The Study of Stability Control Systems for the Technological Processes

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Abstract

This paper presents an approach to designing a single-input-single-output (SISO) control system with enhanced robust stability characteristics and study of stability control systems for the technological process. The underlying method can be applied to objects with parametric uncertainty and control actions are chosen in the class of structurally-stable mappings with one control factor that are based on catastrophe theory. This facilitates synthesis of highly efficient control systems with a very wide range of robust stability. The latter is investigated using the method of Lyapunov functions. The results are demonstrated by an example of a control system designed in the class of structurally-stable mappings and applied to the technological process of drying in textile industry.

Keywords
Control system, parametric uncertainty, robust stability, structurally-stable mappings and stability range.

1. Introduction
Control system design is one of the key tasks in automation of all branches of industry, including machine manufacturing, energy sector, electronics, chemical and biological, metallurgical, textile, transportation, robotics, aviation, space systems, high-precision military systems, etc. It is widely known that most of the practical systems operate with some degree of parametric uncertainty that can be a result of poorly known parameters of controlled plants or unpredictable changes of their values in the process of operation. Therefore, robust stability can be viewed as one of the outstanding issues in control theory, which is also of a great practical interest. In general, the concept can be used to define the range of parameter variations that keep the system within stability boundaries. These boundaries are defined with respect to uncertain parameters of the controlled plant and can be affected by the settings of a control system.

Known methods of control system design for objects with parametric uncertainty are usually based on determination of robust stability of systems with a given structure and with linear control laws or inertia-free (relay) characteristics [1]. They are usually not suitable for designing control systems with wide range of robust stability when the range of parametric uncertainty or parametric drift is significant. Therefore, developing novel approaches to designing such systems is highly desirable.

The results reported in [2] are obtained for dynamic systems that describe self-organizing processes in physical-chemical and biological systems. The models of these systems are presented in the form of structurally-stable mappings based on catastrophe theory [3, 4] and they can be used as universal models of development and self-organization in nature. Therefore, it is of great interest to consider great uncertainty of such systems and develop an automatic control system in the class of structurally stable mappings that uses the mathematical models reflecting complex behavior of the system with many possible stable solutions [9, 10].

In this paper, we address the outstanding issues of designing robust control systems for dynamic linear objects with uncertain parameters. Special attention is paid to the selection of a control law in the class of structurally-stable mappings with one control factor [10, 15-16] that allows maximizing the range of robust stability and improves characteristics of the system.

2. Robust stability discussions
The theory of robust control began in the late 1970s and early 1980s and soon developed a number of techniques for dealing with bounded system uncertainty. Today we see many works in this field [3-14]. Robust stability is

discussed in many previously published papers. However, many of them address either linear systems or nonlinear systems with specific constraints. Successful results were reported when Lyapunov theory [4-5], [7-8] was employed to achieve robust stability of control systems with uncertain parameters. An example of a nonlinear control system considered in [6-12] shows that, without time-delay, all solutions asymptotically approach zero. When the method of Lyapunov functions is applied, it is possible to demonstrate that asymptotic stability of zero solutions can be achieved in time-delay systems as well. The results reported in [9] and [10] are of particular interest, where increased robustness based on catastrophe theory lead to structurally stable systems. In particular, [9] presents both analysis and synthesis steps of the process. However, linear systems are still emphasized and most of the non-linear systems with uncertainties are considered only as special cases. In addition, the examples discussed in these papers demonstrate stability when some system parameters (or their ratios) are negative and the systems become unstable when the values are positive. This limits the range of robustness [11-12]. Therefore this paper presents the approach of the construction of Lyapunov functions based on the geometric interpretation of the Lyapunov's direct method (also called the second method of Lyapunov) [13] and on gradient of dynamical systems in the state space of systems.

3. Materials and methods

Our approach to obtaining enhanced robust stability characteristics of a control system for dynamic objects is based on catastrophe theory [3, 4] where the main results are obtained in the form of structurally stable mappings. They are limited by and defined by the number of control factors.

3.1 Determination and analysis of the equilibrium of the system

Let a time-invariant system be defined by the following state equation

$$\dot{x} = Ax + bu, \; y = cx, \; x \in R^n, \; y \in R, \tag{1}$$

where

$$A = \begin{Vmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & . \\ 0 & 0 & 0 & \ldots & 1 \\ a_n & -a_{n-1} & -a_{n-2} & \ldots & -a_1 \end{Vmatrix}, \; c = \begin{Vmatrix} 1, & 0, & \ldots, & 0 & 0 \end{Vmatrix}, \; b = \begin{Vmatrix} 1, 1, \ldots, & 1 & , 1 \end{Vmatrix}$$

Consider a system driven by control effort $u(t) \in R^n$. Controllability condition of the system states that any *zero* element in matrix b corresponds to uncontrollable state. In our case, all states of the system given by (1) are controllable and the only output coordinate is x1. This means that an object characterized by matrix A can be controlled by a closed loop regulator in the form of a structurally-stable mapping with one control factor [9, 10] in the following form

$$u_i = -x_i^3 + k_i x_i, \; i = 1, \ldots n, \tag{2}$$

to drive the system to any arbitrary state.

Let us demonstrate that for a system described by equations (1) and (2) it is possible to find the range of stability with respect to its varying parameters and to facilitate the maximum range of stability in spite of parametric variations. By combining (1) and (2) our system description can be expanded as follows

$$\begin{cases} \dfrac{dx_1}{dt} = x_2, \\ \dfrac{dx_2}{dt} = x_3, \\ \ldots \; \ldots \; \ldots \; \ldots \; \ldots \; \ldots, \\ \dfrac{dx_n}{dt} = -x_1^3 + (k - a_n)x_1 - a_n - x_2^3 + (k_2 - \alpha_{n-1})x_2 - ,\ldots, -x_n^3 + (k_n - \alpha_1)x_n \end{cases}$$
$$y = x_1. \tag{3}$$

Let us consider equilibrium points of the system. They can be found by setting all time derivatives to zero as shown below.

$$
\begin{cases}
x_{2s} = 0, \\
x_{3s} = 0, \\
\dots\ \dots\ \dots\ \dots\ \dots\ \dots, \\
x_{n\ s} = 0 \\
-x_{1s}^3 + (k_1 - a_n)x_{1s} - x_{2s}^3 + (k_2 - a_{n-1})(x_{2s} - \dots - x_{ns}^3 + (k_n - a_1)x_n = 0
\end{cases}
\tag{4}
$$

Equation (4) can be used to find a trivial solution for the system given by (3)

$$x_{1s} = x_{2s} = \ldots = x_{ns} = 0 . \tag{5}$$

Other possible equilibrium points of (3) can be found by solving the following equation

$$-x_{is}^2 + (k_i - a_{m-i+1}) = 0,\ i = 1,\ldots\ldots, n \tag{6}$$

For negative values of k_i-a_{n-i+1}, i=$1,\ldots,n$ an imaginary solution is obtained, which cannot have any physical meaning.

When k_i-a_{n-i+1}>0, i=$1,\ldots,n$ solution to equation (6) results in the following equilibrium points

$$x_{is} = \sqrt{k_i - a_{n-i+1}},\ x_{is} = -\sqrt{k_i - a_{n-i+1}}, \quad x_{js} = 0 \quad for \quad i \neq j, i = 1,\ldots, n;\ j = 1,\ldots, n \tag{7}$$

All solutions given by equations (7) converge to (5) when the control factor is set in such a way that ki-an-i+1=0.

3.2 Investigation of the stability of equilibrium points

Let us use the method of Lyapunov functions to investigate robust stability of the equilibrium points given by (5) and (7). A sufficient condition for achieving asymptotic stability of the steady state is the existence of a positive-definite function *V(x)* such that its time derivative $\dot{\mathrm{V}}(\mathrm{x})$taken along the state trajectory given by (3) is a negative-definite function, i.e.

$$\dot{\mathrm{V}}(\mathrm{x}) = \frac{\partial \mathrm{V}\ (\mathrm{x})}{\partial \mathrm{x}} \frac{\mathrm{dx}}{\mathrm{dt}} < 0,$$

In our case, a full time derivative of the Lyapunov function can be formulated as a dot product of two vectors: gradient of the Lyapunov function ($\partial \mathrm{v}(\mathrm{x}) / \partial \mathrm{x}$) and velocity ($dx/dt$). It should be noted that the gradient vector is pointed in the direction of increase of the Lyapunov function, i.e. away from the origin. When investigating stability of a system [11, 12] the origin can be selected at one of the equilibrium points. Therefore, state equations given by (1) and (3) are always written for deviations *Δx* from the equilibrium points *Xs (x=Δx=X-Xs).* Then these state equations represent the rate of change of a vector of deviations of *x(t),* and it is fair to say that a vector that represents this rate of change (velocity) is pointed to the origin as long as the system is stable.

Assume that the Lyapunov function *V(x)* is in the form of a vector of functions *(V1 (x), V2 (x), …,Vn (x)),*and, from geometrical interpretation (Fig.1), anti-gradient components for the Lyapunov functions ($\partial \mathrm{V_i}(\mathrm{x}) / \partial \mathrm{x}$, i=$1,\ldots,n$) are equal in their absolute value to the components of the velocity vector *(dx/dt)*, i.e.

$$-\frac{\mathrm{dx}^i}{\mathrm{dt}} = \frac{\partial \mathrm{V}^i(x)}{\partial \mathrm{x}^1} + \frac{\partial \mathrm{V}^i\ (\mathrm{x})}{\partial \mathrm{x}^2} + \ldots + \frac{\partial \mathrm{V}^i\ (\mathrm{x})}{\partial \mathrm{Xn}},\ \mathrm{i} = 1, \ldots, \mathrm{n}$$

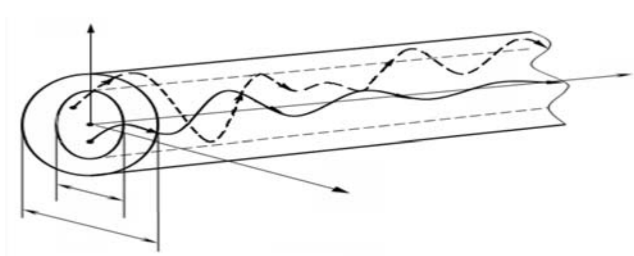

Figure 1. Geometric interpretation of Lyapunov function.

Then it follows that

$$\begin{cases} -\dfrac{dx_1}{dt} = \dfrac{\partial V_1(x)}{\partial x_2} = -x_2 \\ -\dfrac{dx_2}{dt} = \dfrac{\partial V_2\ (x)}{\partial x_3} = -x_3 \\ \ldots \qquad \ldots \qquad \ldots \\ -\dfrac{dx_{n-1}}{dt} = \dfrac{\partial V_{n-1}(x)}{\partial x_n} = -x_n \\ -\dfrac{dx_n}{dt} = \dfrac{\partial V_n(x)}{\partial x_1} + \dfrac{\partial V_n(x)}{\partial x_2} +,\ldots, + \dfrac{\partial V_n(x)}{\partial x_n} = \\ = -x_1^3 + (k_1 - a_n)x_1 - x_2^3 + (k_2 - a_{n-1})x_2 -,\ldots, -x_n^3 + (k_n - a_1)x \end{cases} \tag{8}$$

Then time derivatives of the components of the Lyapunov function defined in the vector form will be as follows for the equilibrium state given by (5):

$$\begin{cases} \dfrac{dV_1(x)}{dt} = -x_2^2 \\ \dfrac{dV_1(x)}{dt} = -x_3^2 \\ \ldots \qquad \ldots \\ \dfrac{dV_{n-1}(x)}{dt} = -x_n^2 \\ \dfrac{dV_n(x)}{dt} = -\left[-x_1^3 + (k_1 - a_n)x_1 - x_2^3 + (k_2 - a_{n-1})x_2 -,\ldots, -x_n^3 + (k_n - a_1)x_n\right]^2 \end{cases} \tag{9}$$

It follows from equation (9) that time derivatives of the components of the Lyapunov vector-function will always be negative. Also, they can be combined together to form the following expression $V(x) = \dot{V}_1(x) + \dot{V}_2(x) + \cdots + \dot{V}_n(x)$.

Vector components of the Lyapunov function *(V_i, i=1,…, n)* are obtained from the components of the gradient vector. Then the Lyapunov function in the scalar form can be derived as follows

$$V(x) = \frac{1}{4}x_1^4 + \frac{1}{2}(a_n - k_1^2)x_1^2 + \frac{1}{4}x_2^4 + \frac{1}{2}(a_{n-1} - k_2 - 1)x_2^2 +,\ldots, + \frac{1}{4}x_n^4 + \frac{1}{2}(a_1 - k_n - 1)x_n^2, \tag{10}$$

Stability of the equilibrium point given by (5) for a system described by equation (3) can be obtained by making sure that the function (9) is negative-definite and the Lyapunov function given by (10) is positive-definite such that

$$\begin{cases} a_n - k_1 > 0 \\ a_{n-1} - k_2 - 1 > 0 \\ \ldots \quad \ldots \quad \ldots \\ a_1 - k_n - 1 > 0 \end{cases} \quad \text{or} \quad \begin{cases} a_n - k_1 > 0 \\ a_{n-1} - k_2 > 1 \\ \ldots \quad \ldots \quad \ldots \\ a_1 - k_n > 1 \end{cases} \tag{11}$$

Next, we investigate stability of the equilibrium point (7) by writing state equations (3) to describe deviation from this point. For the sake of generality, let us assume that all parameters of the system are uncertain, and they can be both in the negative range and in the positive range, then

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \ldots \quad \ldots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = -x_1^3 + 2(a_n - k_1)x_1 - x_2^3 + 2(a_{n-1} - k_2)x_2 \ldots - \ldots - x_n^3 + 2(a_1 - k_n)x_n \\ y = x_1 \end{cases} \tag{12}$$

The Lyapunov function is guaranteed to be positive-definite for the equilibrium point given by (7), which only exists, when $k_i - a_{n-i+1} > 0$, *i=1,…, n,* if

$$a_n - k_1 > 0,\ a_{n-1} - k_2 - \frac{1}{2} > 0,\ a_{n-2} - k_3 - \frac{1}{2} > 0,\ \ldots,\ a_1 - k_n - \frac{1}{2} > 0$$

Therefore, the system described by (1) can be augmented by a control law in the form of structurally-stable mapping with one control factor to assure robust stability for wide range of variation of parameters a_i, $i=1,\ldots n$. It turns out that the equilibrium point $x_{1s}=x_{2s}=\ldots=x_{ns}=0$ is globally asymptotically stable when conditions given by (11) are satisfied und unstable otherwise. When k=0 bifurcation occurs and new equilibrium points are created; therefore, it is possible to design a stable control system for any variation of parameter a_i, $i=1,\ldots n$.

3. Results and Discussion

3.1 Application Example

Let us consider an industrial application example of a material drying system that evaporates moisture from textiles by direct contact with a heating element. Let as assume that the process of drying is affected by both vapor concentration in the surrounding air, which is affected by the speed of the drying process, and by the heating element, which can be viewed as a first-order system [13].

When the speed of drying has an integral relationship $1/s$, the controlled plant can be described by the following transfer function

$$W_0(s) = \frac{k_0}{s(T_0 s + 1)} \tag{13}$$

Where: T_0 – time constant, and k_0 – constant gain of the controlled plant. Both parameters are uncertain and can change in an unpredictable fashion in the process of operation.

A servomechanism that has integral characteristics and integration constant T_1 is often used as an actuating mechanism in the systems that control heating processes. Its transfer function can be written as [14]

$$W_1(s) = \frac{k_1}{T_1 s},$$

where: k_1 – gain of the actuating mechanism.

In the simplest case, an automatic control system for materials drying processes can be implemented with a gain controller k_p, as shown in Fig. 2.

Let us investigate stability of the above system. The closed-loop transfer function for the above system can be found in the following form:

$$\Phi(s) = \frac{k}{T_1 s^2 (T_0 s + 1) + k}, \tag{14}$$

where: $k=kpk_1k_0$ – gain of the closed-loop system.

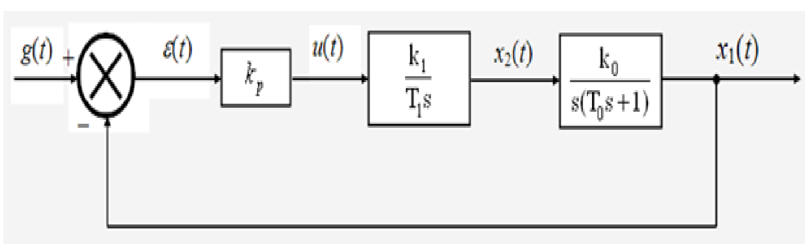

Figure 2.Block diagram of a system for materials drying with proportional gain controller.

Characteristic equation of the closed-loop system can be found as follows

$$T_1 T_0 \lambda^3 + T_1 \lambda^2 + k = 0. \tag{15}$$

Let us obtain Hurwitz matrix from (15)

$$\Delta_3 = \begin{Vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{Vmatrix} = \begin{Vmatrix} T_1 & T_0 T_1 & 0 \\ k & 0 & T_1 \\ 0 & 0 & k \end{Vmatrix}.$$

By using the Hurwitz criterion we can demonstrate that the system given by (14) that has characteristic equation (15) is unstable for any k, i.e.

$$\Delta_1 = T_1 > 0,\ \Delta_2 = -kT_0 T_1 > 0 \text{, for k< 0;}$$

$$\Delta_3 = -k^2 T_0 T_1 < 0 \text{, for any value of } k.$$

The above results demonstrate that the system (18) is unstable for any value of k, since the time constants T_0 and T_1 are always positive.

Let us utilize a controller in the form of a fold catastrophe function, which was demonstrated to assure robust stability, as discussed in the Lyapunov analysis in the previous section. Assume that the state equation describing dynamics of the error as a result of operation of an automatic control system with enhanced robust stability characteristics can be written as

$$\frac{dx}{dt} = Ax + Bu \tag{16}$$

where:

$$x = \begin{Vmatrix} x_1 \\ x_2 \\ x_3 \end{Vmatrix},\quad A = \begin{Vmatrix} 0 & 1 & 0 \\ 0 & \frac{k_0}{T_0} & \frac{k_0}{T_0} \\ 0 & 0 & 0 \end{Vmatrix},\quad B = \begin{Vmatrix} 0 \\ \frac{k_0}{T_0} \\ \frac{k_1}{T_1} \end{Vmatrix},\quad u = -x_1^3 + k_p x_1.$$

In the above equations, A is the inertia matrix of the controlled plant, u is the scalar control input, and B is the control matrix. It is assumed that state variables x_2 and x_3 of the drying process are controllable. The state equation (16) can be expanded as follows

$$\begin{cases} \frac{dx_1}{dt} = x_2, \\ \frac{dx_2}{dt} = -\frac{k_0}{T_0} x_1^3 + \frac{k_p k_0}{T_9} x_1 - \frac{1}{T_0} x_2 + \frac{k_0}{T_0} x_3, \\ \frac{dx_3}{dt} = -\frac{k_1}{T_1} x_1^3 + \frac{k_1 k_p}{T_1} x_1. \end{cases} \tag{17}$$

A system of equations (17) can be represented by a control system whose block diagram is shown in Fig.3.

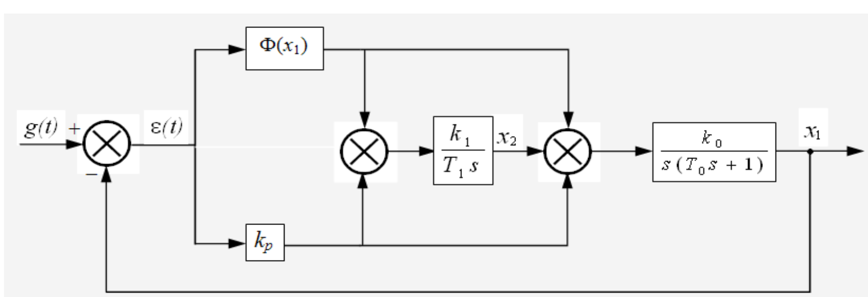

Figure 3. Block diagram of an automatic control system for materials drying with enhanced robust stability characteristics.

In the above figure, a linear function is defined as $\Phi(x_1) = x_1^3$, and the control law is selected in the form of fold catastrophe, a structurally-stable function with one control factor

$$u = -x_1^3 + k_p x_1.$$

Equilibrium points of the system given by (17) can be described by the following equations

$$\begin{cases} x_{2s} = 0, \\ -\frac{k_0}{T_0} x_{1s}^3 + \frac{k_0 k_p}{T_0} x_{1s} - \frac{1}{T_0} x_{2s} + \frac{k_0}{T_0} x_{3s} = 0, \\ -\frac{k_1}{T_1} x_{1s}^3 + \frac{k_1 k_p}{T_1} x_{1s} = 0. \end{cases} \quad (18).$$

A system of equations (18) has the following solutions

$$x_{1s}^1 = x_{2s} = x_{3s} = 0 \quad (19)$$

and

$$x_{1s}^{2,3} = \pm\sqrt{k_p}, \quad x_{2s} = x_{3s} = 0 . \quad (20)$$

Equilibrium points given by (24) converge to (19) when *kp=0* and diverge from it when *kp>0*.

Let us investigate stability of equilibrium points (19) and (20) for the system described by (19) by using the principle of stability of linearized systems.

$$\begin{cases} \frac{dx_1}{dt} = x_2, \\ \frac{dx_2}{dt} = -3\frac{k_0}{T_0}(x_{1s})^2 x_1 + \frac{k_0 k_p}{T_0} x_1 - \frac{1}{T_0} x_2 + \frac{k_0}{T_0} x_3, \\ \frac{dx_3}{dt} = -3\frac{k_1}{T_1}(x_{1s})^2 x_1 + \frac{k_1 k_p}{T_1} x_1. \end{cases} \quad (21).$$

Stability of the first equilibrium point is found from

$$\begin{cases} \frac{dx_1}{dt} = x_2, \\ \frac{dx_2}{dt} = \frac{k_0 k_p}{T_0} x_1 - \frac{1}{T_0} x_2 + \frac{k_0}{T_0} x_3, \\ \frac{dx_3}{dt} = \frac{k_1 k_p}{T_1} x_1. \end{cases} \quad (22).$$

A set of equations (22) corresponds to the following matrix representing the closed-loop system

$$G = \begin{Vmatrix} 0 & 1 & 0 \\ \frac{k_0 k_p}{T_0} & -\frac{1}{T_0} & \frac{k_0}{T_0} \\ \frac{k_1 k_p}{T_0} & 0 & 0 \end{Vmatrix}$$

Its characteristic equation can be found as

$$f(s) = |G - Is| = s^3 + \frac{1}{T_0}s^2 - \frac{k_0 k_p}{T_0}s - \frac{k_0 k_1 k_p}{T_0 T_1} = 0 . \quad (23).$$

For equilibrium points given by (20) linearized system (21) is modified as follows

$$\begin{cases} \dfrac{dx_1}{dt} = x_2, \\ \dfrac{dx_2}{dt} = -2\dfrac{k_0 k_p}{T_0} x_1 - \dfrac{1}{T_0} x_2 + \dfrac{k_0}{T_0} x_3, \\ \dfrac{dx_3}{dt} = -2\dfrac{k_1 k_p}{T_1} x_1. \end{cases} \qquad (24)$$

A necessary condition of stability can be satisfied for any positive value of *kp (kp>0).* In general, a necessary and sufficient condition of stability of system (24) is given by positive values of Hurwitz determinants

$$\Delta_1 = \frac{1}{T_0} > 0,\ \Delta_2 = \begin{vmatrix} \dfrac{1}{T_0} & 1 \\ 2\dfrac{k_0 k_1 k_p}{T_0 T_1} & 2\dfrac{k_0 k_p}{T_0} \end{vmatrix} = 2\frac{k_0 k_p}{T_0}\left(\frac{1}{T_0} - \frac{k_1}{T_1}\right) > 0,$$

$$\Delta_3 = \begin{vmatrix} \dfrac{1}{T_0} & 1 & 0 \\ 2\dfrac{k_0 k_1 k_p}{T_0 T_1} & 2\dfrac{k_0 k_p}{T_0} & \dfrac{1}{T_0} \\ 0 & 0 & 2\dfrac{k_0 k_1 k_p}{T_0 T_1} \end{vmatrix} = 4\frac{k_0^2 k_1 k_p^2}{T_0^2 T_1}\left(\frac{1}{T_0} - \frac{k_1}{T_1}\right) > 0 \quad .$$

Hence, stability of the equilibrium point (20) can be assured if the gain of the nonlinear regulator *kp* is positive (*kp> 0*) and additional condition is also satisfied [15-17]. Fig.4 shows transient response curves of the automatic control system with enhanced robustness characteristics.

4. Simulation Results

Conditions of robust stability discussed earlier in our paper can be demonstrated by simulation experiments [15-17]. Fig. 4 shows trajectories of the state variables of the system with a proportional gain controller, as pictured in Fig. 2, when parameter *kp* assumes both positive and negative values.

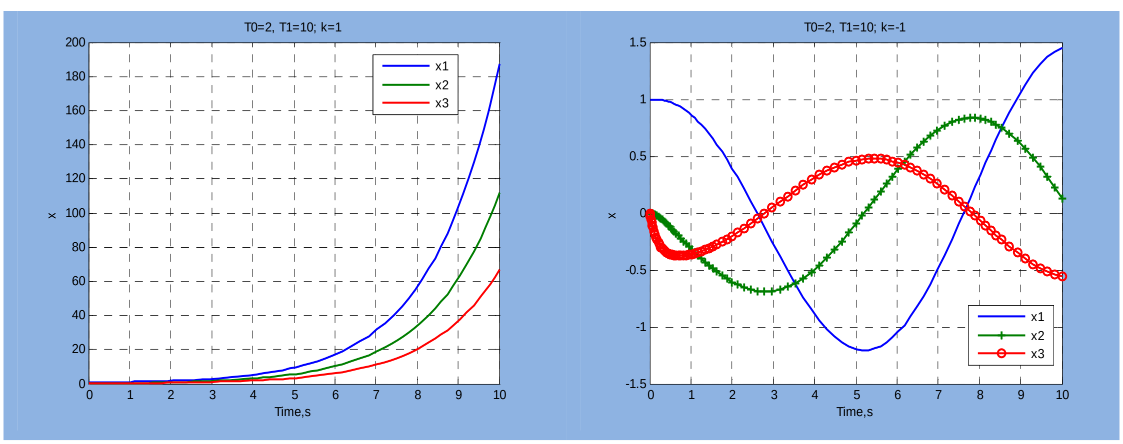

Figure 4.Transient response curves of the state variables of automatic control system with proportional gain controller: a) positive value of *k*; b) negative value of *k*

The results featured in the above figure demonstrate that conditions of stability are not satisfied for either positive or negative values of the proportional gain controller.

The following results (Fig. 5-6) present Simulink outputs for the system that was built in the form of a structurally-stable mapping, whose stability was proven by the Lyapunov method discussed earlier.

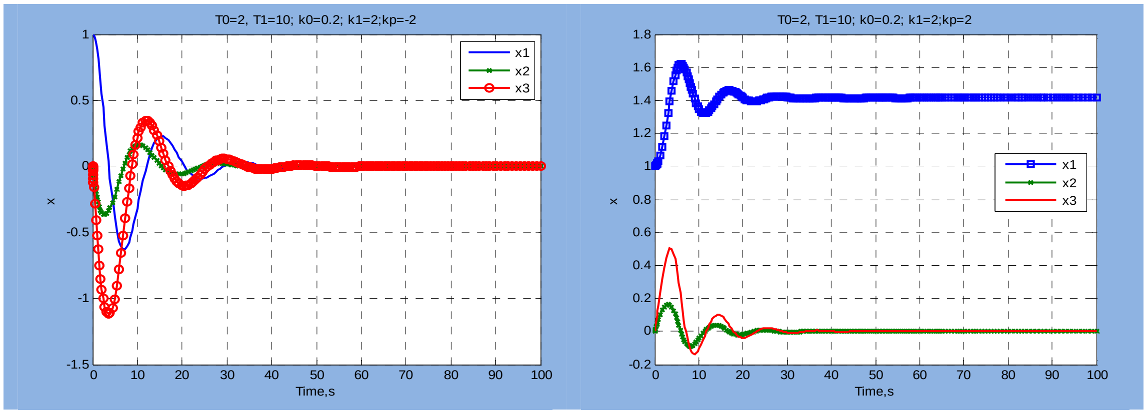

Figure 5. Transient response curves for the state variables of automatic control system with enhanced robust stability characteristics: a) positive value *of k=kokp*; b) negative value of *k=kokp*

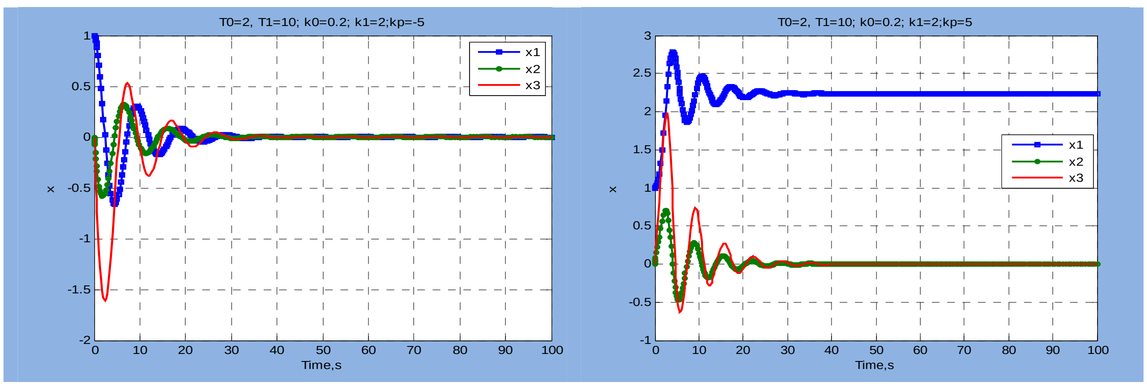

Figure 6. Additional transient response curves for the state variables of automatic control system with enhanced robust stability characteristics: a) positive value of *k=kokp*; b) negative value of *k=kokp*

The above figures demonstrate that a controller can be selected in a special way in the form of a structurally stable function, such that any change in proportional coefficient of the controlled plant or a regulator would still result in a stable system, as long as the condition given by (22) is satisfied.

5. Conclusions

The need for control systems with robust stability is driven by practical scientific and technical applications. The outstanding problems of modeling and control of the processes in technical fields, economics, biology and other areas characterized by substantial uncertainty require enhanced robust stability characteristics to assure that the designed system does not exhibit chaotic behavior and guarantees validity of the models and reliable operation. Practical results obtained for the systems that were designed to assure robust stability demonstrate dynamically safe operation.

In this paper, we have formulated an approach to constructing control systems with enhanced robust stability characteristics for linear controlled plants by selecting control laws in the class of structurally-stable mappings with one control factor and using Lyapunov theory to determine its range of stability. It was demonstrated that the system has asymptotically stable equilibrium points for both positive and negative values of the uncertain parameters of the controlled plant. When the values of uncertain parameters cross zero, bifurcation takes place and new stable points are created. All equilibrium points cannot exist simultaneously; hence we can build a control system that will be stable for any change of uncertain parameters, but can transition from one stable solution to another.

By using an example of a technological process of materials drying we demonstrated that the application of structurally-stable mappings to a system that was unstable for any value of the uncertain parameter not only stabilizes the system, but makes its stability immune to any change of the unknown parameter. Simulation results obtained for both positive and negative values of the uncertain gain support our theoretical conclusions.

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