted orders with and without due date flexibility ($\delta_i = 3$, and $\delta_i = 1$, respectively)

It is worth noting, from Figure 2, that even with very loose capacity (DC=0.6), the no-flexibility model does not accept all orders as delays are not allowed. When model accepts all orders (with $\delta_i > 1$), the average delay was positive between 0.36 (for $\delta_i = 2$) and 0.99 (for $\delta_i = 6$).

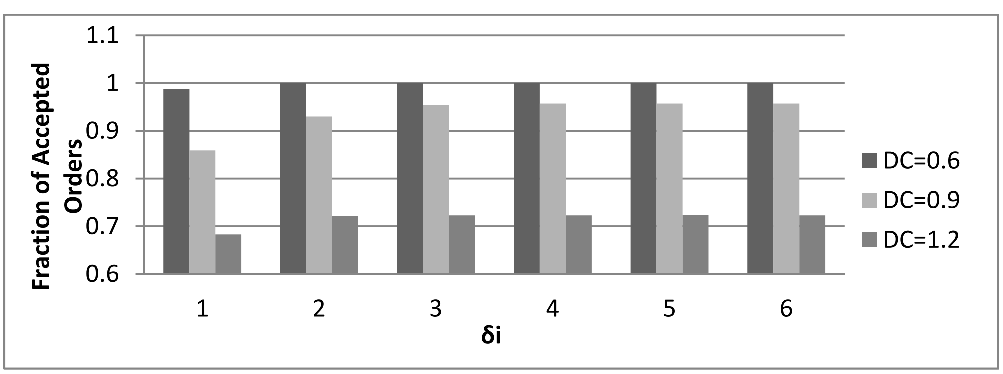

Figure 2. Fraction of accepted orders PP-OA-FDD vs. flexibility coefficient δ_i

4.3 Analyzing the performance of the heuristics

The PP-OA-FDD model was solved using Algorithm 1 by applying the One-By-One heuristic with All Reversals (OBO-AR). The gap between the optimal solution (Opt) obtained using the solver and the heuristic solution (Sol) is calculated as:

$$Gap = 100 \times \frac{Opt - Sol}{Opt}$$

The OBO-AR heuristic was run with Most Profitable First (MPF) and Earliest Due Date (EDD) priority rules to build the initial solution. The results are shown on Table 1 for different values of DC and time window size (δ_i).

Table 1 shows that, on average, EDD yields better profit than MPF. Table 1 also shows that EDD is less sensitive to changes in order time window compared to MPF. However, using any of the two priority rules, the OBO-AR heuristic still yields poor results when DC is high in addition to the fact that it takes too much CPU time even for small values of T and N (40 Seconds on average). This encourages the development of a more elaborate solution method such as a metaheuristic to explore the solution space in a smarter way.

The performance of the relax-and-fix heuristic with reversals is analyzed in Table 2. The parameters of the heuristic are as follows: The interval with integrality constraints is of size $J = 10$ periods. The in which binary variables are permanently fixed is $J' = 8$ in a first setting (RF1) and $J' = 10$ in the second setting (RF2). In Table 2, the first and second columns correspond to the three problem parameters and their values based on which the analysis was done. Problem size is identified by the number of time periods in the planning horizon and the number of orders (T-N) which range from 10 periods and 20 orders to 100 periods and 200 orders. For a fair comparison, we limited the execution time of the solver when applied directly to the PP-OA-FDD formulation to the worst CPU time obtained using the relax-and-fix heuristic. The last six columns in Table 2 present, for three solution approaches, the CPU times and the gap of the best feasible solution (Sol) compared to the best upper bound ($BestUB$). The latter is obtained by running the solver for a maximum CPU time of 5 minutes. *Agg* corresponds to the use of the solver on MILP formulation defined by (1)-(7). Thus, for example:

$$\text{Gap-Agg} = 100 \times \frac{BestUB - Sol}{BestUB}$$

Table 2 shows that the fix-and-relax heuristic with $J' = 8$ (RF1) yields the same results as the solver for much less computational efforts (40 Seconds instead of 174 Seconds). As it can be expected, the solver performs much better than the heuristic for very small problems though it requires a lot of CPU time on average. The effectiveness of the heuristic RF1 becomes more remarkable with longer planning horizons. The more flexibility allowed in the system, the harder is the problem to solve using the solver on formulation (1)-(7) and the relax and fix heuristic starts getting better results in much less CPU time. Though the second relax-and-fix heuristic (RF2) does not obtain better results for any of the different parameters, its results are close to those of the two first approaches as it yields an average gap of 3.96% instead of 3.10% for *Agg* approach and 3.11% for RF1 heuristic. Still the CPU time for RF2 is much smaller (18 Seconds on average). This suggests the use of the parameters of the relax-and-fix heuristic (J, J') to allow the decision maker to choose between quality and CPU time.

Table 1. Gaps between the optimal solution and the heuristics

	DC=0.6		DC=0.9		DC=1.2	
	MPF	EDD	MPF	EDD	MPF	EDD
δ=2	0.21	0.17	2.47	4.32	20.98	24.27
δ=3	0.08	0.28	4.07	3.25	25.62	21.07
δ=4	0.85	0.77	6.85	4.32	26.77	17.22

Table 2. Gaps between the best bound and the best solution of the heuristics

Parameter	Value	**Gap-Agg**	CPU-Agg	Gap-FR1	CPU-FR1	Gap-FR2	CPU-FR2
T-N	10-20	**0.04**	54.94	1.01	0.96	0.45	0.72
	10-50	**0.21**	112.51	0.78	2.29	0.89	0.98
	10-100	**0.15**	142.95	0.39	7.67	0.85	2.55
	50-100	3.69	191.84	**3.30**	21.42	4.13	8.34
	50-300	1.68	201.37	**1.31**	101.84	4.03	46.78
	100-100	**9.90**	256.15	10.27	48.42	10.52	17.25
	100-200	5.27	229.62	**4.17**	91.25	6.00	47.00
δ_i	2	**1.44**	119.75	2.37	35.38	3.16	14.13
	4	**2.81**	182.86	2.89	35.55	3.70	15.66
	8	3.73	190.66	**3.39**	40.25	4.21	17.61
	12	4.65	208.27	**3.90**	52.70	4.91	27.00
DC	0.6	**0.05**	47.60	0.09	34.46	0.43	17.26
	0.9	**1.16**	216.41	1.20	42.36	2.21	17.30
	1.2	8.10	258.50	**8.04**	44.79	9.25	20.30
Average		3.10	174.17	3.11	40.53	3.96	18.29

5. Conclusions

Integrating production planning and sales decisions is crucial for companies to make proper use of available resources and satisfy customer orders without losing profitability. Furthermore, negotiating with customers to have flexible due dates allows companies to accept more orders and give customers firm delivery dates to these customers. In this paper, we have presented a mathematical programming formulation to model the integrated problem of order-acceptance with load dependent lead times and flexible due dates and we have showed, through numerical experiment, the benefits of flexibility. For more realistic problems with a larger number of orders to satisfy over several planning periods, the problem is very hard to tackle and this is why we have developed a relax-and-fix heuristic to solve. The heuristic is very efficient and outperforms the direct application of a commercial solver on the mathematical programming formulation. The relax-and-fix heuristic also offers the decision maker the possibility to adjust some of its parameters to target better solution by scarifying some CPU time. Though the model presented in this paper is very comprehensive, it needs further improvements by considering other details related to production planning decisions such as setup costs and times and multi-products. Furthermore, we believe that it is possible to develop faster solution approaches which do not rely on the solution on integer linear programming problems.

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Biography

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