Proceedings of the 2014 International Conference on Industrial Engineering and Operations Management
Bali, Indonesia, January 7 – 9, 2014

A Stochastic Approach to Hotel Revenue Management Considering Individual and Group Customers

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Abstract

This paper develops a stochastic model for capacity allocation with robust optimization in the quantity-based revenue management. This model considers customers' arrival both individually and in group, limited time horizon and capacity. The policy is to provide early discount during a determined deadline in the booking period in order to provoke demand. This model determines the optimal booking limits for each class of customers by using a two-stage stochastic programming in order to control inventory during booking period. It can be used for optimization of hotel revenue management system, also as a guide for decision makers in developing their own policies for accepting or not accepting booking requests for the hotel.

Keywords
Capacity allocation, Hotel revenue management, Robust optimization, Stochastic programming.

1. Introduction

Revenue management is the art of maximizing profit generating from a limited capacity of a product, over a finite horizon, by selling each product to the right customer with the right price at the right time (Terciyanli 2009). This approach has developed from its origin, the airline industry, to its current position, as one of the most important business activities in a wide range of industries, including hospitality, energy, retail, and manufacturing (Subramanian et al. 1999). This approach has integrated marketing as well as financial and operational activities to maximize revenue from current capacity (Crystal 2007). Depending on the decisions made to manage demand, revenue management is divided into quantity-based or priced-based revenue management. In quantity-based approach, the problem of rejection or acceptance of an order, and the capacity allocation to different classes of customers are expressed, while priced-based approach expresses the problem of determining the price of different product types or the product price over time. Some industries may use the combination of price-based and quantity-based revenue management. Among quantity-based decisions, the ones related to capacity allocation and inventory control, allow managers to look for optimal balance between customer demand and the systems' ability to provide services that meet the demand.

Revenue management, in the hotel industry, is a selection process, in which the acceptance or rejection of customers is based on the price rates, arrival date, and the length of stay in order to maximize revenue. This is done by matching the available facilities with customer demands to get the most profit by combining customers. The main purpose of this study is to present a mathematical programming model for the optimization step in applying revenue management in hotels. The model is also an appropriate capacity allocation and inventory control. This study is an attempt to develop a practical model based on the literature in this area. It also familiarizes the reader with the operational environment and the existing situation of reservation. In this study the model of determining the booking limit and the optimal allocation were obtained by using a two-stage stochastic programming with regard to arrival of customers is individual or group, limited time horizon and capacity. In this model, the early discount strategy for individual customers has been added and two price classes for two booking periods were also considered.

The paper is organized as follows: section 2 describes the notations, parameters and assumptions of the model. The basic stochastic programming model for hotel revenue management is presented in Section 3. In Section 4, stochastic programming model with robust optimization is developed. A numerical illustrative example is presented in section 5. Finally, in Section 6, conclusions and recommendations for future research are discussed.

2. Mathematical Modeling

Before presenting stochastic programming model, this section introduces the assumptions, notations, parameters and variables of the model.

2.1. Model Assumptions

In the presented model, planning is done for a period of T days. No customer is in hotel before the first day (0) and all customers have left the hotel by end of period. Minimum residing is one night. Customers arrive individually or in groups and the hotel provide only one class of rooms. In this model, the early discount strategy has been considered. Customers can reserve rooms with lower price in a certain time period. Two classes of price are considered, normal price with booking limit Bl_1 and discount price with booking limit Bl_2. Two booking periods are also considered, the first time period, offering early discount, thus, it is possible to book with two different rates: discount price and normal price. Booking with discount price is not allowed to cancel and the change must be paid fully in advance; but booking with normal price is allowed to cancel before starting the period. In the second time period, booking is only possible with the normal price. If part of the related capacity to discount price is not sold out in the prior period, it could be sold with a normal price in the second period.

In addition, the possibility to change booking class has been considered for customers. If the demand for class 2, in the first period, is more than its related capacity, the model gives chance to the customer to take the other option.

With respect to operational environment and stochastic demand, there are different scenarios for demand with a certain probability on each day of the period. Different demand scenarios can have different or the same prices. The possibility of canceling the booking is also considered in the model.

2.2. Notations and Definitions

i: The arrival day in the considered period, (i=0,1,…,T-1)
j: The check-out day in the considered period, (j=1,2,…,T)
m: Price class, (m=1,2)
g: Group number, (g=1,2,3)
k: The considered day (k=1,2,…,T-1)
T: Length of the considered period, (T=1,2,…,T)
sc: The set of different demand scenarios, (sc=1,2,…,Sc)

2.3. Parameters

C: total capacity
P_m: Selling price for individual booking to class m
p': Selling price for group booking
S_{ijg}: The size of group g for check-in on day i and check-out on day j
U_{ijm}: booking demand for check-in on day i and check-out on day j in class m
q_{21}: Probability of acceptance of normal price by class 2 customer
e_m: Probability of cancelation for class m
S^{sc}_{ijg}: The size of group g for check-in on day i and check-out on day j under Scenario sc
U^{sc}_{ijm}: booking demand for check-in on day i and check-out on day j in class m under Scenario sc
M: The upper range for L
pr_{sc}: Probability of occurrence of scenario sc
λ: Risk aversion
W_{mi}: Penalty for the deviation of certain demand for class m on day i
w'_i: : Penalty for the deviation of certain demand for group customers on day i

D. Decision Variables

Bl_{mi}:Booking limit for selling capacity to class m on day i
Blg_i: Booking limit for selling capacity to group customers on day i
x_{ijm}: The number of booking capacity of class m for check-in on day i and check-out on day j
y_{ijg}: The probability of acceptance of group g for check-in on day i and check-out on day j
x^{sc}_{ijm}: The number of booking capacity of class m for check-in on day i and check-out on day j under Scenario sc
y^{sc}_{ijg}: The probability of acceptance of group g for check-in on day i and check-out on day j under Scenario sc

3. Problem Formulation

The basic mathematical programming model is presented as follows.

$$\text{Max } \mathbf{Z} = \sum_{m=1}^{2}\sum_{i=0}^{T-1}\sum_{j=i+1}^{T}(j-i)\,p_m x_{ijm} + \sum_{g=1}^{G}\sum_{i=0}^{T-1}\sum_{j=i+1}^{T}(j-i)\,p' s_{ijg} y_{ijg} \tag{1}$$

s.t.

$$\sum_{m=1}^{2}\sum_{i=0}^{k}\sum_{j=k+1}^{T}(1-e_m)x_{ijm} + \sum_{g=1}^{G}\sum_{i=0}^{k}\sum_{j=k+1}^{T} y_{ijg} s_{ijg} \le C \qquad \forall\ k=1,2,\dots,T-1 \tag{2}$$

$$\sum_{m=1}^{2}\sum_{j=1}^{T}(1-e_m)x_{0jm} + \sum_{g=1}^{G}\sum_{j=1}^{T} y_{0jg} s_{0jg} \le C \tag{3}$$

$$x_{ij2} \le U_{ij2} \qquad \forall\ i=0,\dots,T-1 \quad j=1,\dots,T \tag{4}$$

$$x_{ij1} \le U_{ij1} + Max\ \{0,(U_{ij2}-x_{ij2})\} * q_{21} \qquad \forall\ i=0,\dots,T-1 \quad j=1,\dots,T \tag{5}$$

$$x_{ijm} \ge 0\ , y_{ijg} = \{0,1\}$$

$$x_{ijm} \text{ is integer.}$$

The objective function (1) seeks to maximize revenue and it has two parts. The first part shows revenue from capacity allocation to individual customer and the second part shows revenue from group customers. Constraint (2) ensures that the number of customers on day k do not exceed the maximum capacity of the hotel. In this constraint, the likelihood of cancellation has been considered and assumed that the probability of cancellation by customer class m is e_m; so this assumption has been used in the capacity constraints. Constraint (3) limits the number of customers' arrival to the maximum capacity on day zero. Two constraints (4) and (5) ensure the number of admitted customers for check-in on day i and check out on day j are not more than expected demand. Constraint (5) has been added demand of class 2, to expected demand of class 1, if capacity related to class 2 has been sold out.

Because constraint (5) is nonlinear, the above model is converted to the following linear model with some modifications on the constraint:

$$x_{ij1} \le U_{ij1} + h_{ij} * q \qquad \forall\ i=0,\dots,T-1 \quad j=1,\dots,T \tag{6}$$

$$L_{ij} = U_{ij2} - x_{ij2} \qquad \forall\ i=0,\dots,T-1 \quad j=1,\dots,T \tag{7}$$

$$L_{ij} - h_{ij} + M\delta_{ij} \le M \qquad \forall\ i=0,\dots,T-1 \quad j=1,\dots,T \tag{8}$$

$$-L_{ij} + h_{ij} \le 0 \qquad \forall\ i=0,\dots,T-1 \quad j=1,\dots,T \tag{9}$$

$$h_{ij} - M\,\delta_{ij} \le 0 \qquad \forall i=0,\dots,T-1 \quad j=1,\dots,T \tag{10}$$

$$-L_{ij} + h_{ij} + M\delta_{ij} \le M \qquad \forall\ i=0,\dots,T-1 \quad j=1,\dots,T \tag{11}$$

$$L_{ij} - h_{ij} \le 0 \qquad \forall\ i=0,\dots,T-1 \quad j=1,\dots,T \tag{12}$$

$$\mathrm{x}_{ijm} \geq 0 \quad , h_{ij} \geq 0 \ , \mathrm{y}_{ijg} = \{0,1\} \quad , \delta_{ij} = \{0,1\}$$

x_{ijm} *is* integer.

L is free.

In the above constraints h is equal to $Max\left\{0,\left(U_{ij2} - Bl_2\right)\right\}$ and takes zero or L; if L is negative it takes zero.

4. Stochastic Programming Model and Robust Optimization

Uncertainty is a key element in many decision-making problems. Financial planning, airlines planning, and scheduling are the examples of areas that ignoring may lead to errors in decision-makings. There are often various ways in which uncertainty can be formulated and several approaches have been developed for optimizing under uncertainty over recent years.

In traditional methods, sensitivity analysis approaches are used to consider the uncertainty of the data. In such approaches, professionals and model designers ignore the effect of the uncertainty of the data at first and subsequently use sensitivity analysis to endorse the obtained results. However, sensitivity analysis is the only tool to analyze the answers. It cannot be used to produce robust answers. In addition, the use of sensitivity analysis in the models that have a large number of uncertain data is not practical.

Another approach that has been recently developed to cope with uncertainty in data is robust optimization. This approach seeks solutions close to optimum that are feasible with high probability. In other words, by ignoring the objective function, the possibility of answers obtained will be ensured. Here, a unique approach based on probabilistic uncertainty models is considered. By considering the average of feasible outcomes or possibility of occurrence of events, objectives and constraints of the corresponding mathematical programming model can be defined.

In the real world, uncertainty has not been considered in many models or if used it usually leads to nonlinear and complex models that cannot be optimized. The aim of this study is to present capacity allocation in revenue management for uncertain data that have an acceptable performance. The designed stochastic programming and robust optimization model is presented below:

$$\begin{aligned}
\text{Max } \mathbf{Z} = & \sum_{sc=1}^{Sc} pr_{sc} \sum_{m=1}^{2}\sum_{i=0}^{T-1}\sum_{j=i+1}^{T} (j-i)\, p_m^{sc} x_{ijm}^{sc} + \sum_{sc=1}^{Sc} pr_{sc} \sum_{g=1}^{G}\sum_{i=0}^{T-1}\sum_{j=i+1}^{T} (j-i)\, p'^{sc}\ s_{ijg}^{sc} y_{ijg}^{sc} \\
& -\lambda \sum_{sc=1}^{Sc} pr_{sc} \left| \sum_{m=1}^{2}\sum_{i=0}^{T-1}\sum_{j=i+1}^{T} (j-i)\, p_m^{sc} x_{ijm}^{sc} + \sum_{g=1}^{G}\sum_{i=0}^{T-1}\sum_{j=i+1}^{T} (j-i)\, p'^{sc}\ s_{ijg}^{sc} y_{ijg}^{sc} - \left(\sum_{sc=1}^{Sc} pr_{sc} \sum_{m=1}^{2}\sum_{i=0}^{T-1}\sum_{j=i+1}^{T} (j-i)\, p_m^{sc} x_{ijm}^{sc} + \sum_{sc=1}^{Sc} pr_{sc} \sum_{g=1}^{G}\sum_{i=0}^{T-1}\sum_{j=i+1}^{T} (j-i)\, p'^{sc}\ s_{ijg}^{sc} y_{ijg}^{sc} \right) \right| \\
& - \sum_{sc=1}^{Sc} pr_{sc} \left[\sum_{i=0}^{T-1} w_{1i} \left| \sum_{j=i+1}^{T} U_{ij1}^{sc} + \max\left\{0, \left(U_{ij2}^{sc} - x_{ij2}^{sc}\right)\right\} \mathrm{q}_{21} - Bl_{1i} \right| + \sum_{i=0}^{T-1} w_{2i} \left| \sum_{j=i+1}^{T} U_{ij2}^{sc} - Bl_{2i} \right| + \sum_{i=0}^{T-1} w'_i \left| \sum_{g=1}^{g}\sum_{j=i+1}^{T} s_{ijg}^{sc} - B\,\mathrm{lg}_i \right| \right]
\end{aligned} \tag{13}$$

s.t.

$$\sum_{m=1}^{2}\sum_{i=0}^{k}\sum_{j=k+1}^{T} (1-e_m) x_{ijm}^{sc} + \sum_{g=1}^{G}\sum_{i=0}^{k}\sum_{j=k+1}^{T} y_{ijg}^{sc} s_{ijg}^{sc} \leq C \qquad \forall\ \mathrm{k}=1,2,\ldots,\mathrm{T}-1 \quad \mathrm{sc}=1,\ldots,\mathrm{Sc} \tag{14}$$

$$\sum_{m=1}^{2}\sum_{j=1}^{T} (1-e_m) x_{0jm}^{sc} + \sum_{g=1}^{G}\sum_{j=1}^{T} y_{0jg}^{sc} s_{0jg}^{sc} \leq C \qquad \forall \mathrm{sc}=1,\ldots,\mathrm{Sc} \tag{15}$$

$$\sum_{j=i+1}^{T} x_{ijm}^{sc} \leq Bl_{mi} \qquad \forall\ \mathrm{m}=1,2 \quad \mathrm{i}=0,1,\ldots,\mathrm{T}-1 \quad \mathrm{sc}=1,\ldots,\mathrm{Sc} \tag{16}$$

$$\sum_{g=1}^{G}\sum_{j=i+1}^{T} y_{ijg}^{sc} s_{ijg}^{sc} \leq Bl\,\mathrm{g}_i \qquad \forall\ \mathrm{i}=0,1,\ldots,\mathrm{T}-1 \quad \mathrm{sc}=1,\ldots,\mathrm{Sc} \tag{17}$$

$$\mathrm{x}_{ij2}^{sc} \leq U_{ij2}^{sc} \qquad \forall\ \mathrm{i}=0,\ldots,\mathrm{T}-1 \quad \mathrm{j}=1,\ldots,\mathrm{T} \quad \mathrm{sc}=1,\ldots,\mathrm{Sc} \tag{18}$$

$$\mathrm{x}_{ij1}^{sc} \leq U_{ij1}^{sc} + \max\left\{0, \left(U_{ij2}^{sc} - x_{ij2}^{sc}\right)\right\} \mathrm{q}_{21} \qquad \forall\ \mathrm{i}=0,\ldots,\mathrm{T}-1 \quad \mathrm{j}=1,\ldots,\mathrm{T} \quad \mathrm{sc}=1,\ldots,\mathrm{Sc} \tag{19}$$

$$\mathrm{Bl}_{2i} \leq \max\left\{ \sum_{j=i+1}^{T} U_{ij2}^{sc} \right\} \qquad \forall\ \mathrm{i}=0,1,\ldots,\mathrm{T}-1 \quad \mathrm{sc}=1,\ldots,\mathrm{Sc} \tag{20}$$

$$\mathrm{Bl}_{1i} \le \max\left\{\sum_{j=i+1}^{T} U_{ij1}^{sc}\right\} + \left[\max\left\{\sum_{j=i+1}^{T} U_{ij2}^{sc}\right\} - Bl_{2i}\right] \mathrm{q}_{21} \qquad \forall\ \mathrm{i=0,1,...,T\text{-}1 \quad sc=1,...,Sc} \tag{21}$$

$$\mathrm{Blg}_i \le \max\left\{\sum_{g=1}^{g}\sum_{j=i+1}^{T} s_{ijg}^{sc}\right\} \qquad \forall\ \mathrm{i=0,1,...,T\text{-}1 \quad sc=1,...,Sc} \tag{22}$$

$$0 \le x_{ijm}^{sc} \ ,\ 0 \le \mathrm{Bl}_{mi} \le C\ ,\ 0 \le B\lg_i \le C\ , y_{ijg}^{sc} = \{0,1\} \ ,\ \delta_{ij} = \{0,1\}$$

$$x_{ijm}^{sc}, \mathrm{Bl}_{mi}, B\lg_i \text{ are integer.}$$

The first two terms, in the objective function, represent revenue from servicing to customers; the next statement shows the mean absolute deviation from the average revenue for different scenarios. λ is a non-negative weighting parameter that is brought as a risk aversion factor; the more the rate of management risk , the smaller the value of this coefficient will be. If the difference of revenue from various scenarios be high, the amount of penalties will increase, too. Thus, the model seeks to minimize this difference. In This method, the objective function looks for maximizing solution robustness. The next term represents the deviation of the various demand scenarios and w, w' are non-negative weighting parameters that is used as penalty for deviation from the specific demand. Thus, the objective function seeks to maximize model robustness.

With regard to the presented model which is non-linear, the theory devised by Yu and Li (2000) can be used to linearize it.

By using the proposed theory, our stochastic model is also converted to linear form. The non-negative variables γ^{sc} ‘ ξ_k^{sc} ‘ ζ_k^{sc} and φ_k^{sc} were presented for the set of scenarios and the model with the changes in the objective function and adding some constraint comes in the form below.

$$\text{Max } \mathbf{Z} = \sum_{sc=1}^{Sc} pr_{sc} \sum_{m=1}^{2}\sum_{i=0}^{T-1}\sum_{j=i+1}^{T} (j-i)\, p_m^{sc} x_{ijm}^{sc} + \sum_{sc=1}^{Sc} pr_{sc} \sum_{g=1}^{G}\sum_{i=0}^{T-1}\sum_{j=i+1}^{T} (j-i)\, p'^{sc}\ s_{ijg}^{sc} y_{ijg}^{sc}$$

$$-\lambda \sum_{sc=1}^{Sc} pr_{sc}\left[\sum_{m=1}^{2}\sum_{i=0}^{T-1}\sum_{j=i+1}^{T} (j-i)\, p_m^{sc} x_{ijm}^{sc} + \sum_{g=1}^{G}\sum_{i=0}^{T-1}\sum_{j=i+1}^{T} (j-i)\, p'^{sc}\ s_{ijg}^{sc} y_{ijg}^{sc} - \left(\sum_{sc=1}^{Sc} pr_{sc}\sum_{m=1}^{2}\sum_{i=0}^{T-1}\sum_{j=i+1}^{T} (j-i)\, p_m x_{ijm}^{sc} + \sum_{sc=1}^{Sc} pr_{sc}\sum_{g=1}^{G}\sum_{i=0}^{T-1}\sum_{j=i+1}^{T} (j-i)\, p'^{sc}\ s_{ijg}^{sc} y_{ijg}^{sc}\right) + 2\gamma^{sc}\right]$$

$$-\sum_{sc=1}^{Sc} pr_{sc}\left[\sum_{i=0}^{T-1} w_{1i}\left[\sum_{j=i+1}^{T} U_{ij1}^{sc} + h_{ij}^{sc} * q_{21} - Bl_{1i} + 2\zeta_i^{sc}\right] + \sum_{i=0}^{T-1} w_{2i}\left[\sum_{j=i+1}^{T} U_{ij2}^{sc} - Bl_{2i} + 2\xi_i^{sc}\right] + \sum_{i=0}^{T-1} w'_i\left[\sum_{g=1}^{g}\sum_{j=i+1}^{T} s_{ijg}^{sc} - B\lg_i + 2\varphi_i^{sc}\right]\right] \tag{23}$$

s.t.

$$\sum_{m=1}^{2}\sum_{i=0}^{k}\sum_{j=k+1}^{T} (1-e_m)\, x_{ijm}^{sc} + \sum_{g=1}^{G}\sum_{i=0}^{k}\sum_{j=k+1}^{T} y_{ijg}^{sc} s_{ijg}^{sc} \le C \qquad \forall\ \mathrm{k=1,2,...,T\text{-}1 \quad sc=1,...,Sc} \tag{24}$$

$$\sum_{m=1}^{2}\sum_{j=1}^{T} (1-e_m)\, x_{0jm}^{sc} + \sum_{g=1}^{G}\sum_{j=1}^{T} y_{0jg}^{sc} s_{0jg}^{sc} \le C \qquad \forall \mathrm{sc=1,...,Sc} \tag{25}$$

$$\sum_{j=i+1}^{T} x_{ijm}^{sc} \le Bl_{mi} \qquad \forall\ \mathrm{m=1,2 \quad i=0,1,...,T\text{-}1 \quad sc=1,...,Sc} \tag{26}$$

$$\sum_{g=1}^{G}\sum_{j=i+1}^{T} y_{ijg}^{sc} s_{ijg}^{sc} \le Bl\ \mathrm{g}_i \qquad \forall\ \mathrm{i=0,1,...,T\text{-}1 \quad sc=1,...,Sc} \tag{27}$$

$$\mathrm{x}_{ij2}^{sc} \le U_{ij2}^{sc} \qquad \forall\ \mathrm{i=0,...,T\text{-}1 \quad j=1,...,T \quad sc=1,...,Sc} \tag{28}$$

$$\mathrm{x}_{ij1}^{sc} \le U_{ij1}^{sc} + h_{ij}^{sc} * q_{21} \qquad \forall\ \mathrm{i=0,...,T\text{-}1 \quad j=1,...,T \quad sc=1,...,Sc} \tag{29}$$

$$\mathrm{L}_{ij}^{sc} = U_{ij2}^{sc} - \mathrm{x}_{ij2}^{sc} \qquad \forall\ \mathrm{i=0,...,T\text{-}1 \quad j=1,...,T \quad sc=1,...,Sc} \tag{30}$$

$$\mathrm{L}_{ij}^{sc}\text{-}\mathrm{M}\delta_{ij}^{sc} \le 0 \qquad \forall\ \mathrm{i=0,...,T\text{-}1 \quad j=1,...,T \quad sc=1,...,Sc} \tag{31}$$

$$h_{ij}^{sc}\text{-}\mathrm{M}\delta_{ij}^{sc} \le 0 \qquad \forall\ \mathrm{i=0,...,T\text{-}1 \quad j=1,...,T \quad sc=1,...,Sc} \tag{33}$$

$$\text{-}\mathrm{L}_{ij}^{sc}\text{-}\mathrm{M}\left(1-\delta_{ij}^{sc}\right) \le 0 \qquad \forall\ \mathrm{i=0,...,T\text{-}1 \quad j=1,...,T \quad sc=1,...,Sc} \tag{32}$$

$$\text{-}\mathrm{L}_{ij}^{sc} + h_{ij}^{sc} + \mathrm{M}\delta_{ij}^{sc} \le \mathrm{M} \qquad \forall\ \mathrm{i=0,...,T\text{-}1 \quad j=1,...,T \quad sc=1,...,Sc} \tag{34}$$

$$\mathrm{L}_{ij}^{sc} - h_{ij}^{sc} \le 0 \qquad \forall\ \mathrm{i=0,...,T\text{-}1 \quad j=1,...,T \quad sc=1,...,Sc} \tag{35}$$

$$Bl_{2i} \le \max\left\{\sum_{j=i+1}^{T} U_{ij2}^{sc}\right\} \qquad \forall\ i=0,1,...,T-1 \quad sc=1,...,Sc \tag{36}$$

$$Bl_{1i} \le \max\left\{\sum_{j=i+1}^{T} U_{ij1}^{sc}\right\} + \left[\max\left\{\sum_{j=i+1}^{T} U_{ij2}^{sc}\right\} - Bl_{2i}\right] q_{21} \qquad \forall\ i=0,1,...,T-1 \quad sc=1,...,Sc \tag{37}$$

$$Blg_i \le \max\left\{\sum_{g=1}^{g}\sum_{j=i+1}^{T} s_{ijg}^{sc}\right\} \qquad \forall\ i=0,1,...,T-1 \quad sc=1,...,Sc \tag{38}$$

$$\sum_{sc=1}^{Sc} pr_{sc}\sum_{m=1}^{2}\sum_{i=0}^{T-1}\sum_{j=i+1}^{T}(j-i)\,p_m x_{ijm}^{sc} + \sum_{sc=1}^{Sc} pr_{sc}\sum_{g=1}^{G}\sum_{i=0}^{T-1}\sum_{j=i+1}^{T}(j-i)\,p'^{sc}\ s_{ijg}^{sc} y_{ijg}^{sc} - \gamma^{sc} \le \sum_{m=1}^{2}\sum_{i=0}^{T-1}\sum_{j=i+1}^{T}(j-i)\,p_m x_{ijm}^{sc} + \sum_{g=1}^{G}\sum_{i=0}^{T-1}\sum_{j=i+1}^{T}(j-i)\,p'^{sc}\ s_{ijg}^{sc} y_{ijg}^{sc}$$
$$\forall sc=1,...,Sc \tag{39}$$

$$Bl_{2i} - \xi_i^{sc} \le \sum_{j=i+1}^{T} U_{ij2}^{sc} \qquad \forall\ i=0,...,T-1 \quad sc=1,...,Sc \tag{40}$$

$$Bl_{1i} - \zeta_i^{sc} \le \sum_{j=i+1}^{T} U_{ij1}^{sc} + h_{ij}^{sc} * q_{21} \qquad \forall\ i=0,...,T-1 \quad sc=1,...,Sc \tag{41}$$

$$B\lg_i - \varphi_i^{sc} \le \sum_{g=1}^{g}\sum_{j=i+1}^{T} s_{ijg}^{sc} \qquad \forall\ i=0,...,T-1 \quad sc=1,...,Sc \tag{42}$$

$$0 \le x_{ijm}^{sc}\ ,\ 0 \le Bl_{mi} \le C\ ,\ 0 \le B\lg_i \le C\ ,\ 0 \le h_{ij}^{sc}\ ,\ y_{ijg}^{sc} = \{0,1\}\ ,\ \delta_{ij}^{sc} = \{0,1\}$$

x_{ijm}^{sc}, Bl_{mi}, $B\lg_i$, h_{ij}^{sc} are integer.

L is free.

5. Illustrative Example

In this section we consider a numerical example in order to carry on a preliminary test of the extended model. We assume a case where a hotel would like to take four different demand scenarios into its planning with the probability of occurrence 0.2, 0.4, 0.25 and 0.15, respectively. The prices per residing night for individual customers are 0.84 and 0.69 (normal price and discount price) and it is 0.65 for group customers. The total capacity of hotel is 500. The planning horizon is set to be 5 days. The risk aversion factor is equal to 1. The demands are shown as the following tables 1 and 2 for the four scenarios. In each scenario, each row represents a customer's arrival day and each column represents the check-out day and the class of customer for the reservation. For example, number 18 in the first row of the first scenario, column 1.1, shows the customer demand of class 1 for check-in on day 0 and check-out on day 1. Similarly, Table 2 shows the size of the customer of group g for check-in on day i and check-out on day j.

Table 1: Demands for individual customers (U^{sc}_{ijm})

	Scenario 1											Scenario 2									
	1.1	1.2	2.1	2.2	3.1	3.2	4.1	4.2	5.1	5.2		1.1	1.2	2.1	2.2	3.1	3.2	4.1	4.2	5.1	5.2
0	18	20	21	35	67	90	80	120	54	100	0	11	15	8	30	24	75	75	100	22	60
1			15	20	20	25	80	120	90	140	1			12	14	11	16	78	100	80	120
2					13	17	40	30	80	120	2					11	13	20	25	70	100
3							34	50	70	140	3							38	40	74	120
4									12	26	4									15	25
	Scenario 3											Scenario 4									
	1.1	1.2	2.1	2.2	3.1	3.2	4.1	4.2	5.1	5.2		1.1	1.2	2.1	2.2	3.1	3.2	4.1	4.2	5.1	5.2
0	4	10	13	15	56	60	64	80	78	80	0	2	5	7	15	12	40	36	60	14	40
1			9	8	9	12	35	70	70	100	1			8	5	4	12	31	50	50	60
2					5	8	10	12	60	80	2					1	5	8	12	30	50
3							15	25	24	60	3							10	20	36	40
4									9	22	4									12	20

Table 2: Demands for group customers (S^{sc}_{ijg})

	Scenario 1														
	1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3	4.1	4.2	4.3	5.1	5.2	5.3
0		15				12	22		17				20		
1							18	15		22	15	33		12	
2										12				18	

Table 2 continued: Demands for group customers (S^{sc}_{ijg})

Scenario 1															
	1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3	4.1	4.2	4.3	5.1	5.2	5.3
3												23			
4													12		
Scenario 2															
	1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3	4.1	4.2	4.3	5.1	5.2	5.3
0				17	10			14			15	20			
1									21	12	17				
2													22		30
3											17		12	18	22
4															
Scenario 3															
	1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3	4.1	4.2	4.3	5.1	5.2	5.3
0		15		12		18		16							
1							14			22	18	14			
2													42		
3														22	16
4													27		
Scenario 4															
	1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3	4.1	4.2	4.3	5.1	5.2	5.3
0							22		27				45		
1							18	25	10		12			18	
2										32				16	20
3															
4														18	

Table 3: Optimal solution – First stage

			0	1	2	3	4
First stage	Individual customers' booking limit	Class 1	240	140	86	88	15
		Class 2	70	60	20	40	0
	Group customers' booking limit		76	14	110	133	104

Table 4: Optimal solution – Second stage (Individual)

X^{1}_{ij} m	0	1	2	3	4	X^{2}_{ij} m	0	1	2	3	4	X^{3}_{ij} m	0	1	2	3	4	X^{4}_{ij} m	0	1	2	3	4
1-1						1-1						1-1						1-1	5				
1-2						1-2	1					1-2	2					1-2					
2-1	33	25				2-1	8	18				2-1	13	13				2-1	17	11			
2-2	13					2-2	30					2-2	15	2				2-2					
3-1	88	14				3-1	68	22				3-1	27	4				3-1	40	12			
3-2	3					3-2	12		8			3-2	5	1	8			3-2			4		
4-1	19	1				4-1	100					4-1	100	23		2		4-1	59	66	16	24	
4-2	20	20				4-2	27					4-2						4-2	27				
5-1	100	100	86	88	15	5-1	64	100	86	88	15	5-1	100	100	86	66	15	5-1	14	51	65	64	15
5-2	34	40	20	40		5-2		60	3	37		5-2	48	57	18			5-2	40	58			

The optimal first stage solutions obtained are then summarized in Tables 3-5; Namely, the booking limit for each class of customers, individual customers and group customers, for each arrival day. For example, the booking limit on day 1 for class 1 is 140 and for class 2 is 60; also accepting 14 group customers is optimal. Table 4 shows the optimal allocation for the arrival day and the check-out day and customer's class in the event any of the scenarios. The value of the variable y^{sc}_{ijg} for days that are equal to 1 in table 5, this means the acceptance of group customers on these days.

Table 5: Optimal solution – Second stage (groups)

y^1_{ijg}	0	1	2	3	4	y^2_{ijg}	0	1	2	3	4	y^3_{ijg}	0	1	2	3	4	y^4_{ijg}	0	1	2	3	4
1.1						1.1						1.1						1.1					
1.2	1					1.2						1.2	1					1.2					
1.3						1.3						1.3						1.3					
2.1						2.1	1					2.1	1					2.1					
2.2						2.2	1					2.2						2.2					
2.3	1					2.3						2.3	1					2.3					
3.1	1					3.1						3.1		1				3.1					
3.2						3.2	1					3.2	1					3.2					
3.3						3.3						3.3						3.3	1				
4.1						4.1						4.1						4.1					
4.2						4.2	1					4.2						4.2		1			
4.3						4.3	1					4.3						4.3					
5.1	1					5.1						5.1						5.1	1				
5.2		1				5.2						5.2						5.2					
5.3						5.3						5.3						5.3					

6. Conclusions and Future Study

Hotel industry has provided a necessary platform for the implementation of revenue management approach to manage the demand and maximize the revenue. In this paper, a mathematical programming model has been developed in context of capacity allocation and inventory control, for applications in this industry. This model, aiming to maximize revenue, determines the booking limits for each class of customers (individual and group) and also specifies the amount of allocated capacity to each class according to the check-in and check-out date. In this model customers' arrival is assumed as individual and group and a specific policy is also considered to offer early discount for individual customer if booked before a certain date.

It is possible to change the booking class when the discounted price class is sold out. Because of the special characteristics of the operating environment and unspecified demand, the stochastic model under various scenarios was developed and robust optimization approach with minimizing the deviation from the average revenue and the deviation from demand for different scenarios was used to in order to provide solutions close to the optimum that are feasible with high probability.

In the presented model, in order to avoid the loss of potential profit caused by room cancellation, a limited overbooking is added to the model. In this case, there is always a risk that passengers may arrive more than the available capacity. In these cases the hotel has to provide another place which is usually in higher class. Therefore, a high penalty for booking cancellation should be considered in the model to minimize the overbooking costs. This is stated as a suggestion for future research.

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