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Optimal Ordering Policies for Inventory Problems in Supermarkets under Stochastic Demand: A Case Study of Milk Powder Product

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Abstract

Supermarkets continually face the challenge of optimizing ordering decisions of products in a stochastic demand environment. In this paper, a stochastic inventory model is proposed for selected supermarkets that optimize inventory ordering decisions and costs of milk powder under demand uncertainty. In our model, the distribution of demand over successive periods is dependent on a Markov chain that represents possible states of demand for milk powder product. The decision of when to order is made using dynamic programming over a finite period planning horizon. The approach demonstrates the existence of an optimal state-dependent ordering decision and the corresponding total inventory costs in supermarkets.

Keywords
Inventory problems, optimal ordering policies, stochastic demand, supermarkets

1 Introduction

Optimizing inventory ordering decisions is a considerable challenge when the demand for items in inventory follows a stochastic trend. Two major problems are usually encountered: (i) determining the most desirable period during which to order additional units of the item in question and (ii) determining the optimal ordering decision given a periodic review inventory system when demand is uncertain. In this paper, an inventory system is considered whose goal is to optimize the decision to order and total costs associated with ordering and holding inventory. At the beginning of each period, a major decision has to be made, namely whether to order additional units of milk powder or postpone ordering and utilize the available units in inventory. The paper is organized as follows. After reviewing the previous work done, a mathematical model is proposed where initial consideration is given to the process of estimating the model parameters. The model is solved thereafter and applied to special case studies. Some final remarks lastly follow.

Tadashi and Takeshi (1993) formulated the stochastic EOQ type models with discounting using Gaussian processes in the context of the classical EOQ model. Numerical properties of the order quantities that minimize expected costs for various model parameters were examined. However, optimality guidelines on when to order or not order were not specified. Hein and Wessels (1998) provide an analysis of the demand for dairy products. The structure of dairy product demand is estimated using household food consumption survey data. Under the assumption of a two-stage budgeting procedure, a complete demand system for food incorporating demand graphic effects is explained. Next, using the demand relations estimated from cross-cutting section data, prediction interval tests utilizing time-series data are performed for milk. In this model, optimality guidelines in terms of the ordering decisions are insufficient. The stochastic EOQ-type models to establish inventory policies were examined by Berman and Perry (2001). Output can be interpreted by a random demand and the input by a deterministic production plus random returns. In this study, optimality guidelines on the decision of when to order and inventory costs are not explicit. Research results also explore the structure of optimal ordering policies for stochastic inventory system with minimum order quantity Yao & Kateshaki (2006). In this article, ordering quantity is either zero or at least a minimum order size. The impact of such inventory policies on the cost structures of the stocked item is however not explicit. In a similar context, Broekmeullen (2009) proposed a replenishment policy for a perishable inventory system based on estimated aging and retrieval behavior. The model takes into account the age of inventories and which requires only very simple calculations. The model has profound insights especially in terms of the randomness of demand. However, the model is restricted to perishable products. According to Roychowdhury (2009), an optimal policy for a stochastic inventory model for deteriorating items with time-dependent selling price is feasible. The rate of deterioration of the items is assumed to be constant over time. The selling price decreases monotonically at a constant rate with the deterioration of items. The demand and lead time both are random. A profit-maximization model is formulated and solved for optimum order quantity. The model provides some intriguing insights to the problem. However, the holding and shortage costs of inventory as well as the corresponding optimal inventory policies must be embedded in the model to provide optimal results.

In this paper, an inventory system is considered whose goal is to optimize the ordering policy and the total costs associated with inventory in supermarkets. At the beginning of each period, a major decision has to be made, namely whether to order units of the stocked item or to postpone ordering and utilize the available units in stock. The paper is organized as follows. After describing the mathematical model in §2, consideration is given in §3 to the process of estimating the model parameters. The model is solved in §4 and applied to a special case study in §5.Some final remarks lastly follow in §6.

2 Model Development

We consider a designated number of supermarkets where the demand during each time period over a fixed planning horizon is classified as either *favorable* (denoted by state F) or *unfavorable* (denoted by state U) and the demand of any such period is assumed to depend on the demand of the preceding period. The transition probabilities over the planning horizon from one demand state to another may be described by means of a Markov chain. Suppose one is interested in determining an optimal course of action, namely to order additional stock units (a decision denoted by Z=1) or not to order additional units (a decision denoted by Z=0) during each time period over the planning horizon, where Z is a binary decision variable. Optimality is defined such that the lowest expected total inventory costs are accumulated at the end of N consecutive time periods spanning the planning horizon under consideration. In this paper, a two-period (N=2) planning horizon is considered.

2.1 Model variables and parameters

Varying demand is modeled by means of a Markov chain with *state transition matrix* $Q^Z(S)$ where the entry $Q^Z_{ij}(S)$ in row i and column j of the transition matrix denotes the probability of a transition in demand from state iϵ**{**F , U**}** to state jϵ**{**U , F**}** at supermarket S ϵ{1,2,3}under a given ordering policy Zϵ{0,1}. The number of customers observed in the system and the number of units demanded during such a transition is captured by the *customer matrix* $N^Z(S)$ and *demand matrix* $D^Z(S)$ respectively. Furthermore, denote the number of units in inventory and the total (ordering, holding and shortage) cost during such a transition by the *inventory matrix* $I^Z(S)$ and the *cost matrix* $C^Z(S)$ respectively. Also, denote the *expected future cost* , the already *accumulated total cost* at the end of period *n* when the demand is in state iϵ{ F, U }for a given ordering policy Zϵ{0,1} by respectively $e^Z_i(S)$ and $a^Z_i(S,n)$ and let $e^Z(S) = [e^Z_F(S), e^Z_U(S)]^T$ and $a^Z(S,n) = [a^Z_F(S,n), a^Z_U(S,n)]^T$ where "T" denotes matrix transposition.

2.2 Finite period dynamic programming formulation

Recalling that the demand can either be in state F or in state U, the problem of finding an optimal ordering policy may be expressed as a finite period dynamic programming model.
Let $C_n(i,S)$ denote the optimal expected total inventory costs accumulated during the periods *n,n+1,…...,**N*** given that the state of the system at the beginning of period *n* is
*i*ϵ{ F,U }.The recursive equation relating C_n and C_{n+1} is

$$C_n(i,S) = min_Z\{Q^Z_{iF}(S)(C^Z_{iF}(S) + C_{n+1}(F,S)), Q^Z_{iU}(S)(C^Z_{iU}(S) + C_{n+1}(U,S))\} \quad (1)$$

$$i\epsilon\{F,U\} \qquad n = 1, 2, \ldots\ldots, N$$

together with the final conditions

$$C_{N+1}(F,S) = C_{N+1}(U,S) = 0$$

This recursive relationship may be justified by noting that the cumulative total inventory costs

$$C^Z_{ij}(S) + C_{n+1}(j)$$

resulting from reaching state j ϵ{ F,U } at the start of period n+1 from state i ϵ{ F,U } at the start of period n occurs with probability $Q^Z_{ij}(S)$.

The dynamic programming recursive equations become:

$i\epsilon\{F,U\}$ $\quad n = 1, 2\ldots\ldots\ldots N$ (2)

$S = \{1, 2, 3\}$

$$C_N(i,S) = min_Z\{c_i^Z(S)\} \quad (3)$$

result where (3) represents the Markov chain stable state.

3 Computing $Q^Z(S)$ and $C^Z(S)$

The demand transition probability from state iϵ{ F, U } to state j ϵ{ F, U },given ordering policy Z ϵ{ 0,1 } may be taken as the number of customers observed at supermarket S with demand initially in state i and later with demand changing to state j, divided by the sum of customers over all states. That is,

$$Q_{ij}^Z(S) = N_{ij}^Z(S)/[N_{iF}^Z(S) + N_{iU}^Z(S)] \;,\; i\epsilon\{F,U\}\;,\; S\epsilon\{0,1\} \quad (4)$$

When demand outweighs on-hand inventory, the inventory cost matrix $C^Z(S)$ may be computed by means of the relation

$$C^Z(S) = (c_0 + c_h + c_s)(D^Z(S) - I^Z(S)).$$

where c_0 denotes the unit ordering cost, c_h denotes the unit holding cost and c_s denotes the unit shortage cost. Therefore,

$$C_{ij}^Z(S) = \begin{cases} (c_0 + c_h + c_s)(D_{ij}^Z(S) - I_{ij}^Z(S) & if \;\; D_{ij}^Z(S) > I_{ij}^Z(S) \\ 0 & if \;\; D_{ij}^Z(S) \le I_{ij}^Z(S) \end{cases} \quad (5)$$

for all i,jϵ{ F, U } and Zϵ{0,1}.

A justification for expression (5) is that $D^Z_{ij}(S) - I^Z_{ij}(S)$ units must be ordered in order to meet the excess demand. Otherwise ordering is cancelled when demand is less than or equal to the on-hand inventory.
The following conditions must however, hold.

1. Z=1 when $c_0 > 0$ and Z = 0 when $c_0 = 0$
2. $c_s > 0$ when shortages are allowed, and $c_s = 0$ when shortages are not allowed.

4 Computing an Optimal Ordering Policy

The optimal ordering policy and inventory costs are found in this section for each time period separately.

4.1 Optimization during period 1

Favorable demand (state F)

$$Z = \begin{cases} 1 & if \;\; c_F^1(S) < c_F^0(S) \\ 0 & if \;\; c_F^1(S) \ge c_F^0(S) \end{cases}$$

$$C_1(F,S) = \begin{cases} c_F^1(S) & if Z = 1 \\ c_F^0(S) & if Z = 0 \end{cases}$$

Unfavorable demand (state U)

$$Z = \begin{cases} 1 & if \;\; c_U^1(S) < c_U^0(S) \\ 0 & if \;\; c_U^1(S) \ge c_U^0(S) \end{cases}$$

$$C_1(U,S) = \begin{cases} c_U^1(S) & if Z = 1 \\ c_U^0(S) & if \; Z = 0 \end{cases}$$

Using (2),(3) and recalling that $a^Z_i(S,2)$ denotes the already accumulated total inventory costs at the end of period 1 as a result of decisions made during that period, it follows that

$$a_i^Z(S,2) = c_i^Z(S) + Q_{iF}^Z(S) min\{c_F^1(S), c_F^0(S)\} + Q_{iU}^Z(S) min\{c_U^1(S), c_U^0(S)\}$$

$$= c_i^Z(S) + Q_{iF}^Z(S)C_2(F,S) + Q_{iU}^Z(S)C_2(U,S)$$

4.2 Optimization during period 2

Favorable demand (state F)

$$Z = \begin{cases} 1 & if \;\; a_F^1(S,2) < a_F^0(S,2) \\ 0 & if \;\; a_F^1(S,2) \geq a_F^0(S,2) \end{cases}$$

$$C_2(F,S) = \begin{cases} a_F^1(S,2) & if Z = 1 \\ a_F^0(S,2) & if Z = 0 \end{cases}$$

Unfavorable demand (state U)

$$Z = \begin{cases} 1 & if \;\; a_U^1(S,2) < a_U^0(S,2) \\ 0 & if \;\; a_U^1(S,2) \geq a_U^0(S,2) \end{cases}$$

$$C_2(U,S) = \begin{cases} a_U^1(S,2) & if Z = 1 \\ a_U^0(S,2) & if \; Z = 0 \end{cases}$$

5 Case Study

In order to demonstrate use of the model in §3, real case applications from *Shoprite supermarket, Game supermarket* and *Uchumi supermarket* in Uganda are presented in this section. The demand for milk powder fluctuates every week in each respective supermarket. The supermarkets want to avoid excess inventory when demand is Unfavorable (state U) or running out of stock when demand is Favorable (state F) and hence seek decision support in terms of an optimal inventory policy of when to order and the associated inventory costs of milk powder product. Specifically, a recommendation as to the ordering policy of milk powder over the next two-week period is required in each supermarket. The following matrices were captured at each supermarket:

5.1 Data collection

Samples of customers were taken at individual supermarkets. Past data revealed the following demand pattern and inventory levels of milk powder product over the first week of the month when demand was Favorable (F) or Unfavorable (U).At Shoprite supermarket, when additional units were ordered (Z=1),

$$N^1(1) = \begin{bmatrix} N_{FF}^1(1) & N_{FU}^1(1) \\ N_{UF}^1(1) & N_{UU}^1(1) \end{bmatrix} \qquad N^1(1) = \begin{bmatrix} 91 & 71 \\ 64 & 13 \end{bmatrix}$$

$$D^1(1) = \begin{bmatrix} D_{FF}^1(1) & D_{FU}^1(1) \\ D_{UF}^1(1) & D_{UU}^1(1) \end{bmatrix} \qquad D^1(1) = \begin{bmatrix} 156 & 115 \\ 107 & 11 \end{bmatrix}$$

$$I^1(1) = \begin{bmatrix} I_{FF}^1(1) & I_{FU}^1(1) \\ I_{UF}^1(1) & I_{UU}^1(1) \end{bmatrix} \qquad I^1(1) = \begin{bmatrix} 95 & 93 \\ 93 & 94 \end{bmatrix}$$

When additional units were not ordered (Z=0),

$$N^0(1) = \begin{bmatrix} N_{FF}^0(1) & N_{FU}^0(1) \\ N_{UF}^0(1) & N_{UU}^0(1) \end{bmatrix} \qquad N^0(1) = \begin{bmatrix} 82 & 50 \\ 56 & 25 \end{bmatrix}$$

$$D^0(1) = \begin{bmatrix} D_{FF}^0(1) & D_{FU}^0(1) \\ D_{UF}^0(1) & D_{UU}^0(1) \end{bmatrix} \qquad D^0(1) = \begin{bmatrix} 123 & 78 \\ 78 & 15 \end{bmatrix}$$

$$I^0(1) = \begin{bmatrix} I_{FF}^0(1) & I_{FU}^0(1) \\ I_{UFF}^0(1) & I_{UU}^0(1) \end{bmatrix} \qquad I^0(1) = \begin{bmatrix} 43.5 & 45 \\ 46.5 & 45.5 \end{bmatrix}$$

At Game supermarket, when additional units were ordered (Z=1),

$$N^1(2) = \begin{bmatrix} N^1_{FF}(2) & N^1_{FU}(2) \\ N^1_{UF}(2) & N^1_{UU}(2) \end{bmatrix} \qquad N^1(2) = \begin{bmatrix} 48 & 55 \\ 59 & 13 \end{bmatrix}$$

$$D^1(2) = \begin{bmatrix} D^1_{FF}(2) & D^1_{FU}(2) \\ D^1_{UF}(2) & D^1_{UU}(2) \end{bmatrix} \qquad D^1(2) = \begin{bmatrix} 93 & 60 \\ 59 & 11 \end{bmatrix}$$

$$I^1(2) = \begin{bmatrix} I^1_{FF}(2) & I^1_{FU}(2) \\ I^1_{UF}(2) & I^1_{UU}(2) \end{bmatrix} \qquad I^1(2) = \begin{bmatrix} 145 & 145 \\ 78.5 & 79.5 \end{bmatrix}$$

When additional units were not ordered (Z=0),

$$N^0(2) = \begin{bmatrix} N^0_{FF}(2) & N^0_{FU}(2) \\ N^0_{UF}(2) & N^0_{UU}(2) \end{bmatrix} \qquad N^0(2) = \begin{bmatrix} 54 & 46 \\ 45 & 11 \end{bmatrix}$$

$$D^0(2) = \begin{bmatrix} D^0_{FF}(2) & D^0_{FU}(2) \\ D^0_{UF}(2) & D^0_{UU}(2) \end{bmatrix} \qquad D^0(2) = \begin{bmatrix} 72 & 77 \\ 75 & 11 \end{bmatrix}$$

$$I^0(2) = \begin{bmatrix} I^0_{FF}(2) & I^0_{FU}(2) \\ I^0_{UF}(2) & I^0_{UU}(2) \end{bmatrix} \qquad I^0(2) = \begin{bmatrix} 81 & 78.5 \\ 79.5 & 78.5 \end{bmatrix}$$

At Uchumi supermarket, when additional units were ordered (Z=1),

$$N^1(3) = \begin{bmatrix} N^1_{FF}(3) & N^1_{FU}(3) \\ N^1_{UF}(3) & N^1_{UU}(3) \end{bmatrix} \qquad N^1(3) = \begin{bmatrix} 57 & 62 \\ 62 & 9 \end{bmatrix}$$

$$D^1(3) = \begin{bmatrix} D^1_{FF}(3) & D1_{FU}(3) \\ D^1_{UF}(3) & D1_{UU}(3) \end{bmatrix} \qquad D^1(3) = \begin{bmatrix} 82 & 93 \\ 84 & 9 \end{bmatrix}$$

$$I^1(3) = \begin{bmatrix} I^1_{FF}(3) & I^1_{FU}(3) \\ I^1_{UF}(3) & I^1_{UU}(3) \end{bmatrix} \qquad I^1(3) = \begin{bmatrix} 67.5 & 68.5 \\ 72 & 68.5 \end{bmatrix}$$

When additional units were not ordered (Z=0),

$$N^0(3) = \begin{bmatrix} N^0_{FF}(3) & N^0_{FU}(3) \\ N^0_{UF}(3) & N^0_{UU}(3) \end{bmatrix} \qquad N^0(3) = \begin{bmatrix} 36 & 53 \\ 56 & 9 \end{bmatrix}$$

$$D^0(3) = \begin{bmatrix} D^0_{FF}(3) & D^0_{FU}(3) \\ D^0_{UF}(3) & D^0_{UU}(3) \end{bmatrix} \qquad D^0(3) = \begin{bmatrix} 51 & 70 \\ 72 & 10 \end{bmatrix}$$

$$I^0(3) = \begin{bmatrix} I^0_{FF}(3) & I^0_{FU}(3) \\ I^0_{UF}(3) & I^0_{UU}(3) \end{bmatrix} \qquad I^0(3) = \begin{bmatrix} 105 & 100 \\ 100 & 50 \end{bmatrix}$$

The following unit ordering, holding and shortage costs (in UGX) of milk powder were captured at the three supermarkets:

Shoprite:
$c_0 = 4500, \quad c_h = 1200, \quad c_s = 300$

Game:
$c_0 = 4800, \quad c_h = 900, \quad c_s = 300$

Uchumi:
$c_0 = 5100, \quad c_h = 600, \quad c_s = 300$

5.2 Computation of Model parameters

Using (4) and (5), the state transition matrices and inventory cost matrices at each respective supermarket for week 1 are

$$Q^1(1) = \begin{bmatrix} 0.5697 & 0.4303 \\ 0.8312 & 0.1688 \end{bmatrix} \qquad C^1(1) = \begin{bmatrix} 0.366 & 0.132 \\ 0.084 & 0.025 \end{bmatrix}$$

$$Q^1(2) = \begin{bmatrix} 0.4660 & 0.5340 \\ 0.8429 & 0.1571 \end{bmatrix} \qquad C^1(2) = \begin{bmatrix} 0.047 & 0.077 \\ 0.018 & 0.062 \end{bmatrix}$$

$$Q^1(3) = \begin{bmatrix} 0.4790 & 0.5210 \\ 0.8732 & 0.1268 \end{bmatrix} \qquad C^1(3) = \begin{bmatrix} 0.087 & 0.147 \\ 0.072 & 0.054 \end{bmatrix}$$

respectively, for the case when additional units are ordered during week 1, while these matrices are given by

$$Q^0(1) = \begin{bmatrix} 0.6212 & 0.3722 \\ 0.6914 & 0.3086 \end{bmatrix} \qquad C^0(1) = \begin{bmatrix} 0.477 & 0.198 \\ 0.189 & 0.037 \end{bmatrix}$$

$$Q^0(2) = \begin{bmatrix} 0.5400 & 0.4600 \\ 0.8036 & 0.1964 \end{bmatrix} \qquad C^0(2) = \begin{bmatrix} 0.008 & 0.001 \\ 0.004 & 0.061 \end{bmatrix}$$

$$Q^0(3) = \begin{bmatrix} 0.404 & 0.596 \\ 0.862 & 0.138 \end{bmatrix} \qquad C^0(3) = \begin{bmatrix} 0.049 & 0.027 \\ 0.025 & 0.036 \end{bmatrix},$$

respectively, for the case when additional units are *not* ordered during week 1.
When additional units are ordered (Z = 1), the matrices Q^1 (1), $C^1(1)$, Q^1 (2) , $C^1(2)$, Q^1 (3) and $C^1(3)$ yield the costs(in million UGX)

$$e_F^1(1) = (0.562)(0.366) + (0.438)(0.132) = 0.2634$$
$$e_U^1(1) = (0.831)(0.084) + (0.169)(0.025) = 0.074$$
$$e_F^1(2) = (0.4660)(0.047) + (0.5340)(0.077) = 0.0627$$
$$e_U^1(2) = (0.8194)(0.0176) + (0.1804)(0.0617) = 0.0256$$
$$e_F^1(3) = (0.479)(0.087) + (0.521)(0.147) = 0.118$$
$$e_U^1(3) = (0.8732)(0.072) + (0.1268)(0.0536) = 0.0697$$

However, when additional units are *not* ordered (Z= 0), the matrices Q^0 (1), $C^0(1)$, Q^0 (2) , $C^0(2)$, Q^0 (3) and $C^0(3)$ yield the costs (in million UGX)

$$e_F^0(1) = (0.6212)(0.477) + (0.3788)(0.198) = 0.3713$$
$$e_U^0(1) = (0.6914)(0.189) + (0.3086)(0.037) = 0.1421$$
$$e_F^0(2) = (0.540)(0.008) + (0.460)(0.0014) = 0.005$$
$$e_U^0(2) = (0.804)(0.004) + (0.196)(0.0608) = 0.0152$$
$$e_F^0(3) = (0.4045)(0.0486) + (0.5955)(0.027) = 0.0357$$
$$e_U^0(3) = (0.8615)(0.017) + (0.1385)(0.036) = 0.0196$$

The results are summarized in Tables 1 and 2 below:

Table 1: Values of Z and $e^Z_i(S)$ at Supermarkets during week 1

Week (n)	Supermarket (S)	Z	$e^Z_F(S)$	$e^Z_U(S)$
1	Shoprite (1)	1 0	0.265 0.371	0.087 0.142
	Game (2)	1 0	0.063 0.005	0.026 0.015
	Uchumi (3)	1 0	0.118 0.024	0.067 0.018

The cumulative total costs $a^Z_i(S,n)$ are computed using (1) for week 2 and results are summarized in Table 2 below:

Table 2: Values of Z and $a^Z_i(S,n)$ at supermarkets during week 2

Week (n)	Supermarket (S)	Z	$a^Z_F(S,2)$	$a^Z_U(S,2)$
2	Shoprite (1)	1 0	0.449 0.568	0.320 0.531
	Game (2)	1 0	0.073 0.015	0.032 0.022
	Uchumi (3)	1 0	0.139 0.044	0.090 0.041

5.3 The Optimal ordering policy

Week1
Shoprite supermarket

Since 0.265 < 0.371, it follows that Z=1 is an optimal ordering policy for week 1 with associated total inventory costs of 0.265 million UGX when demand is favorable. Since 0.087<0.142, it follows that Z=0 is an optimal ordering policy for week 1 with associated total inventory costs of 0.087 million UGX if demand is unfavorable.

Game supermarket

Since 0.005 < 0.063, it follows that Z=0 is an optimal ordering policy for week 1 with associated total inventory costs of 0.005 million UGX when demand is favorable. Since 0.015 < 0.026, it follows that Z=0 is an optimal ordering policy for week 1 with associated total inventory costs of 0.015 million UGX if demand is unfavorable.
Uchumi supermarket

Since 0.024 < 0.118, it follows that Z=0 is an optimal ordering policy for week 1 with associated total inventory costs of 0.024 million UGX when demand is favorable. Since 0.018 < 0.067, it follows that Z=0 is an optimal ordering policy for week 1 with associated total inventory costs of 0.018 million UGX if demand is unfavorable.

Week 2
Shoprite supermarket

Since 0.449 < 0.568, it follows that Z=1 is an optimal ordering policy for week 2 with associated accumulated inventory costs of 0.449 million UGX when demand is favorable. Since 0.320 < 0.531, it follows that Z=0 is an optimal ordering policy for week 2 with associated accumulated inventory costs of 0.320 million UGX if demand is unfavorable.

Game supermarket

Since 0.015 < 0.073, it follows that Z=0 is an optimal ordering policy for week 2 with associated accumulated inventory costs of 0.015 million UGX when demand is favorable. Since 0.022 < 0.032, it follows that Z=0 is an optimal ordering policy for week 2 with associated total inventory costs of 0.022 million UGX if demand is unfavorable.

Uchumi supermarket

Since 0.044 < 0.139, it follows that Z=0 is an optimal ordering policy for week 2 with associated accumulated inventory costs of 0.044 million UGX when demand is favorable. Since 0.041 < 0.090, it follows that Z=0 is an optimal ordering policy for week 1 with associated accumulated inventory costs of 0.041 million UGX if demand is unfavorable

6 Conclusion

An inventory model with stochastic demand was presented in this paper. The model determines an optimal ordering policy and inventory costs of a given product with stochastic demand. The decision of whether or not to order additional stock units is modeled as a multi-period decision problem using dynamic programming over a finite planning horizon. The working of the model was demonstrated by means of a real case study. It would however be worthwhile to extend the research and examine the behavior of ordering policies under non stationary demand conditions in supermarkets. In the same spirit, our model raises a number of salient issues to consider: Lead time of milk powder during replenishment and customer response to abrupt changes in price of the product. Finally, special interest is thought in further extending our model by considering ordering policies in the context of Continuous Time Markov Chains (CTMC).

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