

Optimizing Runway Departure Scheduling Based on a Max-Plus Algebra TSAT Decision Model

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Abstract

Airport departure operations are often affected by surface congestion caused by interactions among airline readiness, ground traffic conditions, and runway capacity constraints. To improve departure flow predictability, the Target Start-up Approval Time (TSAT) has been introduced as a key decision variable for regulating pushback timing. This study investigates a runway departure scheduling problem based on a TSAT decision model and proposes a modeling framework using max-plus algebra. The departure process, including pushback, taxiing, runway queuing, and take-off, is represented as a discrete-event system in which synchronization constraints and processing times are explicitly described by max and addition operations. This formulation provides a transparent representation of event-time propagation from pushback to take-off and clarifies the temporal structure underlying airport departure operations. Numerical examples and sensitivity analysis are conducted under a simplified airport scenario inspired by Tokyo International Airport (Haneda). The results show that departure performance is highly sensitive to departure sequencing decisions and wake turbulence separation constraints, and that TSAT primarily influences runway queue formation rather than directly determining take-off order. These findings clarify the operational role of TSAT in structured departure flow management.

Keywords

Airport departure scheduling, Target Start-up Approval Time (TSAT), Max-plus algebra, Discrete-event systems, Runway sequencing.

1. Introduction

Airport surface congestion remains a critical challenge for large hub airports, leading to increased taxi-out times, excessive fuel consumption, and reduced schedule reliability (Idris et al. 2002). In particular, departure operations are strongly affected by the interaction between airline readiness at gates, surface traffic conditions, and runway capacity constraints. These factors often cause unstable departure queues and unpredictable take-off times (Bennell et al. 2013).

To improve the predictability of departure operations and mitigate surface congestion, several operational concepts have been introduced (EUROCONTROL 2017). Among them, the Target Start-up Approval Time (TSAT) has been adopted as an important decision variable to regulate pushback timing by coordinating gate-side readiness with downstream runway capacity (ICAO 2018). Proper TSAT-related decisions can contribute to smoother departure flows and more stable runway queue formation.

From an operational perspective, TSAT has been widely implemented within Airport Collaborative Decision Making (A-CDM) frameworks at major capacity-constrained airports. Recent empirical analyses based on operational A-CDM milestone data have examined the role of TOBT and TSAT updates in improving departure predictability and identifying start-up phase irregularities (Zuniga and Herrero 2024). In addition, updated A-CDM specifications

summarize accumulated implementation experience across European airports and emphasize the importance of milestone-based information sharing for enhancing coordination between airport stakeholders (EUROCONTROL 2024). These recent developments illustrate the growing integration of structured scheduling concepts with real-world airport operations.

From a modeling perspective, airport departure operations can be viewed as a discrete-event system, where events such as pushback, taxiing, runway queuing, and take-off occur sequentially under synchronization and separation constraints (Cohen et al. 1985), (Baccelli et al. 1992), (Komenda, 2018). While a variety of optimization-based scheduling approaches have been proposed, many of them rely on complex formulations that make it difficult to clearly interpret the temporal structure underlying departure processes (Barnhart and Smith 2012).

Recent studies have examined dynamic surface and departure scheduling under operational uncertainty, incorporating wake turbulence separation and real-time coordination mechanisms (Park and Lee 2024), (Aditya et al. 2024). In addition, updated A-CDM specifications summarize accumulated operational experience and reinforce the role of TSAT in structured departure control (EUROCONTROL, 2024).

This study addresses a runway departure scheduling problem based on a TSAT decision model and proposes a modeling framework using max-plus algebra. By explicitly representing waiting and processing relationships through max and addition operations, the proposed approach provides a transparent and interpretable description of event-time propagation from pushback to take-off.

The main contributions of this study are summarized as follows: (1) a TSAT-based departure scheduling model is formulated as a discrete-event system using max-plus algebra; (2) the temporal structure of runway departure operations is clarified through event-time propagation equations; and (3) numerical examples and sensitivity analysis are conducted to demonstrate the applicability of the proposed framework.

The remainder of this paper is organized as follows. Section 2 describes the problem formulation and operational assumptions. Section 3 presents the max-plus-based event-time model. Section 4 provides numerical examples and engineering interpretation of the results. Finally, Section 5 concludes the paper and discusses future research directions.

2. Problem Formulation

2.1 Operational Assumptions

We consider a set of departing flights within the same operational time window. A simplified airport layout is assumed: multiple gates connect to a single departure runway through a common taxiway system. The analysis focuses on temporal dependencies among departure events rather than detailed taxiway geometry.

Aircraft are classified into three wake turbulence categories—Heavy (H), Medium (M), and Light (L)—following standard ICAO practice. The minimum runway separation time between two consecutive departures depends on their wake turbulence categories. (In this paper, the separation parameter is treated as an input table indexed by leader–follower categories.)

2.2 Key Time Variables and Interpretation

For each flight i , the following time variables are defined:

- $TOBT_i$: target off-block time provided by the airline or ground handler,
- t_i^{pb} : pushback start time, interpreted as the modeled target corresponding to TSAT,
- t_i^{tx} : taxiway entry time,
- t_i^q : runway queue entry time,
- t_i^{to} : take-off time.

The pushback start time is constrained not to be earlier than $TOBT_i$, reflecting airline readiness information, while subsequent event times are governed by operational durations and runway separation constraints.

For interpretation, $TOBT_i$ is treated as an input variable representing airline readiness, while the subsequent event times represent internal states of the departure process.

In the proposed framework, TSAT is influenced by several operational factors. These include airline readiness information reflected by the Target Off-Block Time (TOBT), runway capacity constraints, wake turbulence separation requirements between consecutive departures, and the planned departure sequence within the considered time window. In addition, operational durations such as pushback and taxi times indirectly affect TSAT feasibility by shaping downstream temporal constraints.

3. Methods

Figure 1 illustrates the overall framework of the proposed TSAT-based departure scheduling model. Airline readiness information, represented by TOBT, is used as an input to the departure management process, where TSAT-related pushback decisions are made. The determined pushback start time then propagates through taxiing and runway queuing stages, eventually leading to the take-off event. The proposed framework focuses on how TSAT-based decisions shape the formation of departure flows under runway separation constraints and operational processing times.

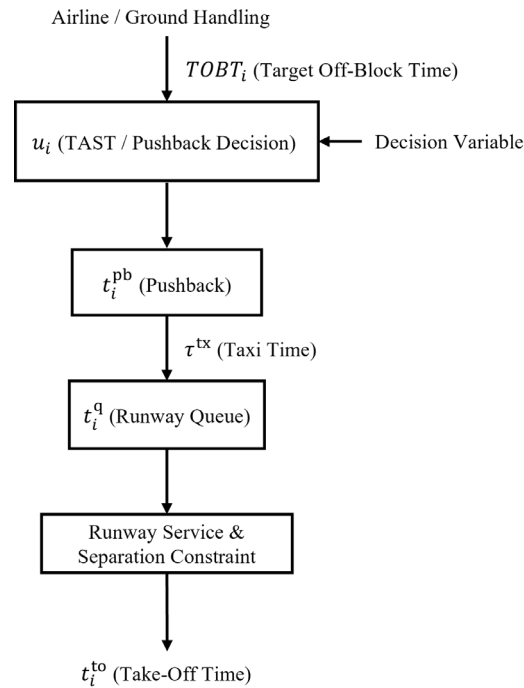


Figure 1. Overall framework of the proposed TSAT-based departure scheduling model

3.1 Event-Time Modeling Using Max-Plus Algebra

The departure process is modeled as a discrete-event dynamic system using max-plus algebra (Heidergott et al. 2006), (Goto 2014). In this algebra, addition and multiplication are defined as

$$a \oplus b = \max(a, b), a \otimes b = a + b.$$

For clarity, all event-time variables in this study represent the earliest feasible occurrence times of the corresponding departure events. The notation is used consistently throughout the paper. The max operator captures synchronization constraints between events, indicating that a given event can occur only after all relevant preceding conditions are satisfied. Additive terms represent deterministic processing or travel times such as pushback and taxi durations. The pushback start time is constrained by airline readiness and scheduling decisions:

$$t_i^{\text{pb}} = \text{TOBT}_i \oplus u_i,$$

where u_i represents a scheduling adjustment associated with TSAT.

Subsequent event times are given by

$$t_i^{\text{tx}} = t_i^{\text{pb}} \otimes \tau^{\text{pb}}, t_i^{\text{q}} = t_i^{\text{tx}} \otimes \tau^{\text{tx}},$$

where τ^{pb} and τ^{tx} denote pushback and taxi durations, respectively.

The take-off time is determined by both aircraft readiness at the runway queue and the minimum separation from the preceding departure:

$$t_i^{\text{to}} = (t_i^{\text{q}} \otimes s_i) \oplus (t_j^{\text{to}} \otimes \Delta_{ji}),$$

where s_i denotes runway service time and Δ_{ji} is the minimum separation depending on wake turbulence categories.

Figure 2 presents the event-time propagation structure of the departure process expressed using max-plus algebra. In this representation, the max operator ($\oplus$) captures synchronization constraints, indicating that an event can only occur after all relevant conditions have been satisfied (De Schutter et al. 2019). The additive operator ($\otimes$) represents deterministic processing or travel times, such as pushback duration and taxi time (Butkovič 2010). By combining these operators, the temporal structure from pushback to take-off can be described in a compact and interpretable manner.

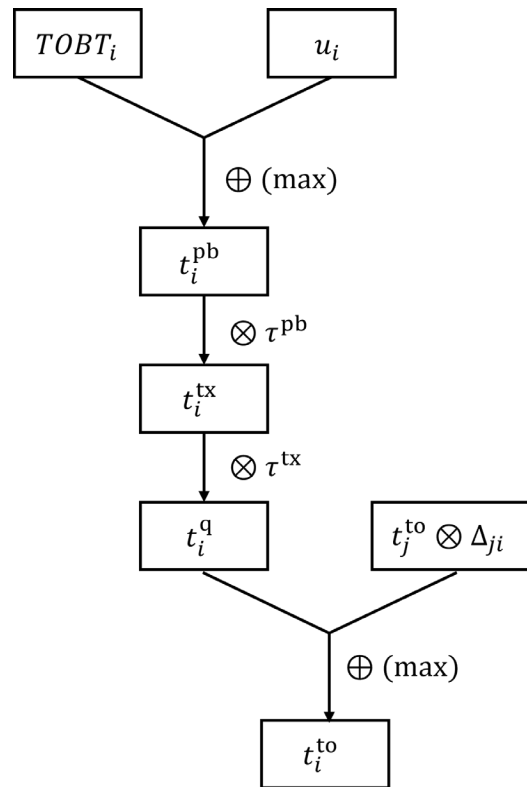


Figure 2. Event-time propagation structure of the departure process expressed in max-plus algebra.

3.2 Departure Sequencing and Optimization Objective

Let $\pi = (\pi_1, \pi_2, \dots, \pi_n)$ denote a departure sequence within the considered time window. For a given sequence π , the take-off times are recursively determined by the max-plus event-time equations.

The scheduling problem is formulated as an optimization problem aimed at minimizing total departure delay while improving runway utilization:

$$\min_{\pi, u} \left\{ \alpha \sum_i (t_i^{\text{to}} - TOBT_i) + \beta \max_i t_i^{\text{to}} \right\}.$$

4. Numerical Example

This section presents a small numerical example to illustrate the behavior of the proposed max-plus-based departure scheduling model. The purpose of this example is to demonstrate how departure sequences and wake turbulence separation affect take-off times and overall performance, rather than to replicate a full-scale airport operation.

4.1 Experimental Setup

A set of four departing flights within the same operational time window is considered. Aircraft are classified into Heavy (H), Medium (M), and Light (L) wake turbulence categories. The target off-block times (TOBT) are set to $[0, 2, 4, 6]$ minutes. Pushback duration, taxi time, and runway service time are assumed to be constant and set to $\tau^{pb} = 2$, $\tau^{tx} = 5$, and $s = 1$ minute, respectively.

Table 1 summarizes the experimental settings used in the numerical example.

Table 1. Experimental settings for the numerical example

Item	Value
Number of flights	4
Wake turbulence categories	H / M / L
TOBT (min)	0, 2, 4, 6
Pushback time τ^{pb}	2 min
Taxi time τ^{tx}	5 min
Runway service time s	1 min

Four aircraft are considered in the numerical example and are assigned to wake turbulence categories (M, M, H, L), respectively. The minimum runway separation times are specified based on wake turbulence categories.

In particular, the Heavy-to-Medium, Heavy-to-Light, and Medium-to-Light separation times, denoted by $\Delta_{H \rightarrow M}$, $\Delta_{H \rightarrow L}$, and $\Delta_{M \rightarrow L}$, are set to 2, 3, and 3 minutes, respectively (ICAO 2016).

4.2 FCFS Departure Sequence

As a baseline, a first-come-first-served (FCFS) departure sequence based on TOBT is evaluated. The take-off times are computed recursively using the max-plus event-time equations. Under the FCFS sequence, the resulting take-off times are $[8, 10, 12, 15]$ minutes, yielding a makespan of 15 minutes. The corresponding total delay is 33 minutes. The take-off times are obtained by recursively applying the max-plus event-time equations introduced in Section 3.

4.3 Alternative Sequence

An alternative departure sequence is then examined to reduce the impact of wake turbulence separation. The resulting take-off times are $[8, 12, 14, 17]$ minutes, with a makespan of 17 minutes. The total delay is 39 minutes. This comparison illustrates that departure sequence decisions significantly influence system performance and that naive reordering does not necessarily lead to improvement. Table 2 compares the departure performance under the FCFS and alternative departure sequences.

Table 2. Comparison of departure performance under different sequences

Sequence	Order	Makespan (min)	Total delay (min)
FCFS	(1,2,3,4) = (M, M, H, L)	15	33
Alternative	(1,3,2,4) = (M, H, M, L)	17	39

4.4 Sensitivity Analysis on Wake Turbulence Separation

To further investigate the robustness of the proposed departure scheduling model, a sensitivity analysis is conducted with respect to wake turbulence separation parameters. In particular, the minimum separation time from Heavy to Light aircraft is varied, while all other parameters and the departure sequence remain unchanged.

Three cases are considered by adjusting the Heavy-to-Light separation time $\Delta_{H \rightarrow L}$ to 3, 4, and 5 minutes, respectively. The FCFS departure sequence is used for all cases to isolate the impact of parameter variation. Table 3 summarizes the resulting makespan and total departure delay under different separation settings.

Table 3. Sensitivity of departure performance to wake turbulence separation

$\Delta_{H \rightarrow L}$	Makespan (min)	Total delay (min)
3	15	33
4	16	34
5	17	35

The results indicate that increased wake turbulence separation leads to longer take-off completion times and higher overall departure delay. This confirms that runway separation constraints significantly influence departure performance and highlights the importance of scheduling strategies that explicitly account for such operational parameters.

4.5 Engineering Interpretation of Results

The numerical results and sensitivity analysis provide several important engineering insights into TSAT-based departure scheduling.

First, the comparison between FCFS and alternative departure sequences demonstrates that departure order strongly interacts with wake turbulence separation constraints. Although sequence adjustment is often considered a straightforward way to reduce congestion, the results show that reordering aircraft without explicitly accounting for wake turbulence effects can increase overall delay and makespan. This occurs because mandatory separation times propagate through the event-time structure, amplifying downstream waiting and reducing effective runway utilization. Therefore, departure sequencing decisions must be evaluated in conjunction with safety-driven separation requirements.

Second, the sensitivity analysis on wake turbulence separation confirms that runway separation parameters play a dominant role in shaping departure performance. An increase in the Heavy-to-Light separation time leads to a systematic increase in both makespan and total departure delay. From an operational perspective, this implies that stricter safety constraints inevitably reduce runway throughput, and that TSAT-based scheduling must adapt to such conditions rather than attempting to eliminate their impact.

Third, the results indicate that TSAT-related decisions primarily influence the formation and timing of runway queues, rather than directly controlling take-off order. By adjusting pushback timing, TSAT indirectly regulates when aircraft arrive at the runway queue, thereby shaping the interaction between demand and runway capacity. This suggests that TSAT is most effective when used as a flow-shaping mechanism, complementing runway-level sequencing decisions.

Overall, these findings demonstrate that the proposed max-plus-based framework provides a transparent and systematic way to analyze the interplay between operational constraints, scheduling decisions, and departure performance.

4.6 Discussion

The numerical example confirms that the proposed max-plus model can systematically evaluate different departure sequences and quantify their impact on departure delays and runway utilization. These results highlight the importance of structured optimization approaches for TSAT-based departure scheduling.

It should be noted that the numerical example presented in this study is based on a simplified airport configuration with an aggregated taxiway representation and deterministic processing times. In real-world airport operations, additional factors such as stochastic taxi times, multiple runway configurations, dynamic airline operating policies, and interactions with arrival traffic may further influence departure dynamics. These factors can affect the quantitative outcomes of scheduling decisions; however, they do not alter the fundamental event-time relationships captured by the proposed max-plus formulation. As such, the simplified setting adopted here is intended to highlight the underlying temporal structure of TSAT-based departure control rather than to replicate the full complexity of operational environments.

5. Conclusions and Discussion

5.1 Conclusions

This paper presented a TSAT-based departure scheduling framework using max-plus algebra to model the temporal structure of airport departure operations. By representing pushback, taxiing, queuing, and take-off events as a discrete-event system, the proposed model captures key synchronization and separation constraints in a compact and interpretable form.

A numerical example and sensitivity analysis demonstrated that departure performance is strongly influenced by both sequencing decisions and wake turbulence separation parameters. The results showed that naive sequence adjustments do not necessarily improve performance and that TSAT-related decisions primarily shape runway queue formation rather than directly controlling take-off order.

The proposed framework provides a transparent analytical basis for evaluating TSAT-based scheduling strategies and can serve as a foundation for adaptive departure management under operational constraints. In practical implementation, the proposed framework can be embedded within a rolling-horizon decision process, where TSAT adjustments are periodically updated as new operational information becomes available.

Furthermore, the proposed event-time formulation can be extended to multi-runway airport environments by incorporating additional runway-specific separation matrices and resource synchronization constraints. Such an extension would enable coordinated departure scheduling across parallel or intersecting runways while preserving the algebraic structure of the model.

5.2 Extension to MPC-Based TSAT Control

As shown in Figure 3, the proposed framework can be embedded into a rolling-horizon MPC structure, where TSAT adjustments are updated based on predicted system states and operational disturbances.

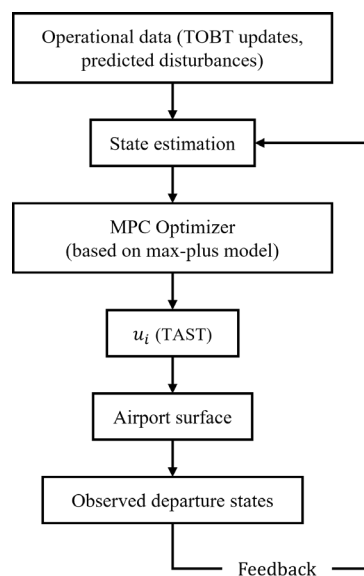


Figure 3. Conceptual framework of MPC-based TSAT control using the proposed max-plus event-time model.

The proposed max-plus-based departure scheduling framework can be naturally extended to a model predictive control (MPC) setting. In such a formulation, TSAT-related decisions are updated over a rolling horizon to account for evolving operational conditions.

In the MPC framework, the system state consists of predicted runway queue arrival times and take-off times, while TSAT adjustments serve as control variables. Airline readiness updates, variations in taxi time, and capacity fluctuations are treated as disturbances. At each decision epoch, a finite-horizon scheduling problem is solved using the max-plus event-time model, and only the near-term TSAT decisions are implemented.

This rolling-horizon structure enables adaptive departure scheduling without altering the underlying event-time model. As a result, the proposed framework provides a transparent and computationally efficient foundation for real-time TSAT control under uncertainty.

From a computational perspective, the max-plus event-time equations admit efficient recursive evaluation, as take-off times are determined through simple max and addition operations along a given departure sequence. For moderate planning horizons, the resulting scheduling problem can be solved within short computation times. This suggests that the proposed framework is compatible with practical implementation as part of a rolling-horizon TSAT decision support system.

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Biographies

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