Intermittent Demand Forecast: Robustness Assessment for Group Method of Data Handling

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Abstract

Demand forecasting is a key ingredient of supply chain process and plays an important role in synchronized planning and reduced bullwhip effect. Intermittent demand forecasting is a special case of demand forecasting when there are several periods of zero and uneven demand for a product in historical time. Developing a model to forecast intermittent demand has been a challenge, and models of various vintages have been proposed. In this paper, we develop a model for intermittent demand forecasting invoking the Group Method of Data Handling (GMDH) and perform a comparative evaluation of the robustness of this model in comparison to classical models in vogue.

Keywords
Intermittent Demand Forecasting, Moving Average, Exponential Smoothing, GMDH

1. Introduction

Intermittent demand implies a 'demand archetype' of a product in a specific time frame in which there are several interval in total span of time during which there is zero demand and even if there is a demand that is irregular. Intermittent demand forecasting plays a pivotal role in manufacturing and supply chain management, leading to optimal capacity utilization, cost optimization and superior services to business. There is a significant need for employing modelling techniques leading to superior forecast. The modeling of intermittent demand pattern is an exacting task, and it warrants identification and exercise of requisite modeling tools. Diverse methodologies exercising time series, moving average, and smoothening have been widely published to achieve intermittent demand forecast using historical information of demand with several interspersed zero demand in given time frame. Each model has some element of advantages and disadvantages. In our work, we have developed a model for intermittent demand forecasting by bringing into play the Group Method of Data Handling (GMDH). This paper documents the output of GMDH model and a comparative assessment of the results of this model with classical models. It has been deduced that the GMDH model outperforms the other models.

Forecasting intermittent demand is a difficult and uncertain task because intermittent demand is irregular with a large proportion of zero values (Silver 1981). Managing this uncertainty is a tricky issue and largely unexplored, is affected by different market characteristics like numerousness of customers, heterogeneity, and frequency of order placing (Bartezzaghi, Verganti and Zotteri 1999). The data set used for this paper consists of 1500 remanufactured products with intermittent demand patterns. This paper is to forecast intermittent demand of these products using the GMDH model and establishes its superiority over classically used methods.

The paper is arranged in the following sequential manner. The literature review section summarizes the review of the work accomplished by researchers in the field of intermittent demand forecasting. Next section provides thumbnail sketch of various modeling methodologies employed by previous researchers. Acquaintance, awareness, knowledge and directive from these works facilitated us to articulate a sequence of actions to model intermittent demand forecasting which is discussed in the subsequent section. The upshot of various models has been summarized and comparative assessments of results have been made. Finally, concluding remarks are submitted.

2. Literature Review

Forecasting intermittent demand has been a very challenging issue in the literature of forecasting. Croston (1970-1977) addressed the problem of intermittent demand, and prescribed a new method which uses separate estimates of size and frequency of the demand. The thirteen methods including moving average, weighted moving average were evaluated and compared. It was found that exponential smoothing and Croston method outperformed other methods (Ghobbar and Friend 2003). However, Croston method has its own limitations as Croston's estimates of demand size and inter demand interval are correct but fail to produce the accurate output when combined as a ratio, because there is always a bias which increases in the value of α, smoothing parameter (Syntetos and Boylan, 2001). Hence, various other methods were explored to forecast intermittent demand. An alternate forecasting method using simple moving average instead of simple exponential smoothing to forecast the size of demands and the inter-arrival time between demands was proposed. This provides a significant improvement over Croston's method of intermittent demand forecasting (Yuan and Cai 2008). An appealing alternate strategy to tackle intermittent demand problems is to aggregate demand in small frequency pockets to avoid zero demands; however, it often results in loss of information as the exact frequency of a particular demand is lost (Nikolopoulos et al 2011). CID forecaster was developed which presents object oriented framework for intermittent demand forecasting and inventory management (Medal, Rossetti, Varghese, and Pohl 2009).

A neural network based methodology was also proposed and it also suggested that a combination of neural and classical methods could also be used (Gutierrez et al 2008). Group Method of Data Handling (GMDH) is a neural network algorithm used for forecasting purposes (Ivakhnenko and Ivakhnenko 2008). It was used to develop a trading system that stimulates a trading portfolio of diverse stocks using everyday out of sample price of stock (Lemke and Mueller 1997). GMDH was used as an effective data mining technique for forecasting weather data (Silver 1981). An analysis of the complex rainfall–runoff processes in a heterogeneous watershed in Taiwan was performed using GMDH (Chang and Hwang 1999). GMDH technique was employed to identify and forecast end use consumption sector of energy systems (Srinivasan 2008).

The analysis of literature suggests that GMDH is an effective forecasting technique. However, it has not been used for forecasting intermittent demands. It motivates us to propose a GMDH method to forecast intermittent demand.

3. Group Method of Data Handling

There are a number of modeling techniques used for forecasting. The accuracy of forecast for validation data as well as forecast for future time periods constitutes the underlying argument in support of a particular method. It testifies how well the forecasting model under consideration replicates the known data (i.e., validation data). Mean Error (ME), Mean Absolute Error (MAE) and Mean Squared Error (MSE), Mean Percentage Error (MPE), Mean Absolute Percentage Error (MAPE) are some of the standard statistical measures to check or compare the accuracy of forecasts obtained by several alternative methods.

The descriptive statistics, regression models and decomposition methods are preliminary forecasting techniques. Exponential smoothing methods including single exponential smoothing, Holt's Linear Trend Method, and Holt's – Winters' Trend and Seasonality method have also been implemented to address the issue of intermittent demand forecasting. Various time series methodologies to elucidate the intrinsic pattern in data have been employed, which help in identification of forecasting models. GMDH is a self-organizing methodology which gradually constructs complex models based on the evaluation of their performances on a set of multiple-input and single-output data pairs (Jamali et al 2008).

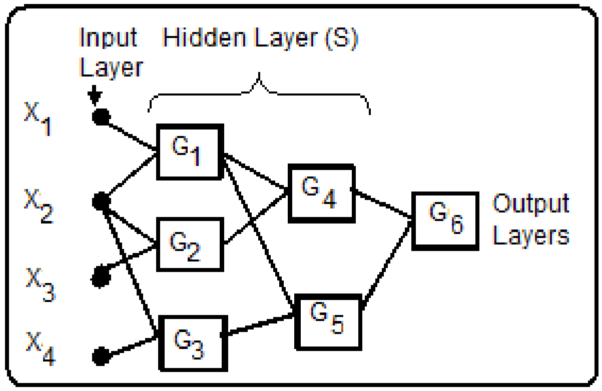

Figure 1: Feed forward GMDH Network

The GMDH algorithm is defined by set of neurons in which different pairs in each layer are connected through a quadratic polynomial and produce new neurons in the next layer. A typical network is depicted in Fig. 1. Here, x1, x2, x3 and x4 are the inputs; G1, G2, G3, G4 and G5 are the outputs of the hidden layers and G6 is the final output. GMDH uses two sets of data, training data and control data which are about 20% of the total data set. The first layer consists of all the predictor variables, which in the present case is the demand of a product. Using equation (1) all possible functions are constructed.

$$y = p_0 + p_1x_1 + p_2x_2 + p_3x_1^2 + p_4x_2^2 + p_5x_1x_2 \quad (1)$$

where

y= output to be calculated;

x_1, x_2= inputs from the previous layers;

p_0, p_1, p_2, p_3, p_4, p_5= weights assigned by the algorithm.

Here, the option to choose inputs from any of the previous layer is employed. Hence, the number of neurons generated is the number of input variables plus the number of neurons generated in pervious layers.

The least squares regression is used to compute optimal parameters for the function in each candidate neuron to make it best fit for the training data. The MSE is calculated for every neuron and is compared with the control data. The candidate neurons are arranged in an increasing order, and the neuron with smallest error is chosen for the next layer. The total number of neurons selected based on model building parameters. The training stops if the error for the best neuron for the current is better than the error for the best neuron from the previous layer, and the maximum layer of layers then has been reached. Otherwise, the process continues.

4. Monte Carlo Simulation

Monte Carlo simulation is able to capture the uncertainties of any model by using probability distributions. By means of combining the distributions and randomly selecting values of the parameters, it works out a simulated model several times and brings out the probability of the output. Several parameters may be used concurrently to generate the probability distribution of one or more outputs. The input parameters of the model may be assigned to different types of probability distributions such as triangular, uniform, exponential, etc. The output of Monte Carlo simulation is a range of values rather which demonstrate the range of occurrence of the output value.

Monte Carlo Simulation has been invoked in order to understand the behavior of the model realized by neural networks. It involved performing iterative exploration on the model represented by the polynomials under typical probability distribution. The Monte Carlo simulation of parameters (coefficients and lags) in the model led to the intermittent demand forecast and the sensitivity analysis reveal the degree of forecast sensitivity to modeling the parameters.

5. Experimental Design

The design of experiment is depicted in Fig. 2. Various forecasting models have been explored and invoked. The classical models include Moving Average, Weighted Moving Average, Exponential Smoothing, Trend Adjusted Exponential Smoothing and Croston, and the contemporary model is GMDH. The mean square errors of models are compared.

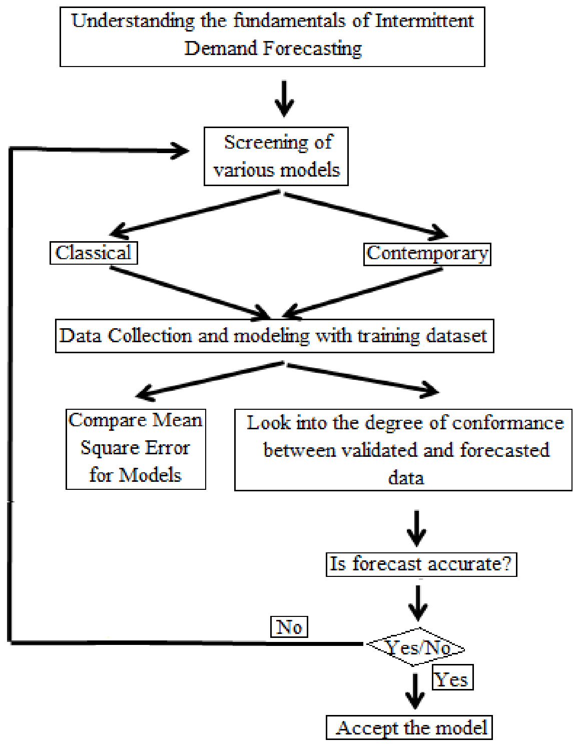

Figure 2: Modeling workflow

The conformity between validated and forecasted data indicates the accuracy forecast and acceptance or rejection of the model. Monte Carlo Simulation leads to probabilistic forecast and an understanding of forecast sensitivity to modelling parameters.

6. Results

The classical and contemporary models discussed in the preceding sections are applied to twenty four months time series data of over 100 products and the outcomes are discussed in this section. For the sake of brevity, the startling revelations are presented by the values of MSE in Table I and the graphs of one product in Figures 3, 4, 5, 6 and 7, respectively. It is evident that there is little or no conformity between the actual and the forecasted values from all the classical methods. This is further proven by the high MSE obtained in the case of all classical methods.

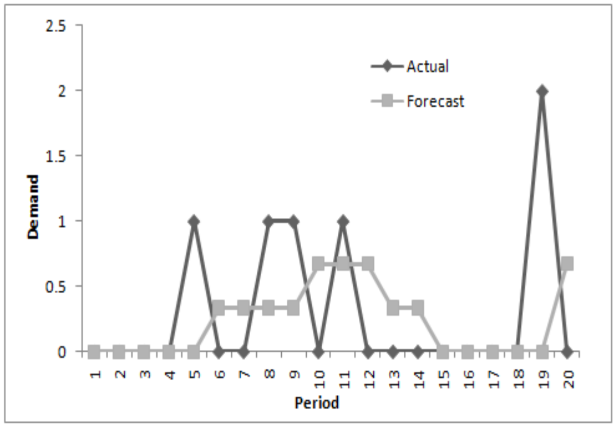

Figure 3: Prod 41 - Moving Average

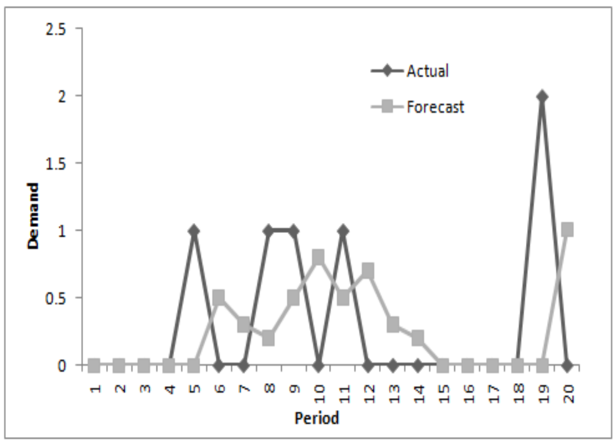

Figure 4: Prod 41 - Weighted Moving Average

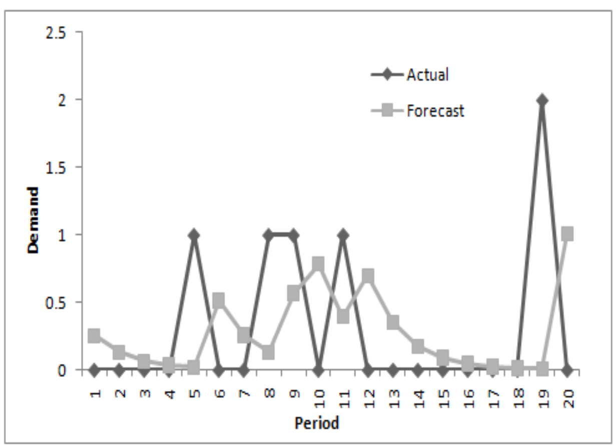

Figure 5: Prod 41 - Exponential Smoothing

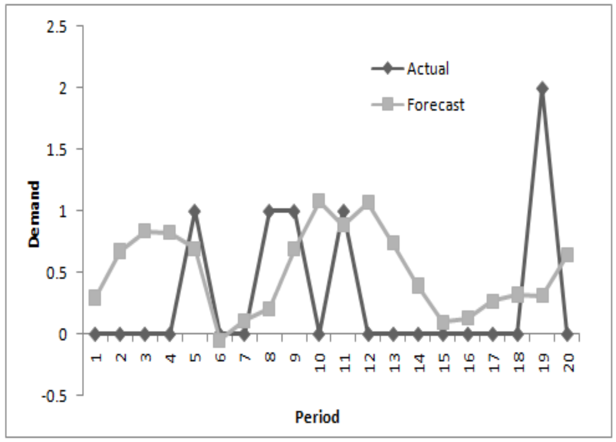

Fig. 6: Prod 41 - Trend Adj. Exp. Smoothing

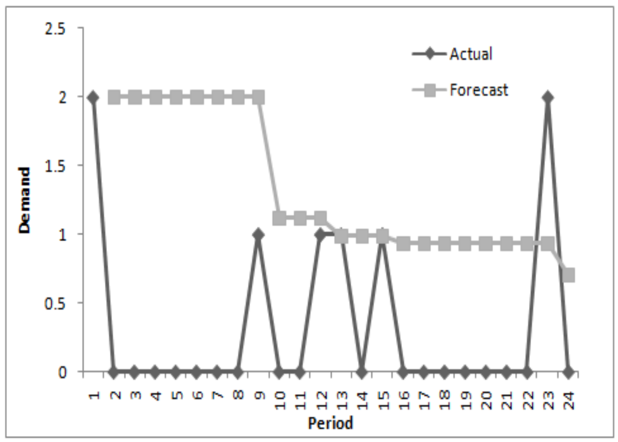

Figure 7. Prod 41 - Croston Method

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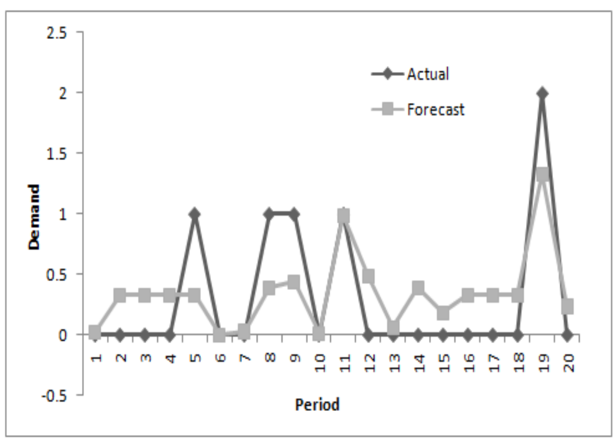

Figure 8 Prod 41 - GMDH

We now construct a neural network. GMDH realizes the fitting network structure without user intervention in respect of learning rules, and hidden layers, etc. The time series data has been split in two sections, Estimation (E) which represents 70% of the total data, and Validation (V) comprising the remaining 30% of the total data. The former is utilized to estimate model coefficients, and the later to evaluate the forecast accuracy of the model. Figure 8 depicts the forecast as well as demand values for Product 41. It can be recognized that the match between values of the actual and forecast by GMDH model is superior as compared to those from the classical models

The values of MSE in respect of GMDH model further corroborate the robustness of this model (Table 1). The findings reveal that GMDH model outperforms the classical models.

The forecasts can be further estimated by the polynomial equations derived by GMDH. For examples, for products 32 and 36, we have equations (2) and (3) as follows.

$$P_{32} = 0.142857 + 2.096459P_{32}z_2 - 2.096459P_{32}\, z_2^2 + 3.805262P_{32}z_1 - 3.805262P_{32}z_1^2 + 3.805262P_{32}z_2\, P_{32} \quad (2)$$

$$P_{36} = -5.551115 - 1.557433P_{36}z_3 + 2.557433\, P_{36}z_3^2 - 3.997661P_{36}z_1 + 3.997661P_{36}z_1^2 - P_{36}z_3P_{36}z_1 \quad (3)$$

$P_{32},\ P_{36}$ = demands of products 32 and 36;
z_1= demand of product lagging by 3;
z_2 = demand of product lagging by 5;
z_3 = demand of product lagging by 11.

For product 32, 0.142857, 2.096459, -2.096459, 3.805262, -3.805262 and 3.805262 are Coefficients 1, 2, 3, 4, 5 and 6, respectively. For product 36, -5.551115, -1.55743, 2.557433, -3.997661, 3.997661 and -1 are Coefficients 1, 2, 3, 4, 5 and 6, respectively.

Table 1: Values of MSE

	Classical Methods					
P	**MA**	**WMA**	**ES**	**TA**	**C**	**GMDH**
P31	0.17	0.21	0.19	0.16	0.35	0.14
P32	2.24	1.26	6.30	10.36	13.85	0.27
P33	1.29	1.37	1.20	1.10	1.61	1.53
P34	0.41	0.40	0.32	0.34	0.40	0.08
P35	0.23	0.23	0.19	0.19	0.32	0.11
P36	0.23	0.26	0.27	0.23	0.23	0.08
P37	7.60	7.66	8.91	8.55	20.85	0.86
P38	0.89	0.90	0.94	1.13	0.95	0.35
P39	0.38	0.39	0.33	0.30	0.25	0.26
P40	0.64	0.59	0.50	0.53	0.46	0.40
P41	0.39	0.42	0.59	0.77	1.65	0.25
P42	0.36	0.34	0.28	0.33	0.25	0.19
P43	0.54	0.56	0.56	0.51	0.43	0.41
P44	0.15	0.16	0.19	0.19	0.19	0.08
P45	4.41	4.03	3.45	4.29	3.15	2.43
P46	0.63	0.58	0.51	0.70	0.81	0.52
P47	1.78	1.70	1.40	1.67	1.27	1.11
P48	0.28	0.28	0.23	0.22	0.20	0.14
P49	0.28	0.26	0.31	0.26	0.24	0.18
P50	0.20	0.20	0.23	0.24	0.23	0.11
PAV	1.16	1.09	1.35	1.60	2.39	0.47
MAV	1.52					0.47
P= Product; PAV= Average MSE over Products; MAV= Average MSE over Methods and Products; MA= Moving Average; WMA= Weighted Moving Average; ES= Exponential Smoothing; TA= Trend Adjusted Exponential Smoothing; GMDH= Group Method of Data Handling						

The probabilistic forecast of intermittent demand for respective products has been achieved by applying Monte Carlo Simulation. The assessment of the intermittent demand forecast and sensitivity forecast of the parameters in equations (2) and (3) have been accomplished. Monte Carlo Simulation (with 10,000 trials), assuming triangular distribution for the coefficients and uniform distribution for lags (Table 2) lead to the realization of the forecasts and the sensitivity analysis (Figure 9), suggests that the demand is more sensitive to lags than to coefficients. It can be seen from the sensitivity chart that the sensitivity for lag 3 is the highest in the cases, 0.85 for product 32 and 0.82 for product 36, whereas it is very low for coefficient 4 in both cases, 0.04 for product 32 and 0.02 for product 36, respectively.

Table 2: Monte Carlo Simulation

	Prod-32	Prod-36
Mean	-76.61	10.44
Median	-62.41	8.48
Standard Deviation	57.71	7.85
Skewness	-1.37	1.01
Kurtosis	5.22	3.54
Coeff. of Variability	-0.75	0.75
Range Minimum	-418.17	0.07
Range Maximum	5.63	41.19
Range Width	423.81	41.12
Mean Std. Error	0.58	0.08

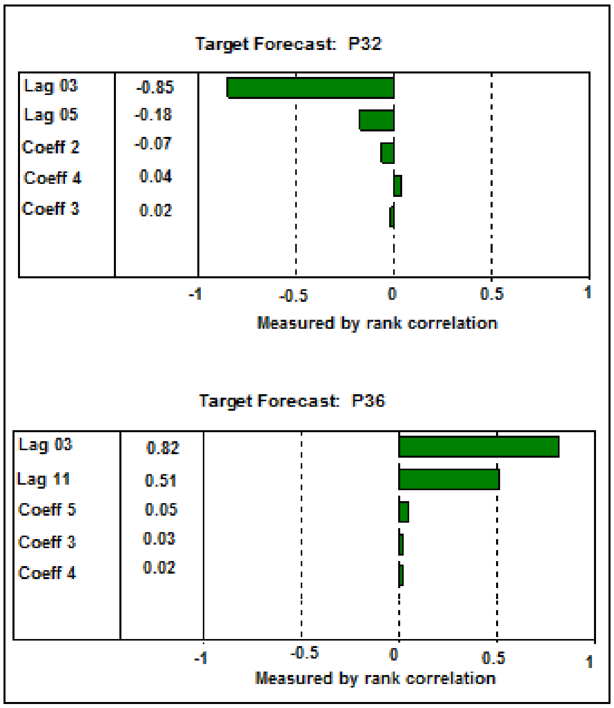

Figure 9: Sensitivity Analysis

7. Conclusion

Intermittent demand is the demand of a product in a specific time frame having several zero or uneven demand. The forecasting of intermittent demand has been widely discussed and attempts have been made to decipher the demand patterns to achieve the best forecast. The most familiar methods to deal with intermittent demand forecasting are moving average, exponential smoothening, trend adjusted exponential smoothening and linear regression. But, the degree of conformance between actual demand and forecast indicates limitations of these models. This paper proposes a GMDH method to undertake the issue of intermittent demand forecasting. GMDH is able to formulate a model that describes the natural structure of complex and non-linear systems by an evolutionary iteration which minimizes the prediction error in each phase. Superior degree of the forecast having outstanding conformance between the data set representing validation demand data and the forecast for the same period as well as lower values of MSE obtained by GMDH as compared to the classical models suggest that the GMDH model outperforms. The average reduction of MSE is over 69.08% in the case of GMDH, compared with the classical methods. The range of MSE reduction is from 56.88% to 80.33%. Moreover, Monte Carlo Simulation has been invoked to have the probabilistic estimate of forecasts by subjecting the polynomials obtained from GMDH and elucidating the forecast sensitivity to lags and coefficients in the equations. It has been observed that the forecast is more sensitive to lags than to coefficients.

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Biography

Ms. Prerna Mishra completed her undergraduate studies from Nanyang Technological University, Singapore with a Bachelor of Engineering Honors in Electrical and Electronic Engineering. She has a few publications during her undergraduate studies. She is currently pursuing Master of Science in Mathematical Finance at Boston University, USA.

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