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System Reliability Optimization: A Fuzzy Genetic Algorithm Approach

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Abstract

System reliability optimization is often faced with imprecise and conflicting goals such as reducing the cost of the system and improving the reliability of the system. The decision making process becomes fuzzy and multi-objective. In this paper, we formulate the problem as a fuzzy multi-objective nonlinear program (FMOOP). A fuzzy multi-objective genetic algorithm approach (FMGA) is proposed for solving the multi-objective decision problem in order to handle the fuzzy goals and constraints. The approach is able flexible and adaptable, allowing for intermediate solutions, leading to high quality solutions. Thus, the approach incorporates the preferences of the decision maker concerning the cost and reliability goals through the use of fuzzy numbers. The utility of the approach is demonstrated on benchmark problems in the literature. Computational results show that the FMGA approach is promising.

Keywords
System reliability optimization, multi-objective optimization, genetic algorithm, fuzzy optimization, redundancy

1. Introduction

Reliable industrial systems are essential for productivity and effectiveness (Kuo and Prasad, 2000; Huang et al., 2005; Wu et al., 2011). As such, these systems are expected to be fully available and operational most of the time so as to maximize productivity. However, an industrial system is composed of a number of complex components, such that the probability of the system survival over time depends directly on the characteristics of its constituent components. Failure is inevitable in industrial systems so much that system reliability optimization has become a very important subject matter in industry. Therefore, the development of effective methods for improving the overall system productivity is imperative. The ever-increasing need for highly reliable systems necessitates the search for improved methods for system reliability optimization. In system reliability design, there are two typical approaches that can enhance system reliability, namely: (i) using redundant elements in the subsystems of the system, and (ii) increasing the reliability of the components that constitute the system.

Industrial systems are designed under several restrictions, including cost, weight, and volume of the resources. With limited resources, the major aim is to find a trade-off between reliability and other resource constraints (Huang et al., 2005). One feasible way is to maximize system reliability via redundancy and component reliability choices, a problem called reliability-redundancy allocation problem (Kuo and Prasad, 2000). However, in designing a highly reliability system, the main problem is to find a trade-off between reliability enhancement and resource consumption.

Real life reliability optimization problems are inundated with several difficulties: (i) the management goals and the constraints are often described with some imprecision or vagueness; (ii) the coefficients or parameters as understood by the decision maker may be characterized with some vagueness; and, (iii) the available historical data, collected under specific conditions, are often imprecise and vague. Variability and changes in the manufacturing processes that produce the components of the systems lead to uncertainties in component reliability. Probabilistic approaches, which essentially deal with uncertainty arising from randomness, cannot adequately address the inherent uncertainties in the data. As such, the concept of fuzzy reliability is more promising (Onisawa, 1990; Cai et al., 1991; Chen, 1994; Chen, 2001). Thus, while probabilistic approaches deal with uncertainties arising from randomness, fuzzy approaches seek to address the uncertainty that arises from vagueness of human judgment and imprecision due to system complexity (Bing et al., 2000; Duque and Morifiigo, 2004; Mohanta et al., 2004; Bag et al., 2009; Huang et al., 2005; Garg and Sharma, 2012; Garg and Sharma, 2013).

Bellman and Zadeh (1970) introduced the fuzzy optimization approach, providing aggregation operators for combining fuzzy goals and fuzzy decision space. Since the inception of the fuzzy optimization approach, numerous methods and applications have been proposed to solve optimization problems that involve vagueness and ambiguity (Slowinski, 1998; Delgado et al., 1993; Huang, 1997; Huang et al., 2006; Mahapatra and Roy, 2006). These approaches treat parameters (coefficients) as fuzzy numerical data. Apart from the fuzziness of the system reliability problem, the presence of conflicting, nonlinear and ambiguous objectives further complicates the problem. In such a fuzzy environment, with multiple objectives, simultaneous reliability maximization and cost minimization calls for a cautious trade-off approach. Thus, finding the optimal solution is almost impossible. Metaheuristic and other intelligent methods are a potential application method for such complex problems (Coit and Smith, 1996; Chen and You, 2005; Michalewicz, 1996). Therefore, the most appropriate procedure is to cautiously find a set of solutions that satisfy the decision maker's expectations to the highest possible degree. Clearly, this calls for an interactive fuzzy multi-objective optimization approach which incorporates the preferences and expectations of the decision maker, allowing for human (expert) judgment. Iteratively, it becomes possible to obtain the most satisfactory solution in a fuzzy environment.

In light of the above issues, the purpose of this paper is to address the problem of system reliability optimization in a fuzzy environment characterized with multiple conflicting objectives. Therefore, in addressing this problem the objectives of this research are as follows:

1. to develop a fuzzy multiple-objective nonlinear programming model for the problem;
2. to use an aggregation method to transform the fuzzy model to a single-objective optimization problem; and,
3. to use a global metaheuristic optimization method to obtain a set of acceptable solutions.

In this work, we use the max-min operator to aggregate the membership functions of the objective functions while incorporating the decision maker's judgment. To this end, we define our acronyms, notations and assumptions.

Acronyms:

FMGA	Fuzzy multi-objective genetic algorithm
GA	Genetic algorithm
MODA	Multi-objective decision analysis
MINLP	Mixed Integer Nonlinear Programming
RRAP	Reliability-redundancy allocation problem
FMOOP	Fuzzy multi-objective optimization problem

Notation:

m	the number of subsystems in the system
n_i	the number of components in subsystem i, $1 \le i \le m$
n	$\equiv(n_1, n_2, \ldots, n_m)$, the vector of the redundancy allocation for the system
r_i	the reliability of each component in subsystem i, $1 \le i \le m$
r	$\equiv(r_1, r_2, \ldots, r_m)$, the vector of the component reliabilities for the system
q_i	$=1 - r_i$, the failure probability of each component in subsystem i, $1 \le i \le m$

$R_i(n_i)$ $= 1 - q_i^{n_i}$, the reliability of subsystem i, $1 \le i \le m$
R_s the system reliability
g_i the i^{th} constraint function
w_i the weight of each component in subsystem i, $1 \le i \le m$
v_i the volume of each component in subsystem i, $1 \le i \le m$
c_i the cost of each component in subsystem i, $1 \le i \le m$
V the upper limit on the sum of the subsystems' products of volume and weight
C the upper limit on the cost of the system
W the upper limit on the weight of the system
b the upper limit on the resource

Assumptions
1. The availability of the components is unlimited;
2. The weight and product of weight and square of the volume of the components are deterministic;
3. The redundant components of individual subsystems are identical;
4. Failures of individual components are independent;
5. All failed components will not damage the system and are not repaired.

2. System Reliability Optimization

The system reliability optimization problem is a maximization problem subject to multiple linear constraints. Thus, the problem can be expressed as a mixed integer nonlinear programming problem. In this study, we present a reliability redundancy problems commonly found in the literature, particularly the series system (Kuo and Prasad, 2000; Hsieh et al. (1998). The series system reliability problem consists of five subsystems as reported in the literature as shown in Figure 1. The problem can be formulated as a nonlinear mixed integer programming problem as follows:

(P1) $\text{Max} f(\text{r},\text{n}) = \prod_{i=1}^{m} R_i(n_i)$

Subject to:

$$g_1(\text{r},\text{n}) = \sum_{i=1}^{m} w_i v_i^2 n_i^2 \le V$$

$$g_2(\text{r},\text{n}) = \sum_{i=1}^{m} \alpha_i \left(-1000/\ln r_i\right)^{\beta_i} \left(n_i + \exp(n_i/4)\right) \le C$$

$$g_3(\text{r},\text{n}) = \sum_{i=1}^{m} w_i n_i \exp(n_i/4) \le W$$

$$0 \le r_i \le 1, \quad n_i \in Z^+, \quad 1 \le i \le m$$

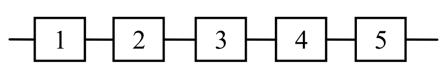

Figure 1: The series system

We present the proposed fuzzy multi-objective optimization approach, based on genetic algorithms in the next section.

3. Fuzzy Multi-objective Optimization Approach

In a fuzzy environment, the objective goal, the constraints and the consequences of the decision taken are inherently imprecise. Thus, in practice, the decision maker seeks to consider a trade-off between reliability, cost, weight and volume. For instance, a common approach may be to maximize reliability and to minimize cost, simultaneously. In this connection, the multi-objective formulation is obtained by transforming constraints to objective functions, such that reliability and other costs functions can be optimized jointly. This is achieved through the use of membership functions for the objective functions. This makes the approach more applicable and adaptable to the real life human decision process. Therefore, the fuzzy multi-objective optimization problem (FMOOP) can generally be represented by the following;

(P2) Min $\tilde{f}(x)$

Subject to:

$g_z(x) \precsim or \equiv or \succsim 0 \qquad z = 1, 2..., p$

$x_q^l \le x_q \le x_q^u \qquad q = 1, 2, ..., Q$

where, $x = (x_1, x_2, ..., x_Q)^T$, is a vector of decision variables that optimize a vector of objective functions, $\tilde{f}(x) = \{\tilde{f}_1(x), \tilde{f}_2(x), ..., \tilde{f}_d(x)\}$ over the decision space *X*; $f_1(x), f_2(x), ..., f_d(x)$ are d individual objective functions; x_q^l and x_q^u are lower and upper bounds on the decision variable x_q, respectively.

3.1 Membership Functions

Fuzzy set theory permits gradual assessment of membership, defined in terms of a suitable membership function that maps to the unit interval [0,1]. A number of membership functions such as Generalized Bell, Gaussian, Triangular and Trapezoidal can be used to represent the fuzzy membership. Though various functions can be used, it has been shown that linear membership functions can provide equally good quality solutions with much ease Sakawa (1993). The triangular and trapezoidal membership functions have widely been recommended (Chen, 2001; Delgado et al., 1993). Therefore, in this study, we use linear functions to define the fuzzy membership functions of the objective functions.

Let m_t and M_t denote the minimum and maximum of the feasible values of each objective function $\tilde{f}_t(x)$, $t = 1, 2, ..., h$, where *h* is the number of objective functions. Let μ_{f_t} denote the membership function corresponding to the objective function f_t. Then, the membership function corresponding to minimization and maximization can be defined, based on the satisfaction degree. Figure 4 illustrates the linear membership functions defined for two optimization cases, that is, minimization and maximization cases. We define the membership functions for both cases.

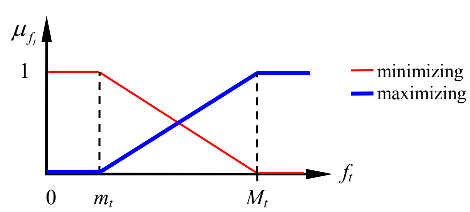

Figure 4: Fuzzy membership function for $f_t(x)$

For the case of minimization, the linear membership function can be formulated according to the following expression;

$$\mu_{f_t}(x) = \begin{cases} 1 & f_t(x) \le M_t \\ \dfrac{M_t - f_t(x)}{M_t - m_t} & m_t \le f_t(x) \le M_t \\ 0 & f_t(x) \ge M_t \end{cases} \qquad (1)$$

Clearly, the function $\mu_{f_t}(x)$ is monotonically decreasing in $f_t(x)$. On the other hand, for the case of maximization, the membership function can be defined as follows;

$$\mu_{f_t}(x) = \begin{cases} 1 & f_t(x) \geq M_t \\ \dfrac{f_t(x) - m_t}{M_t - m_t} & m_t \leq f_t(x) \leq M_t \\ 0 & f_t(x) \geq m_t \end{cases} \tag{2}$$

It can be seen from this analysis that $\mu_{f_t}(x)$ is a monotonically increasing function of $f_t(x)$. The next step is to formulate the corresponding crisp model. The use of fuzzy evaluation in FMGA allows the algorithm to accept inferior which would otherwise be infeasible when using conventional crisp formulation. The advantage of this approach is that it makes the algorithm robust enough to cope with any infeasibility. Allowing the FMGA to pass through inferior solutions gives the algorithm speed and flexibility, which ultimately improves the search power of the optimization approach.

3.2 Corresponding Crisp Model

To further incorporate the decision maker's preferences and to enhance the interactive flexibility of the model, a set of user-defined weights $w = \{w_1, w_2, \ldots, w_h\}$ are introduced. We convert the multi-objective system reliability optimization problem into a single objective optimization problem (Huang, 1997):

$$\text{(P3) Max } \left(\frac{\lambda_1(x)}{w_1} \wedge 1\right) \wedge \left(\frac{\lambda_2(x)}{w_2} \wedge 1\right) \wedge \ldots \wedge \left(\frac{\lambda_h(x)}{w_h} \wedge 1\right)$$

Subject to:

$$\lambda_t(x) = \mu_{f_t}(x) \qquad w_t \in [1,0] \qquad t = 1, \ldots, h$$

$$x_q^l \leq x_q \leq x_q^u \qquad q = 1, \ldots, Q$$

Here, $\mu_{f_t}(x) = \{\mu_{f_1}(x), \mu_{f_2}(x), \ldots, \mu_{f_h}(x)\}$ signifies a set of fuzzy regions that satisfy the objective functions; λ_t denotes the degree of satisfaction of the t^{th} objective, x is a vector of decision variables, w_t denotes the weight of the t^{th} objective function suggested by the expert judgment of the user or decision maker, and the symbol "$\wedge$" is the aggregate min operator or the intersection operator. For instance, the expression $\left(\frac{\lambda_1(x)}{w_1} \wedge 1\right)$ gives the minimum between 1 and $\frac{\lambda_1(x)}{w_1}$. Though the values of $\lambda_1(x)$ are in the range [0,1], the value of $\frac{\lambda_1(x)}{w_1}$ may exceed 1, howbeit, by the min operator the final value of $\left(\frac{\lambda_1(x)}{w_1} \wedge 1\right)$ will always lie in [0,1]. To solve, the optimization problem P5, we employ the genetic algorithm metaheuristic approach, a global optimization approach inspired by the theory of genetics and philosophy of natural selection and survival of the fittest (Goldberg, 1989; Holland, 1975).

3.3 Genetic Algorithm Approach

Genetic algorithm, first introduced by Holland (1975), is a stochastic global optimization technique that attempts to evolve a population of candidate solutions by giving preference of survival to quality solutions, whilst allowing some low quality solutions to survive in order to maintain a level of diversity in the population. Each candidate solution is coded into a string of digits, called chromosomes. New offspring are obtained from probabilistic genetic operators, such as selection, crossover, mutation, and inversion (Goldberg, 1989). A comparison of new and old (parent) candidates is done based on a given fitness function, retaining the best performing candidates into the next population. Thus, characteristics of candidate solutions are passed from generation to generation through probabilistic selection, crossover, and mutation.

Representation

In our implementation, the FMGA for the reliability problem chromosome uses the variable vectors n and r. In this study, a real-coded GA is implemented, where the integer variable n_i is coded as a real variable and transformed to the nearest integer value upon evaluating the objective function.

Initialization and Evaluation
An initial population of the desired size, *pop*, is randomly generated randomly from the solution space. FMGA then computes the objective function for each string. According to the overall objective function in model P4, the objective function is always in the range [0,1].

Selection and Recombination
A number of selection strategies have been suggested by Goldberg (1989), such as deterministic sampling, remainder stochastic sampling with/without replacement, stochastic tournament, and stochastic sampling with/without replacement. The remainder stochastic sampling without replacement is preferred in this study. In this strategy, each chromosome *j* is selected and stored in the mating pool according to the expected count e_j;

$$e_j = \frac{f_j}{\sum_{j=1}^{pop} f_j / pop} \tag{3}$$

Here, f_j is the objective function value of the j^{th} chromosome. Each chromosome receives copies equal to the integer part of e_i, that is, $[e_i]$, while the fractional part is treated as success probability of obtaining additional copies of the same chromosome into the mating pool.

Crossover operator
The crossover operator is applied to selected parent chromosomes for the purpose of exchanging genetic information between the selected chromosomes, thereby producing new offspring. Here, we use the arithmetic crossover operator as in Michalewicz (1996) which defines a linear combination of two chromosomes. A crossover probability of 0.45 was assumed in the application. For instance let p_1 and p_2 be the selected parents, and α represent a random value in [0,1], then the resulting offspring, q_1 and q_2, are given by the following expression;

$$\begin{aligned} q_1 &= \alpha p_1 + (1-\alpha) p_2 \\ q_2 &= (1-\alpha) p_1 + \alpha p_2 \end{aligned} \tag{4}$$

Mutation Operator
The mutation operator is applied to every new chromosome, at a very low probability, so as to maintain diversity of the population and avoid premature convergence. In our implementation we used uniform mutation with a mutation rate of 0.035.

Replacement Strategy
In every generation, new offspring are created, which implies that they may be better or worse. Therefore, nonperforming chromosomes are replaced with better ones using a replacement strategy. A number of replacement strategies have been suggested in the literature, including probabilistic replacement, crowding strategy, and elitist strategy (Michalewicz, 1996). A combination of these has been implemented in this study.

Termination Criteria
Termination conditions are used to stop the FMGA iteration in two ways: when the number of generations exceeds the preset maximum iterations, or when average change in the fitness of the best solution over specific generations is less than a small number, which is 10^{-6}.

Algorithm 1: Pseudo code for FMGA

1: randomly generate initial population
Repeat
    2: evaluation of fitness, objective: $f(x)$, $x = (x_1, x_2, \ldots, x_h)$
    3: selection strategy

4: crossover
5: mutation
6: replacement
7: advance population; *oldpop* = *newpop*
Until (termination criteria is satisfied)

Figure 2: Pseudo code for the overall FMGA

The overall structure of the FMGA for the system reliability problems is summarized in the pseudo code listed in in Figure 2. The next section presents the comparative results of our FMGA computations based on the benchmark problems found in the literature (Kuo and Prasad, 2000; Hsieh et al., 1998; Hikita et al., 1992).

4. Numerical Experiments

In order to evaluate the utility of our fuzzy FMGA for solving mixed integer reliability problems, the set of three reliability systems presented in Section 2 will be solved using the approach. We use the parameter values in Kuo and Prasad (2000) and to define the specific instances of these problems as shown in Tables 1.

Table 1: Basic data used in series system

i	$10^5\alpha_i$	β_i	$w_iv_i^2$	w_i	V	C	W
1	2.330	1.5	1	7	110	175	200
2	1.450	1.5	2	8	110	175	200
3	0.541	1.5	3	8	110	175	200
4	8.050	1.5	4	6	110	175	200
5	1.950	1.5	2	9	110	175	200

The parameters of the FMGA were set as follows: The crossover and mutation were set at 0.45 and 0.035, respectively. A two-point crossover was used in this application. The population size was set to 20. The maximum number of generations or iterations was set at 150. This implies that the termination criterion is either limited to a maximum number of iterations or to the order of the relative error set at 10^{-6}, whichever comes earlier. Specifically, whenever the best fitness f^* at iteration t is such that $|f_t - f^*| < \varepsilon$ is satisfied, then three best solutions are selected; where ε is a small number equal to 10^{-6}. The FMGA was implemented in JAVA, and the program was run 25 times, while selecting the best 3 solutions out of the converged population.

The FMOOP provided by formulation (P4) is used to solve benchmark problems in Kuo and Prasad (2000). A fuzzy region of satisfaction is constructed for each objective function, that is, objective functions corresponding to system reliability, cost, volume, and weight, which are denoted by λ_1, λ_2, λ_3, and λ_4, respectively. By using the constructed membership functions together with their corresponding weight vectors, we obtain the following equivalent crisp optimization formulation for our problem;

$$\text{(P4) Max } \left(\frac{\lambda_1(x)}{\omega_1}\wedge 1\right)\wedge\left(\frac{\lambda_2(x)}{\omega_2}\wedge 1\right)\wedge\left(\frac{\lambda_3(x)}{\omega_3}\wedge 1\right)\wedge\left(\frac{\lambda_4(x)}{\omega_4}\wedge 1\right)$$

Subject to :

$$\begin{array}{ll} \lambda_t(x) = \mu_{f_t}(x) & t = 1,\ldots,4 \\ 0.5 \le r_i \le 1-10^{-6} & r_i \in [0,1] \\ 1 \le n_i \le 10 & n_i \in Z^+ \\ 0.5 \le R_s \le 1-10^{-6} & R_s \in [0,1] \end{array}$$

The weight set ω = {ω_1, ω_2, ω_3, and ω_4} was selected in the range [0.2,1], where the values of the weights indicate the bias towards specific objectives as specified by the expert decision maker. In particular, the weight set ω = [1,1,1,1] implies that the expert user prefers that there should be no bias towards any objective goal, that is, there is no preference at all. Every other case implies that there is some bias towards one or more specific objectives, and the relative importance of objectives is ranked accordingly. For instance, with a weight set defined by ω = [1,0.5,0.5,0.5], the preference is biased towards the region that is closer to the objective corresponding to reliability than to the rest of the objectives that are equally ranked with weight value of 0.5. Therefore, the decision making process takes into account the decision maker's preferences and choices based on expert opinion. In addition, the FMGA approach is a useful decision support tool that can provide a set of good solutions in an interactive manner, rather than prescribe a single solution. Furthermore, the approach enables the decision maker to specify the minimum and maximum values of objective functions in terms of reliability, cost, volume, and weight, denoted by f_1, f_2, f_3, and f_4, respectively. Table 2 provides a list of the selected minimum and maximum values of the objective functions, for the series system, the series-parallel and the complex (bridge) system. This approach makes the FMGA algorithm a more adaptable and flexible method for addressing specific problem situations while accommodating the expert user's managerial preferences. Computational results and discussions are presented in the next section.

Table 2. Minimum and maximum feasible values of objective functions

	Series System			
	f_1	f_2	f_3	f_4
M_i	1	180	120	210
m_i	0.6	60	5	100

5. Results and Discussions

This section presents the comparative results of the numerical experiments. The best three FMGA solutions are compared with the results obtained by other algorithms in the literature, for the series system, series-parallel system and complex bridge system. We specifically compare our results with those in Wu et al. (2011), Chen (2006) and Hsieh et al. (1998).

Tables 3 shows the comparative numerical results in which the best three solutions of each problem compared against solutions from the literature. The results indicate that each of the three solutions is better than the solutions reported previous, specifically in terms of system reliability. In terms of cost, the solutions are no better than the previously reported solutions. However, the difference in cost is quite small. Though there are a few exceptional instances where the cost of the FMGA are slightly higher with differences in the order of 10^{-6}, it can be seen that, overall, FMGA provides better solutions than the approaches reported previously. FMGA approach found high quality solutions, most of which are better than those previously recorded in the literature. In summary, the approach offers a number of practical advantages to the decision maker, including the following:

- FMGA addresses the imprecise and fuzzy nature of the problem;
- The method address the conflicting multiple objectives, giving a trade-off between the objectives;
- The approach accommodates the decision maker's preferences in its procedure;
- The method gives a population of alternative solutions for the decision maker, rather that prescribe a solution;
- The method is practical, flexible and easily adaptable to specific problem situations.

In view of the above advantages, FMGA is a useful decision support tool for the practicing decision maker in industrial system reliability optimization.

Table 3: Comparison of best-3 FMGA solutions with other algorithms for series system

	Best 3 FMGA Solutions			Wu et al. (2011)	Chen Chen (2006)	Hsieh et al.(1998)
No.	(r_i: n_i)	(r_i: n_i)	(r_i: n_i)		(r_i: n_i)	(r_i: n_i)
1	(0.779401321:3)	(0.77940279:3)	(0.77939597:3)	(0.78037307:3)	(0.779266:3)	(0.779427:3)
2	(0.871839015:2)	(0.87181554:2)	(0.87183716:2)	(0.87178343:2)	(0.872513:2)	(0.869482:2)
3	(0.902877370:2)	(0.90287257:2)	(0.90288515:2)	(0.90240890:2)	(0.902634:2)	(0.902674:2)
4	(0.711415792:3)	(0.71141514:3)	(0.71140318:3)	(0.71147356:3	(0.710648:3)	(0.714038:3)
5	(0.787779580:3)	(0.78783097:3)	(0.78780147:3)	(0.78738760:3)	(0.788406:3)	(0.786896:3)
R_s	0.931682387	0.931682384	0.931682388	0.9316800	0.93167800	0.93157800
C_s	175.0000000	175.0000000	175.0000000	174.99899	174.998441	174.878546
W_s	192.4810818	192.4810818	192.4810818	192.48108	192.481082	192.481082
V_s	83.00000000	83.00000000	83.00000000	83.000000	83.0000000	83.0000000

6. Conclusion

In practice, decision makers concerned with system reliability optimization encounter problems of finding a judicious trade-off between maximizing reliability and minimizing cost to an acceptable degree of satisfaction. In such a fuzzy environment, the management goals and constraints are not known precisely. Moreover, the goals are often conflicting, which further complicates the reliability optimization problem. One most viable and useful option is to us a fuzzy satisficing approach that includes the preferences and expert judgments of the decision maker. We provided a multi-objective non-linear mixed integer program for addressing system reliability optimization problems. The fuzzy multi-objective model is transformed into a single-objective model which uses a fuzzy evaluation method. Genetic algorithm uses the fuzzy evaluation method to evaluate the fitness of individuals in each population at every generation. Numerical results demonstrate that the fuzzy multi-objective Genetic Algorithm approach is able to provide high quality solutions while accommodating the preferences of the user.

This work is a useful contribution to practicing decision makers in the field of system reliability design. Practically speaking, FMGA approach provides a trade-off between management goals, contrary to single-objective approaches which seek to optimize system reliability only. Oftentimes, at design stage, the information required for system reliability design is imprecise and incomplete. To that effect, the problem becomes ill-structured such that reliance on expert information is inevitable. Using the FMGA approach, the vagueness and imprecision of the expert knowledge, at the design stage, can be addressed effectively while taking into account the multiple conflicting objectives. Furthermore, FMGA provides a population of good alternative solutions in an interactive manner, which offers the decision maker a wide choice of practicable solutions and an opportunity to consider other practical factors that cannot be included in the formulation. Overall, FMGA approach is a useful platform for decision support for solving system reliability design problems when the parameters, the management goals, the design constraints, and the impact of the possible alternative actions are not precisely known. Therefore, the approach gives a robust method for system reliability optimization.

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