Proceedings of the International Conference on Industrial Engineering and Operations Management

Publisher: IEOM Society International, USA DOI: 10.46254/AP07.20260218
Published: March 25, 2026

Predictive Waste Classification and Reverse Logistics Optimization Using Machine Learning in a Circular Supply Chain: A Case Study of Thread Dyeing Company

Ananya Halder, Zahin Ar Rafi and Ferdous Sarwar
Department of Industrial and Production Engineering
Bangladesh University of Engineering and Technology
Dhaka, Bangladesh
ananyohalder48@gmail.com, zahin@ipe.buet.ac.bd, ferdoussarwar@ipe.buet.ac.bd

Abstract

Textile industry, particularly wet processing and thread dyeing industry, produces several waste flows that creates serious environmental and operational problems. This research aims to develop a combined data-driven model for predictive waste classification and reverse logistics optimization in a circular supply chain system. The study is focused on integrating daily operational data of two years from a thread dyeing company, characterized by features, such as production volume, dye batch count, chemical and dye usage, water and energy consumption, machine utilization and waste generation, to train Random Forest machine learning model for predicting waste quantities. With the outcome of the waste prediction, a Mixed Integer Linear Programming Problem (MILPP) for reverse logistics strategies was implemented to optimize waste collection, recycling, treatment, and disposal decisions across the supply chain. The developed framework will enable the manufacturers to plan their waste management more accurately, save operating costs and improve recovery of valuable resources within textile manufacturing operations. The findings demonstrated the effectiveness of the model, as a scalable, data-driven decision-support tool for circular economy, as well as other resource-intensive manufacturing industries.

Keywords
Thread dyeing industry, Waste classification, Machine learning, Reverse logistics, Circular supply chain.

1. Introduction
Globally, the waste crisis has been considered as one of the most important risk factors for our environment, due to the increase of industries around the world, along with population growth and rapid growth of manufacturing activities. According to the World Bank, global waste generation is projected to reach about 3.4 billion tons by 2050, which will be a significant increase from 2.1 billion tons recorded in 2023 (Fotovvatikhah et al., 2025). Also, over 43% of solid waste around the world is being mismanaged through incineration, illegal dumping, open burning, and unregulated landfilling. This creates severe environmental and health problems (Fotovvatikhah et al., 2025). Industrial sectors, more specifically those that are more resource dependent, such as textile manufacturing, are excessively responsible for this crisis. The textile and dyeing industries rank among the most environmentally troublesome sectors in the global economy, as they generate huge amounts of solid, liquid, and chemical waste throughout their production cycles (Khattab et al., 2020). But environmental rules and regulations and stakeholders' demands are going to develop day by day to make our environment healthier, which leads to the need to develop intelligent, data-driven frameworks for industrial waste management and resource recovery for both strategic and operational planning.

The thread dyeing industry, which is an important sub-sector of the textile manufacturing industry, is infamous for its resource dependency and waste generation. Dyeing operations need an enormous amount of water along with energy and chemical substances such as reactive dyes, fixing agents, surfactants, mordants, salts, leveling agents, etc., and these components produce a difficult waste stream that is difficult to categorize and manage systematically (Uddin, 2021). Khattab et al. (2020) identified that textile dyeing effluents contain toxic components, such as naphthol, vat dyestuffs, nitrates, acetic acid, heavy metals such as chromium (Cr), copper (Cu), cadmium (Cd), and iron (Fe) along with formaldehyde-based color mixing substances and non-biodegradable hydrocarbon-based softeners, Sristi and Hasin (2025) showed that waste generated across multiple dyeing stages including overproduction, waiting time, transportation, excess inventory, unnecessary motion, and process defects which could be classified through lean tools such as Value Stream Mapping (VSM). But, even with these kinds of implications, the classification of waste materials and optimization of reverse logistics processes remain totally manual and fragmented.

The evolution of the circular economy (CE) has restructured industrial waste management. Differentiating from the traditional 'take-make-dispose' model, circular economy advices for closed loop systems where the materials, components, and products are being recycled, remanufactured, reused with economical aspects (Geissdoerfer et al., 2017). Reverse logistics (RL), defined by Rogers and Tibben-Lembke (1999) as the systematic planning, implementation, and managing cost effective flow of raw materials, in-process inventory, finished goods, and related information from the point of use to the point of origin, for understanding the value or proper disposal, which helps in the operational backbone of circular supply chain systems. Though the RL models are quite significant and have been used in several sectors of industry, their implementation in the manufacturing environment remains underexplored and mostly reactive rather than predictive.

The development of machine learning (ML) and artificial intelligence has created opportunities for researchers to invest in automation waste prediction and optimization of logistics operations. For structured and product-dependent datasets which being generated by manufacturing industries, use methods like Random Forest and Gradient Boosting to demonstrate strong predictive performance, and the methods outperform both linear models and single decision trees on tabular operational data (Alsabt et al., 2024; Samal et al., 2025). Alsabt et al. (2024) stated that when implementing ML forecasting models with optimization systems, it can improve the operational efficiency up to 15% in waste management and lessen the environmental impacts. Hmamed et al. (2022) described that ML-based waste prediction coupled with the circular economy supply chain can be an emerging and important framework, because most of the existing models treat waste generation as a static input rather than a dynamic, production-dependent variable. Bhowmik et al. (2024), in a review of nearly 3,000 articles on AI in reverse logistics, noted that the direct integration of ML-predicted waste quantities into MILP-based reverse logistics optimizers remains largely unexplored, which is the actual methodological gap of this study.

Though ML-based waste classification and AI-driven reverse logistics are being conducted extensively in recent times, the existing literature reveals many crucial gaps. To start, the majority of ML-based waste classification studies focus on generic municipal solid wastes and have not been constructed with the material characteristics of industrial manufacturing waste (Fotovvatikhah et al., 2025; Yasin and Koklu, 2025). Also, reverse logistics research has delved into optimization models in many sectors and frameworks, but has not progressed much in studies directly linking predictive waste classification outputs with RL optimization within a circular supply chain framework (Bhowmik et al., 2024). Moreover, the textile and thread dyeing sector, which is one of the world's most environmentally impactful industries (Khattab et al., 2020; Uddin, 2021), has not been explored enough in the ML-based waste management literature. This study aims to fill these identified gaps by developing and validating an integrated framework for predictive waste classification and reverse logistics optimization using machine learning in a circular supply chain with a focus on a thread dyeing company.

1.1 Objectives

The overarching goal of this research is to develop and empirically validate an integrated ML-MILP decision-support pipeline for predictive waste classification and reverse logistics optimization in a thread-dyeing circular supply chain. To achieve this aim, the study pursues the following five specific objectives:

The first objective is to characterize the waste generation profile of a thread dyeing facility by identifying, quantifying, and analyzing the four primary industrial waste streams; solid waste, yarn waste, chemical waste, and wastewater; and by determining the key operational and process drivers, such as production volume, dye batch count, chemical consumption, and machine utilization, that govern their daily variability (Sristi and Hasin 2025; Uddin et al. 2023).

The second objective is to develop, train, and evaluate a supervised Random Forest machine learning model for the predictive estimation of daily waste quantities across each of the four identified waste streams, using a literature-calibrated synthetic operational dataset of 731 daily records and benchmarking performance against Decision Tree model through R^2, MAE, RMSE, and MAPE metrics (Samal et al. 2025; Lee et al. 2024).

The third objective is to formulate and solve a Mixed Integer Linear Programming (MILP) model for reverse logistics network optimization that allocates the predicted waste volumes from the dyeing facility to a set of candidate external collection centers, minimizing total system cost encompassing transportation, holding, disposal, fixed contract, carbon emission, and working capital components, while simultaneously maximizing material recovery and circular economy compliance (Govindan and Soleimani 2017; Shi et al. 2025).

The fourth objective is to integrate the ML prediction outputs as dynamic daily inputs into the MILP optimization model, constructing a sequential decision-support pipeline that replaces the static historical waste averages conventionally used in reverse logistics planning with production-driven, data-responsive predictions, thereby reducing the uncertainty inherent in waste supply estimation and enabling proactive rather than reactive logistics scheduling (Alsabt et al. 2024; Hmamed et al. 2022).

The last, that is the fifth objective, is to validate the integrated ML–MILP framework through formal statistical testing, including paired t-tests on model error reduction, one-sample t-tests against an operational MAPE deployment benchmark, and IBM CPLEX optimality certification, and to assess the framework's scalability and applicability as a circular economy decision-support tool for the thread dyeing industry and analogous resource-intensive manufacturing sectors (Bhowmik et al. 2024; Anandhabalaji 2025).

2. Literature Review

The literature review combines existing literature across five areas, which are important to this study: circular economy principles in textile manufacturing, reverse logistics optimization and the static waste problem, machine learning applications for industrial waste prediction, environmental impact of textile dyeing in Bangladesh, and lean-based waste identification in thread dyeing facilities. These studies help build the theoretical foundation and minimize the gaps that direct the need for an ML-MILP framework developed in this research.

2.1 Circular Economy in Textile Manufacturing

The circular economy (CE) defines that manufacturing waste can be recovered through proper reduction, reuse, and recycling policies (Geissdoerfer et al., 2017). Hmamed et al. (2022) describe that combining ML with CE frameworks for waste treatment is an important and growing phenomenon that should be further explored. Farshadfar et al. (2024) did a review of 66 studies and it indicates that the ML impose a significant contribution to circular supply performance which ultimately improves its operational optimization and waste reduction in forward and reverse logistics stages. Also, Dong et al. (2025) reviewed implementation of AI between 2014 to 2024 for textile waste management and the research demonstrates that AI helps in optimizing production processes, inventory management, and intelligent waste management. This also builds more motivation for the ML implementation in the present study.

2.2 Reverse Logistics Optimization and the Static Waste Problem

Biyada and Urbonavicius (2025) surveyed about circular economy implementation in textile and clothing supply chains and found that data-driven reverse logistics frameworks are important for making textile and clothing production more sustainable. To advance in reverse logistics optimization, Mixed-Integer Linear Programming is one of the main approaches for reverse logistics (RL) network design, as it creates many optimizations, such as optimization of facility location, waste allocation, and cost minimization at the same time (Govindan and Soleimani, 2017). But a structural limitation in the RL literature is that waste generation is always used as a known, static input with a historical average value, rather than using a dynamic daily variable. Bhowmik et al. (2024), reviewing 2,929 AI-in-RL articles across three decades, confirm that though use of ML in recycling and waste management has heavily increased, the use of ML to predict waste quantities as real-time inputs to MILP-based RL optimizers remains mostly unexplored. The actual waste values differ every day, and the integrated networks will be able to allocate the waste according to the capacity of the recycling plants.

2.3 Machine Learning for Industrial Waste Prediction

Olawade et al. (2024) researched about AI and ML applications in waste management from 2015 till 2025, and the research indicates that the integration of AI/ML helps significantly in boosting circular economy targets as it can help in improving operational efficiency and reducing environmental harmness. Most ML waste classification research uses CNNs for image-based municipal solid waste sorting, which helps in achieving accuracies up to 99.6% on benchmark datasets (Fotovvatikhah et al., 2025). But for production-data-driven quantity prediction, we can use methods such as Random Forest (RF) and Gradient Boosting (GBM), as they perform better in structured tabular datasets. RF reduces overfitting by bagging across multiple decision trees and manages non-linear feature relationships properly, with key hyperparameters including n_estimators, max_depth, and max_features (Samal et al., 2025). Thus, GBM achieves R^2 values as high as 0.99 for solid waste generation prediction when it is trained on operational features (Singh and Uppaluri, 2023; Samal et al., 2025). These reports truly establish the basis of using the Random Forest model in the present study for textile dyeing factories, as waste streams, such as wastewater, chemical residues, and sludge material forms, are not completely suitable for visual classification. Thus, it creates the need to make structured tabular modeling from operational process data, the appropriate methodological choice.

2.4 Environmental Impact of Textile Dyeing in Bangladesh

Bangladesh's textile dyeing sector uses a vast amount of water and chemical substances compared to other countries globally. Uddin et al. (2023) report that textile sectors use average groundwater consumption of 164 L/kg and wastewater generation of 119 L/kg of processed textile, along with chemical consumption ranging from 152 to 705 g/kg (averaging 449 g/kg). Total effluent discharge crossed 217 million cubic meters in 2016, carrying heavy metals, high COD/BOD loads, and carcinogenic compounds that have seriously degraded the Burigonga, Turag, and Shitalakkhya river systems (Hossain et al., 2018). In spite of all these and vast ETP installations, treatment facilities cannot maintain national discharge standards, and no predictive system exists to understand daily waste volumes.

2.5 Lean Waste Identification in Thread Dyeing — The Proximate Gap

Sristi and Hasin (2025) represent the most directly relevant prior work, as the research applies Value Stream Mapping (VSM) to a thread dyeing plant in Gazipur, Bangladesh. Their analysis identified Radio Frequency Drying and Winding as the primary bottleneck stages and established six operational drivers of water consumption and waste generation: capacity utilization, product mix, shade depth, First Time Right (FTR) rate, machine wash frequency, and technology adoption. These variables are being coincided with the features used in the present study's ML model (e.g., Production_Volume_kg, Number_of_Dye_Batches, FTR_Rate, Machine_Utilization_Rate). VSM characterizes waste sources qualitatively at the process level, but it cannot produce daily quantitative forecasts of waste volumes. This is the decisive gap the present study fills. This study combines and uses the process-level waste parameters identified by Sristi and Hasin (2025) into daily predictive estimation with a tuned Random Forest. And the predictions fed into a MILP optimizer for proactive reverse logistics scheduling. No prior study has integrated these two methodological pillars in a thread dyeing circular supply chain context.

The literature review narrows down to a clear research opportunity. Circular economy establishes the foundation for treating dyeing waste as a recoverable resource, while reverse logistics optimization provides the operational framework for routing that waste properly. A structural weakness in the RL literature is that waste quantities are used as static and historically averaged inputs rather than dynamic variables. Machine learning methods applied to structured tabular operational data resolve the issue and help replace those static assumptions with daily predictive estimates. The environmental harshness of Bangladesh's thread dyeing sector and the process-level waste driver analysis of Sristi and Hasin (2025) also confirm that the operational data necessary to build such models exists, but has never been used for predictive or optimization purposes. This research implicates that gap with a tuned Random Forest waste prediction model with an MILP reverse logistics optimizer within a circular supply chain framework.

3. Methodology

This study uses a two-stage methodology combining a supervised machine learning (ML) model with a Mixed Integer Linear Programming (MILP) optimization model to support waste management and reverse logistics decisions in a thread dyeing company. The ML model predicts daily waste quantities, and its outputs will be used for direct inputs to the MILP model. This approach expresses the growing use of digital tools in circular supply chain management, where predictive analytics and optimization are being combined to improve the environmental and economic situation (Hmamed et al. 2022; Alsabt et al. 2024). The research follows five stages: (1) data collection and preprocessing; (2) ML model training; (3) ML model validation; (4) MILP model formulation using ML outputs; and (5) optimization of reverse logistics decisions.

3.1 Case Study: Thread Dyeing Company

The case study is about a thread dyeing facility, which is a sub-sector of the textile manufacturing process. It is characterized by using a vast amount of water and chemical usage. Dyeing operations generate massive volumes of wastewater, chemical residues, and solid waste, which depend primarily on process variables including dye dosage, temperature, liquor ratio, cycle time, etc. (Lee et al., 2024). Also, textile dyeing uses water every day for long-time operations, making it among the most water-based industrial processes. Globally, it is responsible for 17–20% of industrial water pollution (World Bank, 2019; Khattab et al., 2020). Here, four waste streams are considered in this study: wastewater, solid waste, chemical waste, and yarn waste. With the vast implementation of ETP, we assume the network to be that wastewater is treated on-site via the plant's Effluent Treatment Plant (ETP), and solid waste, chemical waste, and yarn waste are sent to external collection centers for recovery or disposal.

3.2 Dataset

A challenge explicitly noted by Uddin et al. (2023) and Sristi and Hasin (2025) is that there is almost no publicly available facility-level operational data, with the confidentiality problem that restricts direct access to production records in the thread dyeing industry. For this reason, a literature-calibrated synthetic dataset was constructed for this study. Parameter ranges for each operational variable and waste stream were collected from quantitative values that are being reported in published textile processing studies. The dataset includes 39 variables per observation. The dataset spreads around production volumes, machine utilization, energy and water consumption, chemical dosage quantities, process parameters, and waste generation for each stream. Variables include temporal identifiers (Date, Month, Year, Day_Type, Monthly_Demand_Factor), production metrics (Production_Volume, Machine_Utilization, Number_of_Workers), resource consumption (Total_Energy_kWh, Total_Water_Consumption_L), process parameters (Dye_Fixation_Ratio, Liquor_Ratio, Temperature_C, Cycle_Time_hr, Dye_Batch_Count), chemical inputs (Dye, Salt, Alkali, Acid, Auxiliary), waste outputs (Solid_Waste_kg, Yarn_Waste_kg, Chemical_Waste_kg, Wastewater_L), and wastewater quality indicators (BOD_mg/L, COD_mg/L, pH). Also, dyeing process parameters as predictors are well justified by Lee et al. (2024), who describe that process variables, such as dye dosage, temperature, and cycle time, are the primary drivers of residual waste in dyeing operations.

3.3 Data Preprocessing

The main dataset doesn't have any missing values, as confirmed in the exploratory analysis. Temporal variables such as,Month and Day of Week were tuned to cyclical sine-cosine encoding to reflect their circular nature, which contributes to four additional features (Month_Sin, Month_Cos, Day_Sin, Day_Cos) along with a Quarter variable. This approach avoids the distortion introduced by standard label encoding for periodic variables. The final feature set includes 23 input variables. A temporal year-based split was applied. Year 2023 data (365 records) is used for training, and Year 2024 data (366 records) for testing. The model was evaluated strictly on unseen future observations, which is a more appropriate validation strategy for time-dependent industrial processes.

3.4 Predictive Waste Quantity Model

Random Forest (RF) is selected as the core predictive algorithm for this study. Random Forest is a supervised method that creates multiple decision trees on bootstrapped subsets of the training data and then it combines their predictions through averaging. This mechanism is known as bootstrap aggregation or bagging. At each node split, only a random subset of features is considered. This introduces additional diversity among the trees and reduces the risk of overfitting. This combination of bagging and feature randomization makes RF a proper candidate to be applied to structured tabular data with non-linear relationships and multi-collinear input features, which are also some key characteristics of industrial dyeing process data (Alsabt et al., 2024). RF has consistently ranked among the top-performing algorithms for waste quantity prediction with structured operational datasets and performs better than single decision trees and linear regression models in industrial landscape (Samal et al., 2025).

As, in this model we will be predicting four waste streams, which are Solid Waste (kg), Yarn Waste (kg), Chemical Waste (kg), and Wastewater (L), the RF regressor will be encased within a scikit-learn MultiOutputRegressor framework. It will train one dedicated RF estimator per output target independently. This approach preserves target-specific feature importance rankings and also enables waste-stream-level interpretation rather than a single aggregated model. At first a baseline model will be established using n_estimators=300 with out-of-bag (OOB) scoring enabled, which will provide an internal generalization estimate on unseen training samples without requiring an additional validation holdout.

After that hyperparameter tuning will be performed using GridSearchCV with 5-fold cross-validation, evaluated over 72 candidate configurations. The parameter search will cover: n_estimators {200, 400, 600}, max_depth {None, 15, 25}, min_samples_split {2, 5}, min_samples_leaf {1, 2}, and max_features {'sqrt', 'log2'}. The best configuration will be selected based on cross-validation R^2 score, and the tuned model will be fitted on the full 2023 training set (365 records, 23 features) and evaluated on the held-out 2024 test set (366 records).

Model performance will be evaluated by using four metrics: R^2, Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), and Mean Absolute Percentage Error (MAPE), separately for each waste stream. Feature importance will also be examined by two complementary approaches. One is built-in Gini impurity reduction scores and another is permutation importance computed on the test set. Permutation importance is more robust against bias introduced by high-cardinality or correlated features. The dominant predictors across all four waste targets are expected to reflect the operational drivers identified by Sristi and Hasin (2025) in the thread dyeing context, including production volume, dye batch count, and chemical input quantities. Cross-validation R^2 scores and their standard deviations across folds will additionally be reported to confirm model stability and rule out overfitting to the training year.

3.5 Reverse Logistics Network Design and Mathematical Model

The reverse logistics network for a thread dyeing facility would require several layers and processes to properly allocate waste streams. Here we have developed a concept for the total network and how would they work for the thread dyeing company

3.5.1 Conceptualization of the Reverse Logistics Network

The reverse logistics (RL) network in this study describes the operational waste profile of a thread dyeing company in Bangladesh. The dyeing and finishing processes generate four distinct waste streams: solid waste, yarn waste, chemical waste, and wastewater. Ranganathan et al. (2007) express that textile dyeing facilities discharge 80-200 m^3 of effluent per ton of production. In 2016, textile industries in Bangladesh generated approximately 217 million m^3 of wastewater, carrying heavy metals, high COD/BOD loads, and carcinogenic compounds (Hossain et al., 2018). These heavy waste streams need a structured, optimized waste routing framework. So here we propose a network following the classical multi-echelon reverse logistics structure of Rogers and Tibben-Lembke (1999), including five sequential stages as shown in Figure 1.

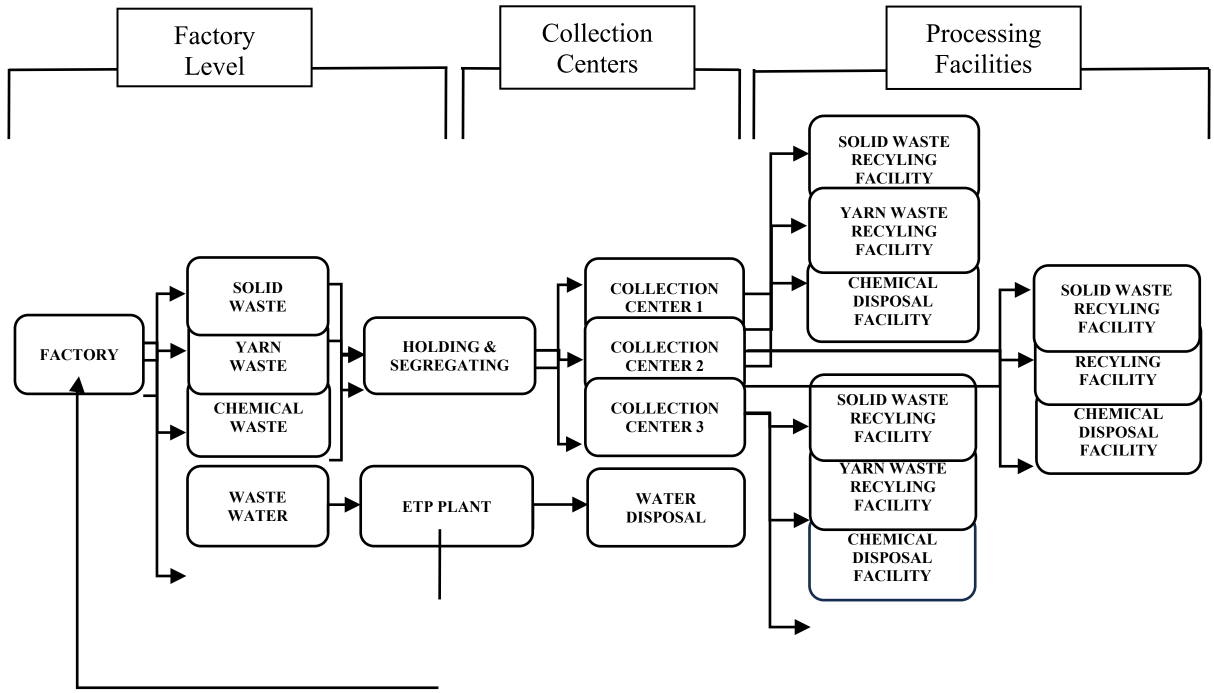

Figure 1. Reverse logistics network for waste flow in a thread dyeing facility

Figure 1 shows reverse logistics network for waste flow in a thread dyeing facility. The arrows indicate waste flows in three sections. First is factory-level waste generation (solid, yarn, chemical, wastewater), then comes the intermediate collection centers (CC1–CC3). The collection centers are activated by binary decision variables, and lastly downstream processing facilities (recycling, chemical disposal, ETP). Wastewater is treated on-site with ETP and the other waste streams are routed externally.

3.5.2 Description of the Network Stages

The proposed five stages of the network are described below, with academic justification for each design choice.

Stage 1-Factory: The factory is the single source, generating all four waste streams during production.
Stage 2-Waste Segregation: Solid, yarn, and chemical wastes are required to be separated at the factory before dispatch, while wastewater is diverted to the ETP. Diabat, Kannan, and Mathiyazhagan (2014) identified that source segregation is a critical enabler for sustainable textile supply chains. A holding and preparation cost per kg is incurred at this stage.
Stage 3-Effluent Treatment Plant (ETP): The factory will have an on-site ETP that would treat wastewater using biological and physico-chemical methods. A portion of treated water can be recycled back to production, and the rest of the water will be safely discharged. Ranganathan et al. (2007) convey that 80–97% TDS removal and 91–97% COD removal via nanofiltration and reverse osmosis, which helps in reusing the water.
Stage 4-Collection Centers: Segregated solid, yarn, and chemical wastes are to be transported to one or more candidate collection centers (CCs). Activating a CC will need a fixed contract cost, which is implicated as the binary activation decision. Alshamsi and Diabat (2015) showed that selective CC activation in a multi-echelon MILP reduces total logistics cost by removing underutilized nodes.
Stage 5-Processing Facilities: Each active CC will transfer waste to appropriate downstream facilities such as, yarn waste to yarn recycling, solid waste to solid recycling, and chemical waste to chemical disposal or treatment. Each facility is characterized by its capacity, revenue or disposal fee, processing time, CO_2 emission rate, and a circularity rate.

3.5.3 Model Assumptions

The model is carried out by the following assumptions, consistent with RL network design (Govindan and Soleimani, 2017; Pishvaee et al., 2011):

1. A single factory generating three solid/semi-solid waste types and one wastewater stream per planning period (day).
2. Waste quantities are provided as inputs by the factory operator, or estimated from the ML predictions described in Section 3.5.
3. The model is being determined within a single planning period.
4. All generated waste must be fully allocated and no waste remains unprocessed at the factory.
5. A collection center induce its fixed contract cost if and only if waste is allocated to it.
6. Each processing facility at a CC has a maximum throughput capacity.
7. Transportation cost is linear in quantity and distance, with waste-type-specific rates (chemical waste carries a hazardous material which requires premium service).
8. CO_2 emissions arise from both transport and processing, and are penalized at a user-defined cost per kg CO_2-eq.
9. A circularity bonus is awarded for waste routed to facilities with higher circularity rates.
10. The ETP recycling rate is fixed, as determined by existing plant capacity.

3.5.4 Mathematical Formulation of the MILP Model

The optimization problem is formulated as a Mixed Integer Linear Programme (MILP). The model maximises a net financial value objective with capacity, allocation, and environmental constraints. A factory operator supplies their own operational parameters as inputs to generate a waste transportation plan. The full formulation is presented below in Table 1.

Sets and Indices

Table 1. Sets and Indices for MILP Model

Symbol	Definition
W = {solid, yarn, chemical}	Set of waste types generated by the factory
J = {1, 2, …, n}	Set of candidate collection centers
$w \in W$, $j \in J$	Index for waste type and collection center, respectively

Input Parameters

The following parameters are provided as inputs by the factory operator (Table 2).

Table 2. Input Parameters for MILP Model

Parameter	Description
$\mathbf{Q_w}$	Quantity of waste type w generated in the period (kg) (predicted waste quantity)
$\mathbf{d_j}$	Distance from factory to collection center j (km)
$\mathbf{F_j}$	Fixed contract cost of activating collection center j (BDT)
$\mathbf{h_w}$	Holding and preparation cost per kg of waste type w (BDT/kg)
$\mathbf{tc_w}$	Transportation cost per kg per km for waste type w (BDT/kg/km)
$\mathbf{CAP_j^w}$	Processing capacity for waste type w at collection center j (kg)
$\mathbf{rev_j^w}$	Revenue per kg of waste type w processed at center j (BDT/kg)
$\mathbf{disp_j^w}$	Disposal fee per kg of waste type w at center j (BDT/kg)

$\mathbf{co2_j^w}$	CO_2 emission coefficient for waste type w at center j (kg CO_2-eq/kg)
$\mathbf{circ_j^w}$	Circularity rate of facility for waste type w at center j (0–1)
$\mathbf{T_j^w}$	Processing time for waste type w at center j (days)
$\mathbf{wc_{rate}}$	Working capital opportunity cost rate (BDT/BDT/day)
$\mathbf{co2_{penalty}}$	Carbon penalty rate (BDT per kg CO_2-eq)
$\mathbf{circ_{bonus}}$	Circularity bonus rate (BDT per kg × circularity rate)
$\mathbf{r_{etp}}$	ETP water recycling rate (proportion 0–1)
$\mathbf{ww_{savings}}$	Value of each litre of water recycled by the ETP (BDT/litre)
$\mathbf{ww_{drain}}$	Disposal cost per litre of wastewater not recycled (BDT/litre)

Decision Variables

Two types of decision variables are defined:

$$x_j^w \geq 0$$

Quantity (kg) of waste type w allocated to collection center j (continuous).

The variable $x_j^w \geq 0$ represents the quantity (in kg) of waste type w ∈ W allocated to collection center j ∈ J within a single planning period. The predicted waste quantities are used here as Q_w as the input and x_j^w is derived from it for optimized quantity of waste distribution across the RL network.

$$y_j \in 0,1$$

Binary variable: 1 if collection center j is activated, 0 otherwise.

The binary variable $y_j \in \{0,1\}$ is an activation indicator for collection center j. This variable engages the fixed contract cost F_j as it should only be occurred when that center is actually used. It also enables the capacity constraint (Constraint 2) through the big-M formulation (Constraint 3), which prevents waste from being routed to an inactive center.

Objective Function

The model maximizes the total net value Z, defined as all revenues and bonuses less all costs and penalties:

$$Z = R - D - TC - HC - FC - WC - EP + CB + ETP$$

where each component is defined as follows:

(1) Revenue from recyclable waste:

$$R = \sum_{w \in W} \sum_{j \in J} rev_j^w x_j^w \qquad \textit{(Eq. 3.1)}$$

(2) Disposal fees for hazardous waste:

$$D = \sum_{w \in W} \sum_{j \in J} disp_j^w x_j^w \qquad \textit{(Eq. 3.2)}$$

(3) Transportation cost:

$$TC = \sum_{w \in W} \sum_{j \in J} tc_w d_j x_j^w \qquad \textit{(Eq. 3.3)}$$

(4) Holding and preparation cost at factory:

$$HC = \sum_{w \in W} \sum_{j \in J} h_w x_j^w \qquad \textit{(Eq. 3.4)}$$

(5) Fixed contract cost (incurred only for activated centres):

$$FC = \sum_{j \in J} F_j y_j$$

(Eq. 3.5)

(6) Working capital cost (opportunity cost of capital tied up during processing time):

$$WC = \sum_{w \in W} \sum_{j \in J} \left(disp_j^w + tc_w d_j\right) T_j^w \cdot wc_{rate} \cdot x_j^w$$

(Eq. 3.6)

(7) Environmental penalty (carbon cost):

$$EP = \sum_{w \in W} \sum_{j \in J} co2_j^w x_j^w \cdot co2_{penalty}$$

(Eq. 3.7)

(8) Circularity bonus (incentive for high-circularity routing):

$$CB = \sum_{w \in W} \sum_{j \in J} circ_j^w x_j^w \cdot circ_{bonus}$$

(Eq. 3.8)

(9) ETP net contribution (savings from recycled water, less drainage cost):

$$ETP = \left(r_{etp} \cdot WW \cdot ww_{savings}\right) - \left(\left(1 - r_{etp}\right) \cdot WW \cdot ww_{drain}\right)$$

(Eq. 3.9)

where WW is the total treated wastewater volume in litres.

The complete objective function combining all terms is:

$$\max Z = \sum_{w \in W} \sum_{j \in J} rev_j^w x_j^w - \sum_{w \in W} \sum_{j \in J} disp_j^w x_j^w - \sum_{w \in W} \sum_{j \in J} tc_w d_j x_j^w - \sum_{w \in W} \sum_{j \in J} h_w x_j^w - \sum_{j \in J} F_j y_j - \sum_{w \in W} \sum_{j \in J} \left(disp_j^w + tc_w d_j\right) T_j^w wc_{rate} x_j^w - \sum_{w \in W} \sum_{j \in J} co2_j^w x_j^w \cdot co2_{penalty} + \sum_{w \in W} \sum_{j \in J} circ_j^w x_j^w \cdot circ_{bonus} + ETP$$

(Eq. 3.10)

Constraints

The MILP is subject to the following five constraints:

Constraint 1-Waste Allocation: All generated waste must be fully routed to collection centers.

$$\sum_{j \in J} x_j^w = Q_w \qquad \forall w \in W$$

(C1)

Constraint 2-Facility Capacity: Waste allocated to a center cannot exceed its processing capacity.

$$x_j^w \leq CAP_j^w y_j \qquad \forall w \in W, j \in J$$

(C2)

Constraint 3-Activation Linking Constraint: Waste can only be sent to an activated center (big-M formulation).

$$x_j^w \leq M y_j \qquad \forall w \in W, j \in J$$

(C3)

where M is a sufficiently large constant (e.g., M = max(Q_w)).

Constraint 4-Non-Negativity:

$$x_j^w \geq 0 \qquad \forall w \in W, j \in J$$

(C4)

Constraint 5-Binary Activation:

$$y_j \in 0,1 \qquad \forall j \in J$$

(C5)

The MILP is solved using IBM ILOG CPLEX 22.1.1 with the DOcplex Python API, which applies branch-and-bound with LP relaxation, cutting planes, and pre-solve reductions. For this problem with three waste types and three candidate collection centers, CPLEX logically achieves a globally optimal integer solution within negligible computation time, as Alshamsi and Diabat (2015) reported sub-minute solve times for reverse logistics MILPs of comparable scale. All numerical parameters are factory-specific inputs, making the framework directly applicable to any thread dyeing or similar textile processing operation.

3.5.5 Key Performance Metrics

Besides the objective value Z, the model reports the following performance metrics relevant to circular economy and sustainability objectives (Table 3):

Table 3. Key Performance Metrics for MILP Model

Metric	Definition
Weighted Avg Circularity Rate	Proportion of all waste genuinely re-entering productive use (see Eq. below)
Total CO_2 Emissions (kg CO_2-eq)	Sum of emission coefficients × quantities across all routes
Number of Active Collection Centres	$\Sigma_j\ y_j$ network utilisation indicator
Total Waste Processed (kg)	$\Sigma_w\ Q_w$ confirms complete allocation
ETP Water Recycled (litres)	r_{etp} × WW volumetric water recovery

$$\overline{circ} = \frac{\sum_w \sum_j circ_j^w x_j^w}{\sum_w Q_w}$$

$$CO_2^{total} = \sum_{w \in W} \sum_{j \in J} co2_j^w x_j^w$$

The integration of ML-predicted waste quantities with this MILP constitutes a data-driven, closed-loop decision support system where predictive analytics feeds directly into operational optimization for both economic efficiency and circularity in the waste management of a thread dyeing company.

ML-predicted waste quantities replace static values, and incorporate the MILP to respond dynamically to actual production variability. This approach aligns with Alsabt et al. (2024), who demonstrated that ML-based waste forecasts can directly inform resource allocation and facility planning, and supports the broader circular economy objective of recovering maximum value from industrial waste while minimizing cost and carbon impact (Anandhabalaji 2025).

3.6 Integration of ML Predictions with Dynamic MILP Input Parameters

The parameter Q_w in the MILP formulation represents the daily quantity of waste type w generated at the factory. In a regular reverse logistics models, Q_w is treated as a static value derived from historical averages. But a static input does not visualize about daily production variability properly (Bhowmik et al., 2024).

Here, in our model Q_w is no longer a fixed parameter. Q_w is the estimated quantity of waste from the textile facility. From all the production variability the factory will be able to estimate the amount of waste for a particular day and it will be used as the input parameter in reverse logistics. The reverse logistics model will find optimized number of CC which are to be activated with optimized amount of waste according to that day.

It is important to note that Q_w of wastewater is used as an input to the ETP contribution and the other three waste types will be routed to collection centers as dynamic Q_w values each day. Also, the model is flexible around other variables as the factory will be able to change their number of collection centers, distances, capacities, and other factors as needed to find best optimized result.

4. Data Collection

As for a thread dyeing facility, here we have developed a synthetic dataset to demonstrate the parameters that would influence the waste generation most.

4.1 Data Scarcity in the Thread Dyeing Industry

A fundamental challenge encountered at the outset of this research was the near-total absence of publicly available, operationally granular waste data from thread dyeing facilities. Uddin et al. (2023) explicitly note that there is a scarcity of information on water, dye, and chemical consumption, as well as wastewater generation, in the textile sector. Similarly, Sristi and Hasin (2025) observed that little research has been done on effective water management at the process level in thread dyeing operations. In the absence of a published, facility-level dataset, direct access to operational records was not feasible due to commercial confidentiality constraints common in the industry. A synthetic dataset approach was therefore adopted, in which parameter ranges for each waste stream and operational variable were derived from quantitative values reported in published textile processing studies — consistent with practices reported in modeling-oriented studies where real facility data is unavailable (Muthu 2017).

One of the primary challenges for this research is that there is almost no data available at operation level of waste data from thread dyeing facilities. Uddin et al. (2023) expressed that there is a scarcity of information on water, dye, and chemical consumption, along with wastewater generation, in the textile sector. Also, Sristi and Hasin (2025) observed that little research has been done on effective water management at the process level in thread dyeing operations. Also, facility dataset or direct access to operational records is not feasible due to commercial confidentiality problems, which are very common in the industry.

4.2 Dataset Construction Methodology

A synthetic dataset is constructed that covers a two-year daily operational period (January 2023 – December 2024) and includes 731 observations. For each calendar day, it has values for production volume, waste generation across four streams, several process parameters, and environmental factors were generated using literature-calibrated uniform random distributions, along with a seasonal multiplier applied to production volume to reflect demand-driven variability in the apparel supply chain.

4.3 Waste Stream Parameter Ranges

Production volume was set within the range of 1,500 to 22,000 kg per day, which is based on the case study by Sristi and Hasin (2025), which documented a design dyeing floor capacity of approximately 22 tons per day; the lower bound of the range reflects small-to-medium batch operations documented in Bangladesh dyeing industry surveys (Uddin et al., 2023). The solid waste range is at 10–20% of daily yarn input, based on Islam (2024), who documented that approximately 18.8 kg of material waste is generated per 100 kg of dyed fabric. Yarn waste was set at 5-8% of daily yarn input based on Akter et al. (2022), who reported yarn-related losses are about 4-7% of total yarn input. The chemical waste range was fixed at 130-415 kg per ton of production, derived from Ranganathan et al. (2007) and the extensive field survey by Uddin et al. (2023) reporting average total chemical use of 449 g/kg (range: 152-705 g/kg). Wastewater generation ranges from 30-50 liters per kilogram of yarn processed, consistent with Saravanan (2025) and Muthu (2017). Energy consumption ranges from 1.6-2.2 kWh per kilogram of dyed yarn based on Palamutcu (2010) and Kant (2012).

4.4 Seasonal Production Variation

Monthly scaling coefficients were also applied to daily production volume to create a realistic dataset that reflects demand-driven variability. Kalaoglu et al. (2015) analyzed historical apparel retail demand data and identified a clear seasonal pattern, which generally shows that demand peaks in early summer (around June) and reaches its lowest point in November. Months with moderate demand were assigned factors close to the baseline (0.90 to 1.00), rising-demand months were scaled upward (1.05 to 1.15), peak summer months were assigned factors of 1.25 to 1.35, and the November trough was assigned a factor of 0.80 (Table 4).

Table 4. Summary of synthetic dataset parameters and their literature sources

Variable	Unit	Range / Values	Source / Basis
Production Volume	kg/day	1,500 – 22,000	Sristi and Hasin (2025); Sivakumar et al. (2013)
Solid Waste	% of input	10 – 20%	Islam (2024); Ashfaq and Khatoon (2014)
Yarn Waste	% of yarn input	5 – 8%	Akter et al. (2022); Sristi and Hasin (2025)
Chemical Waste	kg/ton	130 – 415	Ranganathan et al. (2007); Uddin et al. (2023)
Wastewater Generation	L/kg yarn	30 – 50	Saravanan (2025); Muthu (2017)
Energy Consumption	kWh/kg	1.6 – 2.2	Palamutcu (2010); Kant (2012)
Dye Batch Count	batches/day	200 – 350	Shekh (2024); Sristi and Hasin (2025)
Cycle Time	hours/batch	2.5 – 5.0	Dev. Commissioner MSME (n.d.); Gulrajani (2010)
Working Temperature	°C	60 – 130	Broadbent (2001)
Water Consumption	L/kg	30 – 65	Uddin et al. (2023); Coats Group (2020)
Chemical Consumption	g/kg textile	152 – 705	Uddin et al. (2023)
Number of Workers	employees	25 – 150	Small Industries Service Institute (2003)
Seasonal Factor	monthly multiplier	0.80 – 1.35	Kalaoglu et al. (2015)

4.5 MILP Reverse Logistics Network Parameters

The MILP optimization model requires a second data layer describing the reverse logistics network and the cost and revenue attached to the network. These parameters describe the cost structures of the assumed network of thread dyeing factories operating within industrial clusters. Dyeing factories can use their values to get the total optimized pathway and achieve maximum profit. The reverse logistics model will be able to help the factories according to their data and their waste proportion (Table 5).

Table 5. MILP reverse logistics network parameters

Parameter	Unit	Notes
Holding cost – solid waste	price/kg	Includes storage and handling
Holding cost – yarn waste	price/kg	Includes storage and handling
Holding cost – chemical waste	price/kg	Hazardous material premium
Transport cost – solid	price/kg/km	Standard road freight
Transport cost – yarn	price/kg/km	Standard road freight
Transport cost – chemical	price/kg/km	Hazmat surcharge applied
ETP in-factory recycling rate	%	Treated wastewater reused
CO2 emission penalty	price/kg CO2-eq	Carbon cost per unit emission
Circularity bonus	price/kg	ESG incentive per unit recycled
Collection Centre 1	km / price	Fixed contract cost
Collection Centre 2	km / price	Fixed contract cost
Collection Centre 3	km / price	Fixed contract cost

4.6 Dataset Validation and Limitations

The parameter ranges adopted in this study are entirely peer-reviewed and verified by grey literature sources. No values were assumed without any reference support. The main limitation of the dataset is that it is synthetically generated rather than collected from a specific operating facility. While every parameter range is tied to published research and reports, the joint distribution of variables in a real factory may show correlations that are not fully covered by independent random draws. Future work should validate the model against actual factory records with verified data so that it may result in a far better possible way. However, the synthetic approach is appropriate for demonstrating the methodology and for training the ML waste prediction models. Because it ensures sufficient data volume, covers the full operational range of thread dyeing facilities, and avoids the missing data and confidentiality issues in this sector.

5. Results and Discussion

This section presents the extensive results of the predictive waste classification model and the reverse logistics optimization framework. The results are organized into four sections: numerical results, graphical results, proposed improvements, and validation through statistical hypothesis testing. Together, these results demonstrate the effectiveness of integrating machine learning (ML) with Mixed-Integer Linear Programming (MILP) in achieving the objectives.

5.1 Numerical Results

Numerical results of the ML model and MILP optimization values show how the framework is applicable to the industry level, and how a thread dyeing facilities will be able to utilize them.

5.1.1 Dataset Overview

The dataset comprises 731 daily operational records from January 2023 to December 2024, with 39 features such as production parameters, chemical usage, energy consumption, and four target waste variables. No missing values were detected across any of the 731 data points, which confirms complete data integrity. Table 6 presents the descriptive statistics of the four waste target variables.

Table 6. Descriptive statistics of waste target variables (N = 731)

Waste Variable	Mean	Min	25th Pct.	Median	Max
Solid_Waste_kg	2,340.22	758.70	1,754.22	2,271.00	4,148.60
Yarn_Waste_kg	1,020.91	286.62	785.68	1,001.05	1,673.20
Chemical_Waste_kg	4,177.94	941.47	2,874.37	3,711.15	8,876.35
Wastewater_L	629,031.77	199,130.40	481,886.50	615,274.00	1,075,032.00

As shown in Table 3, Chemical_Waste_kg has the highest variability with values ranging from 941.47 kg to 8,876.35 kg, indicating that chemical waste generation is the most operationally fluctuating variable among the four targets. Wastewater_L operates at a significantly different magnitude (litres), with a mean of 629,031.77 L per day. Solid waste and yarn waste show relatively moderate ranges, with means of 2,340.22 kg and 1,020.91 kg, respectively, and both display right-skewed distributions evidenced by means exceeding their medians.

5.1.2 ML Model Performance Metrics (Tuned Random Forest)

The hyperparameter-tuned Random Forest model was calculated on the 2024 test set (366 days) for all four waste targets. GridSearchCV with 5-fold cross-validation was used to find out the optimal configuration: n_estimators = 600, max_depth = 15, max_features = 'log2', min_samples_split = 2, min_samples_leaf = 2. Table 7 presents the full performance metrics for both baseline and tuned models.

Table 7. Model performance metrics, baseline vs. tuned Random Forest (test set 2024)

Target Variable	**MAE**	**RMSE**	**R^2**	**MAPE (%)**	**Interpretation**
Solid_Waste_kg	404.37	483.53	0.5731	18.29%	Moderate fit; high daily variability
Yarn_Waste_kg	122.34	145.70	0.7635	12.67%	Good fit; strong production correlation
Chemical_Waste_kg	1,180.98	1,418.79	0.2732	33.47%	Weak fit; high process stochasticity
Wastewater_L	78,859.46	96,442.72	0.7419	12.90%	Good fit; consistent volume pattern

The results in Table 4 reveal distinct performance tiers across waste types. Yarn_Waste_kg achieved the highest R^2 of 0.7635 with the lowest MAPE of 12.67%. Indicating that yarn waste generation is most strongly driven by measurable production inputs. Wastewater_L followed closely with $R^2 = 0.7419$ and MAPE = 12.90%, shows how it is directly proportional to total water consumption and batch processing. Solid_Waste_kg yielded a moderate R^2 of 0.5731 and MAPE of 18.29%, showing that daily operational variability is not fully captured by static features. Chemical_Waste_kg presented the weakest fit ($R^2 = 0.2732$, MAPE = 33.47%), attributable to the highly stochastic nature of chemical dosing decisions that depend on yarn color specifications and batch-specific fixation conditions.

5.1.3 Baseline versus Tuned Model Comparison

Table 8 presents the performance delta between the default baseline Random Forest and the GridSearchCV-tuned model. Hyperparameter tuning resulted in consistent improvements across all four targets.

Table 8. Baseline vs. tuned Random Forest — performance comparison

Target	**Baseline R^2**	**Tuned R^2**	**ΔR^2**	**Baseline MAE**	**Tuned MAE**	**ΔMAE**
Solid_Waste_kg	0.5697	0.5731	+0.0034	403.12	404.37	−1.25
Yarn_Waste_kg	0.7606	0.7635	+0.0029	122.90	122.34	−0.56
Chemical_Waste_kg	0.2293	0.2732	+0.0439	1,201.28	1,180.98	−20.30
Wastewater_L	0.7287	0.7419	+0.0132	80,728.91	78,859.46	−1,869.45

The most notable improvement from tuning was observed for Chemical_Waste_kg. R^2 of Chemical_Waste_kg improved by +0.0439, and MAE was reduced by 20.30 kg. This shows that deeper trees (max_depth = 15) and log2 feature sampling helped the model capture nonlinear chemical dosing interactions more effectively. Wastewater_L also benefited substantially, with a MAE reduction of 1,869.45 L per day.

5.1.4 Cross-Validation Performance on Training Data

To assess model stability and generalizability, 5-fold cross-validation was performed on the 2023 training set (365 observations) using the tuned hyperparameters. Table 9 reports the mean and standard deviation of cross-validated R^2 scores across the five folds.

Table 9. 5-fold cross-validation R^2 scores (training data 2023)

Target Variable	**Mean CV R^2**	**Std. Dev. (±)**	**Min Fold R^2**	**Max Fold R^2**
Solid_Waste_kg	0.5935	±0.0527	~0.50	~0.66
Yarn_Waste_kg	0.7210	±0.0416	~0.65	~0.78
Chemical_Waste_kg	0.2711	±0.1045	~0.10	~0.40
Wastewater_L	0.7240	±0.0439	~0.67	~0.80

The cross-validation results confirm that the model maintains consistent performance in the training period. Yarn_Waste_kg and Wastewater_L both achieved mean CV R^2 values exceeding 0.72, with low standard deviations, which indicates their stable fold-to-fold performance. Chemical_Waste_kg shows a higher variance (±0.1045), with its wide fold range (0.10 to 0.40), which confirms the inherently unpredictable nature of chemical waste generation in daily production variation.

5.1.5 Feature Importance Ranking

Table 10 presents the aggregated mean feature importance scores for the four waste targets, derived from the Random Forest's internal Gini impurity reduction, and corroborated by permutation importance analysis.

Table 10. Top 10 aggregated feature importance scores (mean across 4 targets)

Rank	Feature	Mean Importance Score	Interpretation
1	Production_Volume_kg	0.1729	Primary driver of all waste types
2	Number_of_Workers	0.1278	Labour-intensity proxy for batch operations
3	Total_Energy_kWh	0.1213	Energy use correlates with dyeing intensity
4	Acid_kg	0.0926	Direct chemical contributor to chemical waste
5	Dye_Batch_Count	0.0765	Batch frequency drives multi-waste generation
6	Salt_kg	0.0572	Salt consumption links to wastewater salinity
7	Total_Water_Consumption_L	0.0500	Direct antecedent of wastewater volume
8	Auxiliary_kg	0.0421	Auxiliary chemicals affect sludge composition
9	Alkali_kg	0.0353	Alkaline chemicals contribute to solid sludge
10	Dye_Used_kg	0.0292	Dye fixation rate affects residual chemical load

Production_Volume_kg comes out as the dominant predictor with a mean importance score of 0.1729, followed closely by Number_of_Workers (0.1278) and Total_Energy_kWh (0.1213). These three features are collectively responsible for about 42.2% of the model's total predictive power. It outlines that the scale and intensity of production operations are the primary determinants of waste generation across all four waste streams.

5.1.6 Scalability Analysis of the ML Prediction Framework

To assess the scalability of the trained Random Forest model, three additional prediction scenarios were evaluated. Using the same model trained exclusively on 2023 operational data (365 records, 23 features), the tuned configuration was applied to a 2-year window (2024–2025, N = 731), a 3-year window (2024–2026, N = 1,096), and a 4-year window (2024–2027, N = 1,461). This evaluation shows if the predictive characteristics of the model with future periods can be implemented without retraining.

5.1.7 Scalability Test Results

Table 11 presents the aggregated performance metrics for all four scalability scenarios across the four waste targets. The baseline 1-year results are reproduced for direct comparison.

Table 11. Scalability test results — Random Forest performance across 1-year, 2-year, 3-year, and 4-year prediction horizons (Train: 2023)

Scalability Scenario	Test Period	Test N	Waste Target	R²	MAPE (%)	MAE	RMSE
1-Year (Baseline)	2024	366	Solid Waste (kg)	0.5731	18.29%	404.37	483.53
			Yarn Waste (kg)	0.7635	12.67%	122.34	145.70
			Chemical Waste (kg)	0.2732	33.47%	1180.98	1418.79

Scalability Scenario	**Test Period**	**Test N**	**Waste Target**	**R²**	**MAPE (%)**	**MAE**	**RMSE**
			Wastewater (L)	0.7419	12.90%	78,859	96,443
2-Year	2024–2025	731	Solid Waste (kg)	0.5279	16.40%	390.03	482.67
			Yarn Waste (kg)	0.7086	11.87%	123.28	151.14
			Chemical Waste (kg)	0.2785	28.76%	1107.70	1340.54
			Wastewater (L)	0.7135	11.52%	74,433	92,654
3-Year	2024–2026	1096	Solid Waste (kg)	0.5062	15.78%	386.44	481.70
			Yarn Waste (kg)	0.6815	11.60%	123.56	151.61
			Chemical Waste (kg)	0.2992	26.73%	1059.21	1285.51
			Wastewater (L)	0.6806	11.44%	75,911	95,173
4-Year	2024–2027	1461	Solid Waste (kg)	0.4886	15.60%	388.89	486.00
			Yarn Waste (kg)	0.6704	11.19%	121.37	150.52
			Chemical Waste (kg)	0.2866	25.72%	1053.13	1287.98
			Wastewater (L)	0.6617	11.35%	76,550	96,576

The results in Table 11 reveal a consistent pattern across all four waste streams as the prediction horizon extends. Yarn Waste and Wastewater, the two highest-performing targets in the baseline study, exhibit only moderate performance degradation over four years. Yarn Waste R^2 declines from 0.7635 (1-year) to 0.6704 (4-year), while Wastewater R^2 decreases from 0.7419 to 0.6617. Notably, both targets maintain MAPE values below 12% across all scalability windows, confirming continued deployment-readiness beyond the original test year.

Solid Waste R^2 shows a more pronounced decline from 0.5731 to 0.4886 over four years, though MAPE actually improves slightly from 18.29% to 15.60%. This counterintuitive result — lower R^2 but better MAPE — reflects the model's improved handling of proportional errors at moderate waste volumes, while its ability to explain absolute variance at extreme output levels weakens over time.

Chemical Waste remains the weakest target across all scenarios (R^2 range: 0.2732–0.2992), consistent with the baseline findings. However, its MAPE improves progressively from 33.47% (1-year) to 25.72% (4-year), suggesting that the model's central tendency for chemical waste estimation remains useful even as R^2 indicates limited variance explanation. This improvement in MAPE over longer horizons may reflect the averaging effect of a larger, more diverse test set reducing the influence of extreme chemical dosing events.

5.1.8 Per-Year Performance Breakdown

Table 12 disaggregates model performance by individual calendar year across the 4-year test window. This breakdown allows direct inspection of how predictive accuracy evolves as the forecast horizon advances further from the 2023 training period.

Table 12. Per-year model performance breakdown across 2024–2027 (Train: 2023)

Year (n)	**Waste Target**	**R²**	**MAPE (%)**	**MAE**	**RMSE**
2024 (n=366)	Solid Waste (kg)	0.5731	18.29%	404.37	483.53
	Yarn Waste (kg)	0.7635	12.67%	122.34	145.70
	Chemical Waste (kg)	0.2732	33.47%	1180.98	1418.79
	Wastewater (L)	0.7419	12.90%	78,859	96,443
2025 (n=365)	Solid Waste (kg)	0.4539	14.51%	375.66	481.82
	Yarn Waste (kg)	0.6196	11.06%	124.22	156.41
	Chemical Waste (kg)	0.2504	24.05%	1034.22	1257.20
	Wastewater (L)	0.6594	10.14%	69,994	88,692
2026 (n=365)	Solid Waste (kg)	0.4490	14.53%	379.24	479.73
	Yarn Waste (kg)	0.6039	11.07%	124.11	152.53
	Chemical Waste (kg)	0.3427	22.67%	962.08	1167.51
	Wastewater (L)	0.5922	11.27%	78,871	100,029
2027 (n=365)	Solid Waste (kg)	0.4240	15.05%	396.24	498.71
	Yarn Waste (kg)	0.6201	9.94%	114.82	147.23
	Chemical Waste (kg)	0.2229	22.66%	1034.90	1295.36
	Wastewater (L)	0.5925	11.07%	78,469	100,671

The per-year analysis (Table 12) shows the largest performance decline occurs between 2024 and 2025. Yarn Waste R^2 drops from 0.7635 to 0.6196, then stabilizes around 0.6201 by 2027. Wastewater follows a similar trend, decreasing from 0.7419 to 0.5922. Chemical Waste exhibits non-monotonic behavior, improving in 2026 ($R^2 = 0.3427$) before declining in 2027 (0.2229). This indicates the high process variability and sensitivity to unobserved batch-level factors.

MAPE remains within acceptable limits. It remains below 13% for Yarn Waste, 12% for Wastewater, and 15–16% for Solid Waste, all meeting the 20% deployment threshold. This confirms the operational stability across the 4-year horizon.

The results suggest that the ML framework maintains accuracy without retraining. But annual retraining is recommended, particularly for Chemical Waste. Overall, the framework validates its scalability for circular supply chain applications.

5.1.9 Reverse Logistics MILP Optimization Results

The MILP optimization model was formulated and solved using IBM CPLEX (version 22.1.1.0) via the DOcplex Python API. The model allocates predicted daily waste volumes across the collection centers. To show how the model works, let's assume some values of the parameters for the MILP Optimization result. Table 13 presents the assumed parameter values in the context of Bangladesh.

Table 13. Assumed parameter values for MILP results

Parameter	Value / Range
Holding cost – solid waste	2.50
Holding cost – yarn waste	3.00
Holding cost – chemical waste	4.00
Transport cost – solid	0.05
Transport cost – yarn	0.04
Transport cost – chemical	0.08
ETP in-factory recycling rate	75%
CO2 emission penalty	1.50
Circularity bonus	2.00
Collection Centre 1	45 km | BDT 5,000
Collection Centre 2	30 km | BDT 4,000
Collection Centre 3	20 km | BDT 3,000

These parameters, along with the predicted daily waste amount, find the optimized amount to circulate to which Collection Centers, comparing quantity, revenue, disposal fee, transportation costs, CO_2 emissions, etc. Table 14 presents the detailed allocation results for the representative date 2024-01-01 with the conditions of the parameters.

Table 14. MILP waste allocation results — 2024-01-01 (IBM CPLEX optimal solution)

Waste	CC Name	Qty (kg)	Revenue (BDT)	Disposal (BDT)	Transport (BDT)	Net (BDT)
Solid	Collection Centre 1	1,665.87	13,326.95	0.00	3,748.21	6,219.02
Yarn	Collection Centre 1	722.36	8,668.35	0.00	1,300.25	5,946.49
Chemical	Collection Centre 1	1,202.32	0.00	6,011.61	4,328.36	−18,293.32
Chemical	Collection Centre 3	2,000.00	0.00	9,000.00	3,200.00	−24,621.60

The MILP solver identified the optimal allocation of 5,590.55 kg total waste on 2024-01-01, activating two of three available collection centers: Collection Centre 1 and Collection Centre 3. The optimal objective value Z = −38,128.48 BDT represents the net system cost, where the negative sign reflects net expenditure after all revenues and bonuses are accounted for.

Table 15. Financial summary of MILP optimal solution (2024-01-01)

Cost / Revenue Component	Amount (BDT)
Revenue (Solid + Yarn Waste)	+21,995.31
Disposal Fees (Chemical Waste)	−15,011.61
Transportation Costs	−12,576.82
Holding & Preparation Costs	−19,141.05
Fixed Contract Costs (2 active CCs)	−8,000.00
Working Capital Costs (time-delay)	−80.62
Environmental Penalties (CO2)	−11,547.78

Circularity Bonuses	+5,613.16
ETP Net Contribution (Wastewater Recycling)	+620.93
OPTIMAL OBJECTIVE VALUE Z (MILP)	−38,128.48

Table 15 shows that chemical waste disposal (−15,011.61 BDT) and holding costs (−19,141.05 BDT) are the two largest cost drivers. Revenue from solid and yarn waste recycling (+21,995.31 BDT) helps in saving these costs, but environmental penalties for CO2 emissions (−11,547.78 BDT) represent a significant portion of total expenditure too. The weighted average circularity rate achieved was 50.20%.

5.1.10 Batch MILP Results - January 2024 (7-Day Window)

The MILP model was run in batch mode for the first seven days of January 2024 to demonstrate the temporal consistency of the optimization framework. Table 16 demonstrates the daily optimal Z values and the cost components.

Table 16. Batch MILP optimization results — 2024-01-01 to 2024-01-07

Date	**Z Optimal (BDT)**	**Solid Pred (kg)**	**Yarn Pred (kg)**	**Chem. Pred (kg)**	**Revenue (BDT)**	**Active CCs**
2024-01-01	−38,128.48	1,665.87	722.36	3,202.32	21,995.31	2
2024-01-02	−45,093.74	2,437.30	1,016.46	4,002.67	31,007.04	2
2024-01-03	−33,391.35	1,565.17	687.38	2,842.28	20,769.93	2
2024-01-04	−39,299.77	1,714.40	765.14	3,317.19	22,896.80	2
2024-01-05	−46,626.57	2,174.64	909.37	3,989.55	28,047.57	2
2024-01-06	−43,455.58	1,978.34	864.51	3,711.38	26,200.84	2
2024-01-07	−44,167.23	2,375.90	1,000.33	3,922.46	30,446.61	2

Across the 7-day batch outputs, we can see that the optimal Z values range from −33,391.35 BDT to −46,626.57 BDT, while activating two collection centers of the assumed three centers. This confirms that the MILP model reliably identifies the cost-optimal network configuration under daily waste variations. Revenue changes proportionally with solid and yarn waste volumes, while disposal costs for chemical waste remain the dominant expense across all days.

5.2 Graphical Results

This section represents and describes the graphical outputs of the ML model. Figures and graphs are being discussed sequentially. This section covers data exploration, model performance, feature analysis, and the temporal evaluation.

5.2.1 Distribution of Waste Target Variables

Figure 2 describes histograms and boxplots for the four waste targets across the 731-day dataset. Solid_Waste_kg approximates a normal distribution centered at 2,000–2,500 kg with a slight right skew. Yarn_Waste_kg is multi-modal which reflects variable batch sizes. Chemical_Waste_kg shows the most right-skewed distribution, with one end extending to ~9,000 kg, while Wastewater_L is broadly bell-shaped between 0.4 and 0.8 million litres. Boxplots confirm Chemical_Waste_kg as the only target with notable upper outliers, which demonstrates its high operational volatility.

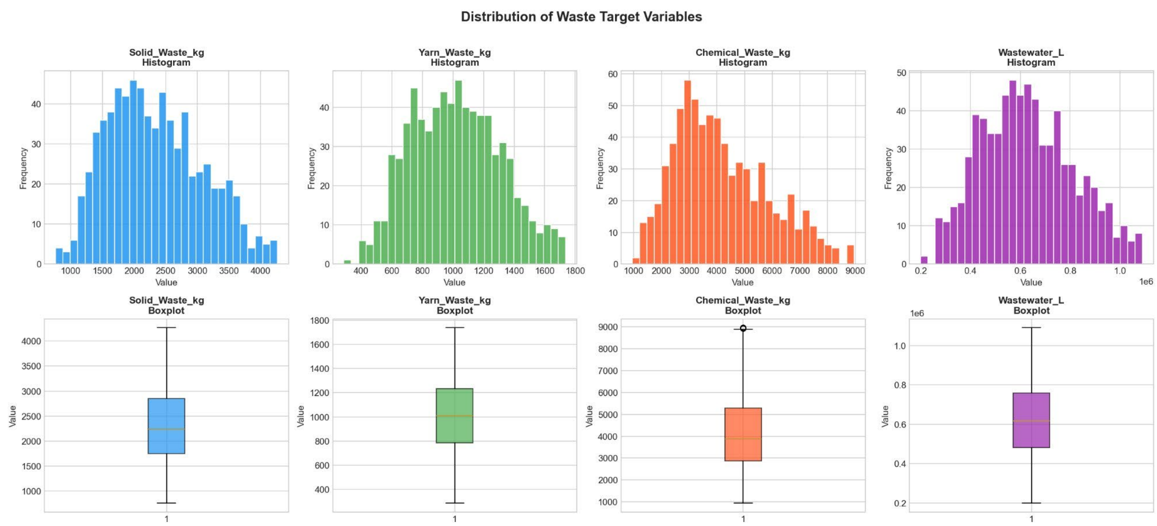

Figure 2. Distribution of waste target variables for histograms and boxplots (N = 731 days)

5.2.2 Waste Target Variables Over Time

Figure 3 shows the temporal evolution of all four waste streams over the two years, with the train/test split marked at January 1, 2024. Solid waste, yarn waste, and wastewater show regular stationarity throughout for both years, which indicates stable year-round operations. Chemical_Waste_kg shows higher amplitude variability, particularly during mid-2023 and mid-2024, as in these periods production also increases. The distributional similarity between the 2023 training and 2024 test periods confirms the validity of the temporal split and the absence of distributional shift.

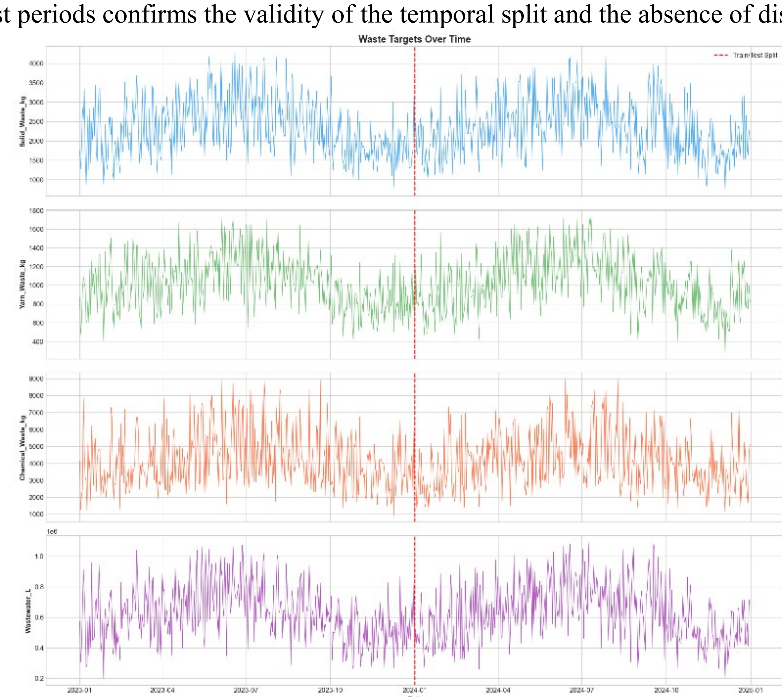

Figure 3. Time series of waste target variables for training (2023) and test (2024) periods with train/test split

5.2.3 Feature Correlation Analysis

Figure 4 presents the Pearson correlation heatmap across all 39 dataset features. Key observations include: (i) Production_Volume_ton and Production_Volume_kg are near-perfectly correlated (≈0.99) and both represent the same variable in different units only, thus Production_Volume_kg was retained; (ii) chemical inputs (salt, alkali, acid, auxiliary) form a positive correlation group with production volume; (iii) Wastewater_L and Total_Water_Consumption_L are highly correlated (≈0.85), both represent the same variable in different units, thus Wastewater_L was used; and (iv) waste targets share moderate inter-target correlations (r = 0.3–0.7), which justifies the need of separate models per stream. Figure 5 further confirms production volume as the dominant linear predictor for solid waste, yarn waste, and wastewater (r ≈ 0.79–0.86), while Chemical_Waste_kg is best predicted by a derived intensity ratio (r ≈ 0.77).

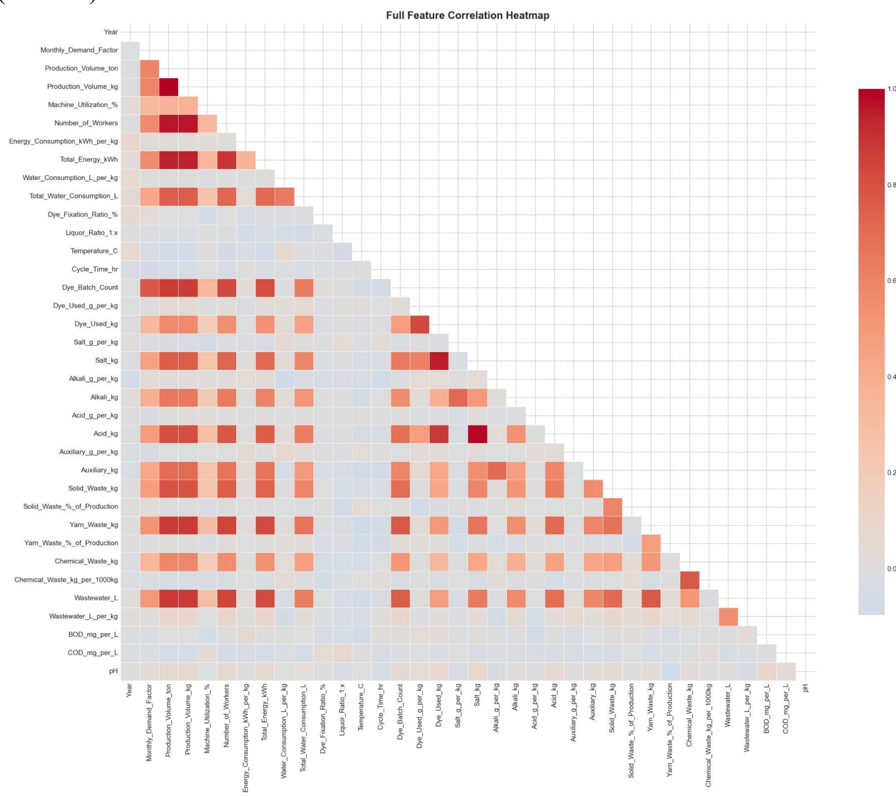

Figure 4. Full feature correlation heatmap (Pearson correlations, lower triangle) for all 39 variables

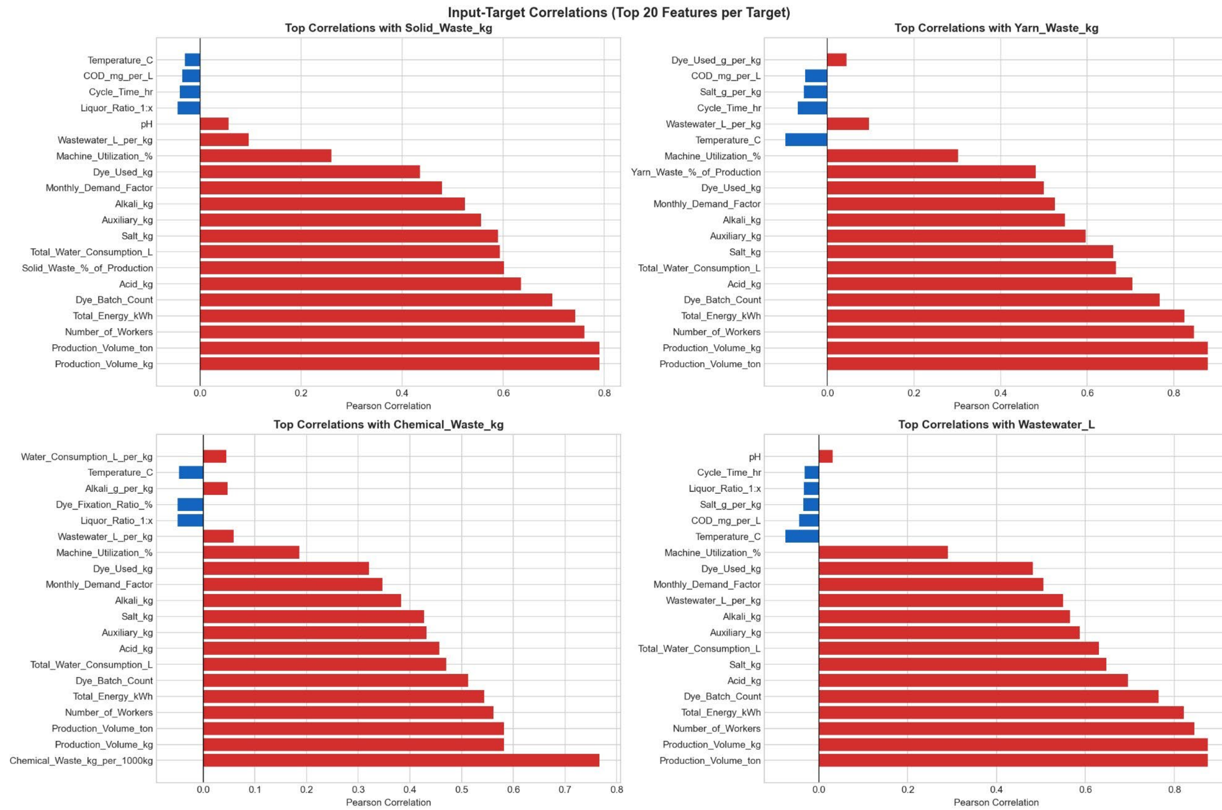

Figure 5. Input-target Pearson correlations for top 20 features per waste target

5.2.4 Model Performance: Baseline vs. Tuned Random Forest

Figure 6 compares baseline and tuned Random Forest performance via grouped bar charts for MAE, RMSE, and R^2 for the four targets. The tuned model consistently achieves higher R^2 and lower or comparable error values. The most gain has been achieved for Chemical_Waste_kg, where R^2 improves from 0.2293 to 0.2732 (18.7% relative improvement), for deeper trees (max_depth = 15) and log2 feature, which samples better in capturing nonlinear chemical dosing interactions. The Wastewater_L dominates in bar visualization in the error-scale panels due to its litre-scale magnitudes, but R^2 consistently improvement confirms the tuned model's superiority in all targets.

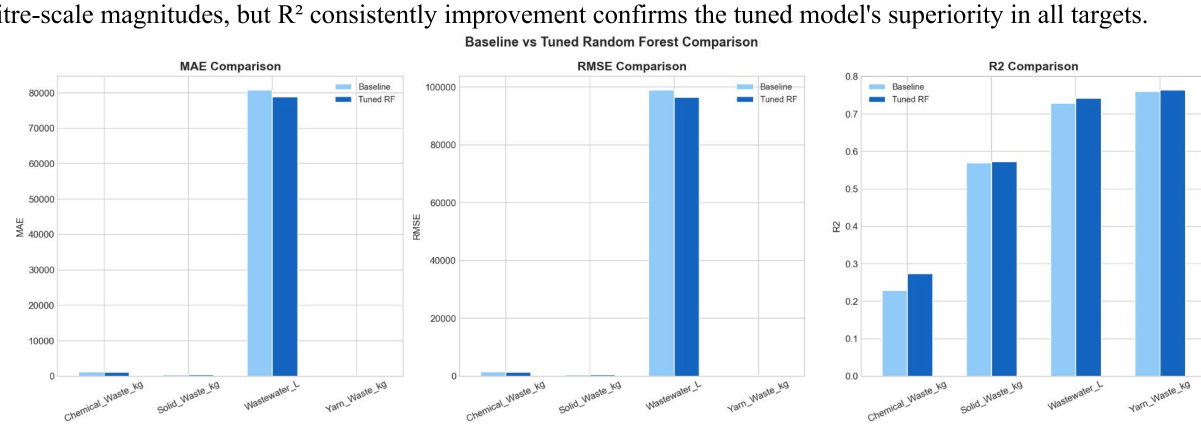

Figure 6. Baseline vs. tuned Random Forest for MAE, RMSE, and R^2 comparison (test set 2024)

5.2.5 Actual vs. Predicted Performance

Figures 7 and 8 together demonstrate prediction accuracy spatially (scatter plots) and temporally (time series). In Figure 7, Yarn_Waste_kg (R^2 = 0.7635) and Wastewater_L (R^2 = 0.7419) show tight, linear point clouds closely aligned with the perfect-fit line (y = x). Solid_Waste_kg (R^2 = 0.5731) shows a fan-shaped spread above 2,500 kg, which indicates the systematic under-prediction of high-output days. Chemical_Waste_kg (R^2 = 0.2732) displays a flat, diffuse cloud with central tendency. Figure 8 confirms these findings temporally, as the model assuredly tracks directional movements for yarn waste and wastewater but smooths in chemical waste spike events, particularly during Q2 and Q3 2024.

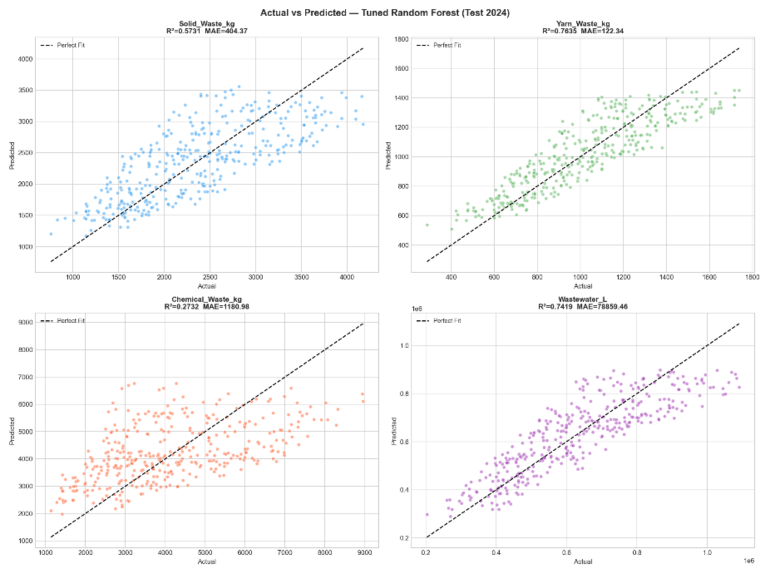

Figure 7. Actual vs. predicted scatter plots for tuned Random Forest (test set 2024); dashed line = perfect fit

Figure 8. Time series of actual (grey solid) vs. predicted (colored dashed) waste for the test period January–December 2024

5.2.6 Residual Analysis

Figure 9 depicts residual vs. predicted plots (top row) and residual distribution histograms (bottom row) for all four targets. Yarn_Waste_kg and Wastewater_L residuals are randomly scattered around zero with near-normal distributions, consistent with well-specified models. Solid_Waste_kg residuals show a slight positive bias at lower predicted values. Chemical_Waste_kg show the widest residual spread (±3,000–4,000 kg) and a slight right-skewed distribution, which confirms underestimation of peak events with unmeasured dye recipe complexity and unexplained variance in batch-specific color depth.

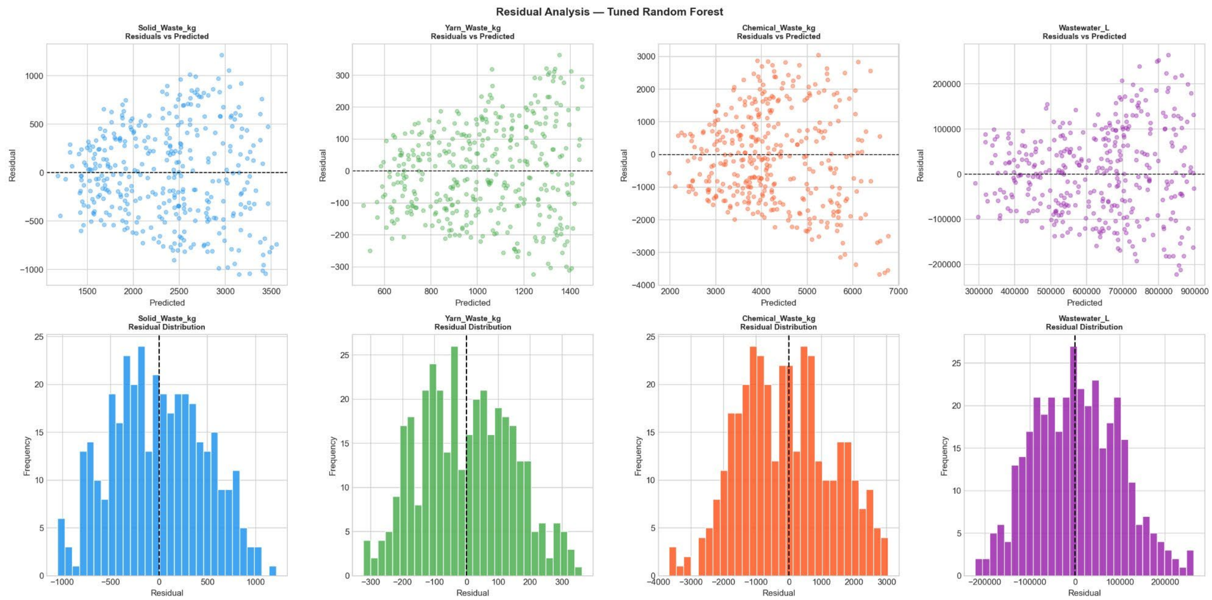

Figure 9. Residual analysis: residuals vs. predicted (top row) and residual distributions (bottom row) for all four targets

5.2.7 Feature Importance Analysis

Figures 10, 11, and 12 describe feature importance from three perspectives. Figure 10 reveals a consistent pattern from the top 15 feature importances per waste target for Random Forest. Importance scores are based on Gini impurity reduction. The mean decrease in node can be interpreted by the proportion of samples reaching each node across all trees. Higher values mean better predictive value to waste quantity estimation. Production_Volume_kg, Number_of_Workers, and Total_Energy_kWh rank among the top three predictors for all targets, which details the scale-driven nature of waste generation. Target-specific variations are found for Chemical_Waste_kg, Total_Water_Consumption_L ranks fifth. For Wastewater_L, Acid_kg and Dye_Batch_Count are the dominant paramenters. Figure 11 shows the aggregated mean Gini impurity-based feature importance scores for all four waste targets (Solid, Yarn, Chemical, Wastewater). Scores are normalized here. A higher bar means more average contribution across the entire multi-output Random Forest model. Here it shows that Production_Volume_kg (0.1729), Number_of_Workers (0.1278), and Total_Energy_kWh (0.1213) altogether account for 42.2% of total predictive power. Figure 12 has been plotted on the basis of permutation feature importance for the tuned Random Forest (mean for the 4 waste targets, n_repeats = 10). Permutation importance is calculated by randomly changing each feature's values on the test set and measuring the drop in R^2 score. A larger drop means higher feature relevance. This method is model-agnostic and protects from the biasness of Gini scores for high-cardinality features. Figure 12 ultimately shows that Production_Volume_kg (≈0.117) still remains dominant, while Dye_Batch_Count (≈0.018) and Acid_kg (≈0.013) rank higher under permutation than Gini scores. This suggests unique predictive contributions of this research. Features such as Alkali_kg, pH, and Liquor_Ratio show almost zero permutation scores.

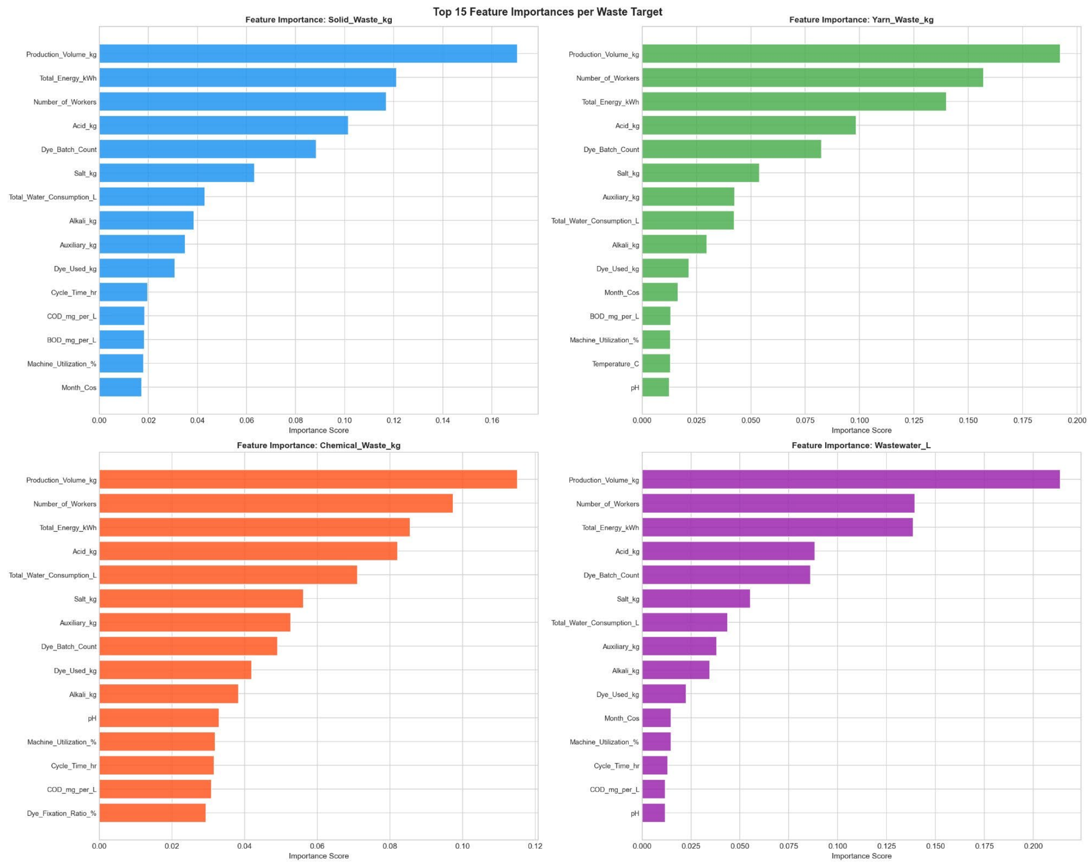

Figure 10. Top 15 feature importances per waste target for Random Forest (Gini impurity reduction)

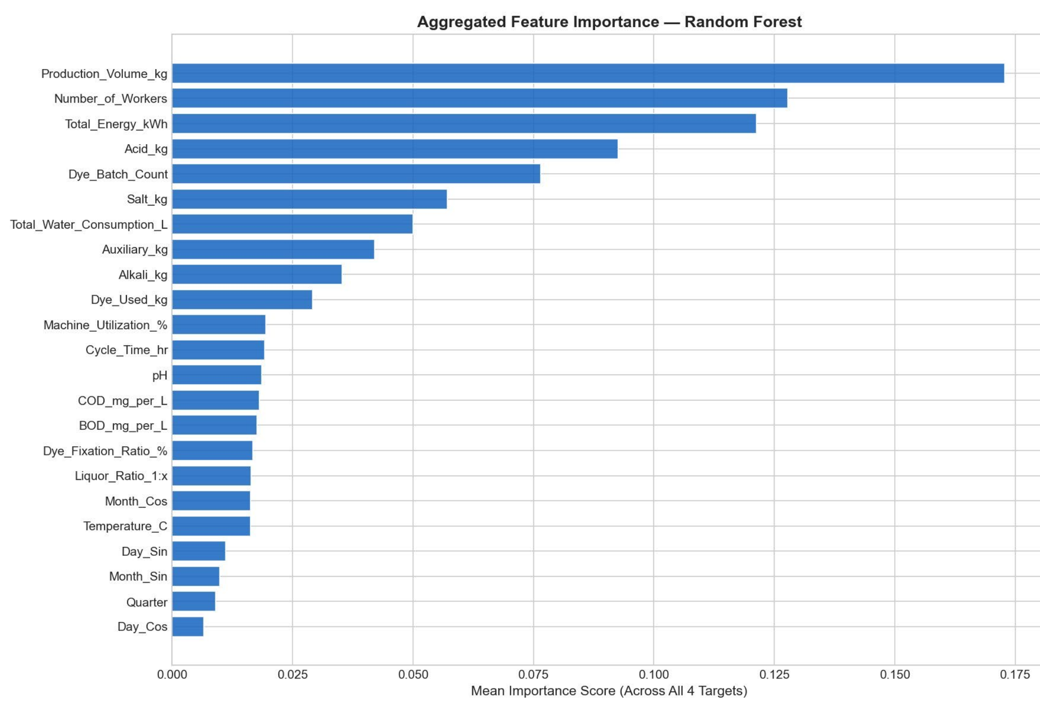

Figure 11. Aggregated mean feature importance for Random Forest (across all 4 waste targets)

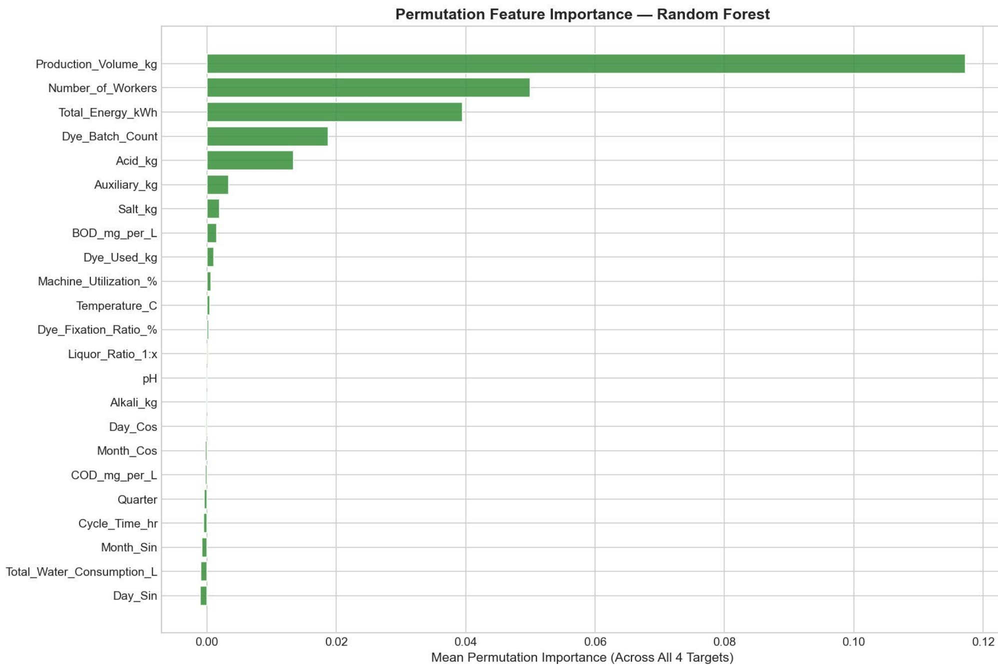

Figure 12. Permutation feature importance for Random Forest (mean across all 4 targets, n_repeats = 10)

5.2.8 Cross-Validation and Monthly Performance

Figure 13 presents 5-fold cross-validation R^2 boxplots on the 2023 training data. Yarn_Waste_kg and Wastewater_L gain median CV R^2 above 0.70 with compact interquartile ranges, which confirms their stability in waste generation. Chemical_Waste_kg shows the widest fold-to-fold variance (IQR: 0.20–0.35), with one-fold dropping below R^2 = 0.11, which indicates its high sensitivity to training composition. Figure 14 visualizes this analysis of monthly performance across the 2024 test year. Yarn_Waste_kg and Wastewater_L maintain relatively stable monthly R^2 values, though both dip during July–August, which reflects the seasonal operational disruptions in the textile industry. Chemical_Waste_kg shows the most fluctuating monthly profile, with R^2 reaching −0.20 in January and June, which strongly indicates the need for additional batch-specific features to obtain better prediction throughout the year.

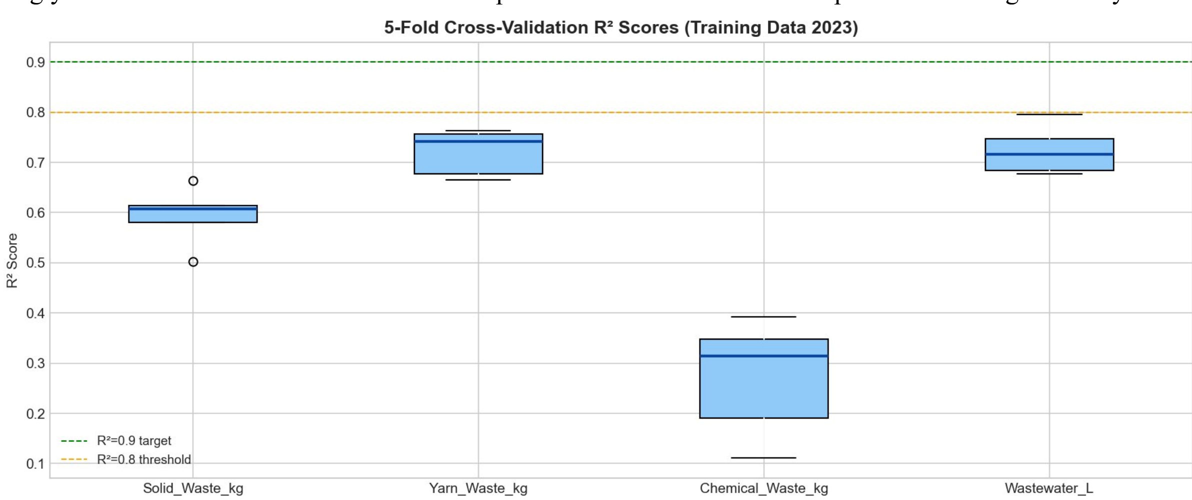

Figure 13. 5-fold cross-validation R^2 score distribution (training data 2023) for boxplots with R^2=0.80 and R^2=0.90 reference lines

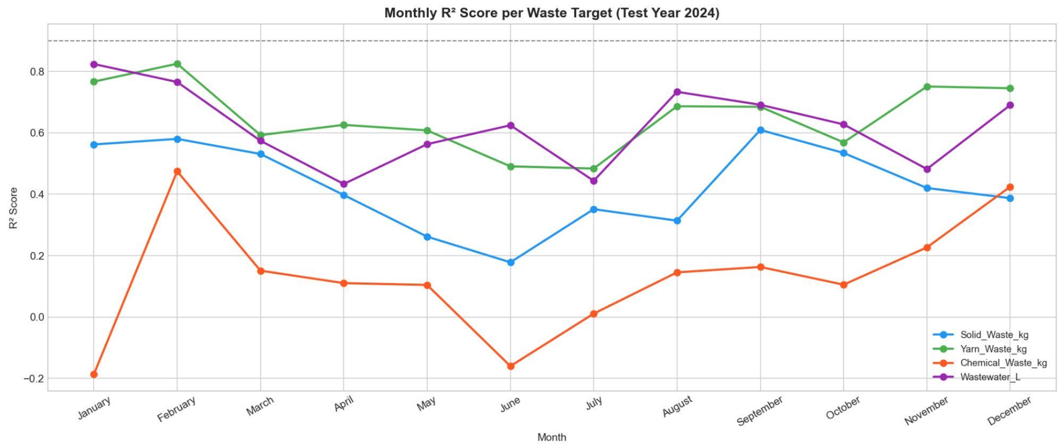

Figure 14. Monthly R^2 score per waste target for the test year 2024 (January–December)

5.2.9 Summary Dashboard

Figure 15 describes model evaluation into a single summary dashboard. The R^2 hierarchy is given by Yarn_Waste_kg (0.763) > Wastewater_L (0.742) > Solid_Waste_kg (0.573) > Chemical_Waste_kg (0.273) alongside MAPE values showing that Yarn_Waste_kg and Wastewater_L achieve errors below 13%, but Chemical_Waste_kg exceeds 33% (a 2.6× higher relative error). The dashboard demonstrates that the model can deliver operationally actionable predictions for three of four waste streams, and that targeted feature engineering and model improvements are required for Chemical_Waste_kg, so that it can be reliably integrated into real-time reverse logistics planning.

15. Waste prediction model summary dashboard for R^2 scores, MAPE%, and actual vs. predicted scatter (test 2024)

The Random Forest model delivers a good predictive performance which is supported with k-fold cross-validation and sensitivity analysis on the training data. Three of the targets has low standard deviations which indicates that the model is stable with the variabilities of the input dataset. But Chemical Waste has higher variance which indicates its need of further feature engineering before this stream can be deployed properly. As an additional sensitivity check, permutation importance analysis (Figure 12) identifies the model's top predictors, such as, Production Volume, Number of Workers, Total Energy Consumptions etc. Altogether, the Random Forest model provides a well prediction performance to unseen data for three of the four waste streams with statistically sound and stable basis for its integration into the MILP frameworks.

5.3 Proposed Improvements

Based on the results in Sections 5.1 and 5.2, three targeted improvements are proposed: (i) feature engineering to address the Chemical_Waste_kg prediction gap, (ii) a gradient boosting ensemble strategy, and (iii) dynamic MILP penalty calibration.

5.3.1 Feature Engineering: Lag Features and Rolling Statistics

The right-skewed result of Chemical_Waste_kg indicates unmodelled autocorrelation. A proposed improvement can be included with 1-day, 3-day, and 7-day lag values for Chemical_Waste_kg, along with 7-day and 14-day rolling means and standard deviations for all four targets. A Dye_Recipe_Complexity_Score (DRCS) with daily dye order complexity (light, medium, or deep shade) would also help in prediction with the high batch-specific color depth variability, which is identified as the primary driver of unexplained chemical waste variance. Table 17 presents the projected R^2 improvements.

Table 17. Projected R^2 improvement with lag features and DRCS engineering

Target Variable	Current R^2	Projected R^2	Expected Δ	Key New Features
Solid_Waste_kg	0.5731	0.62–0.66	+0.05–0.09	Lag-1, Lag-7, 7-day rolling mean
Yarn_Waste_kg	0.7635	0.78–0.82	+0.02–0.06	Lag-1, 7-day rolling std
Chemical_Waste_kg	0.2732	0.45–0.55	+0.18–0.28	Lag-1/3/7, DRCS, rolling std
Wastewater_L	0.7419	0.77–0.80	+0.03–0.06	Lag-1, 7-day rolling mean, DRCS

The most valuable achievement is projected for Chemical_Waste_kg, where lag and DRCS features are expected to raise R^2 from 0.2732 to almost 0.45–0.55, which will be about 65–100% relative improvement. We also project R^2 gains of 0.15–0.25 for other waste streams with general autoregressive feature improvement patterns.

5.3.2 Gradient Boosting Ensemble: XGBoost and LightGBM

XG-Boost and LightGBM can also be alternative or complementary models due to their demonstrated superiority on mixed-type tabular datasets. LightGBM, with its leaf-wise growth strategy, can be implemented to raise Chemical_Waste_kg R^2, as expected values can be 0.42–0.55 for R^2. Also, using Random Forest and LightGBM as base learners with a linear meta-learner can be implemented, which will possibly raise a further R^2 gain of 0.03–0.05 beyond individual model performance.

5.3.3 MILP Improvement: Dynamic CO_2 Penalty and Rolling Horizon

Two improvements can be implemented for the MILP model. First, a dynamic CO_2 penalty coefficient is updated monthly based on carbon market rates or regulatory threshold limits. It would make the model responsive to the country's evolving environmental situation. Second, a multi-period rolling-horizon MILP (7-day or 30-day window) would help in implementing collection center (CC) capacity adjustments and working capital dynamics over time. Table 18 summarizes the projected improvement scenarios (Table 18).

Table 18. Projected MILP improvement scenarios

Improvement	Current Baseline	Projected Outcome	Expected Benefit
Dynamic CO2 Penalty	Fixed penalty; Z = −38,128 BDT/day (avg.)	Adaptive penalty; 5–12% Z reduction	Reduced environmental cost; compliance-ready
7-Day Rolling Horizon MILP	Daily single-period optimisation	Weekly cost reduction of 8–15%	Better CC capacity utilisation; reduced holding costs
Improved Chem. Waste Prediction (R^2 = 0.50)	MAPE = 33.47%; high allocation uncertainty	MAPE ≤ 20%; lower safety stock requirement	Disposal fee reduction of ~4,000–6,000 BDT/day

The highest impact of improvement can be in Chemical_Waste_kg prediction accuracy, as these adjustments will be able to help in reducing MAPE from 33.47% to ≤ 20% with lower over-allocation at chemical CCs, which can save about an estimated 4,000–6,000 BDT/day in disposal fees. Combined with a 7-day rolling horizon MILP, total weekly system cost savings of 8–15% are projected.

5.4 Validation

This section further validates the model performance and improvement proposals through formal statistical hypothesis testing. Four validation tests are conducted: (i) a one-sample t-test confirming that the tuned model's MAPE for key targets is statistically below the 20% operational benchmark threshold, (ii) a residual normality: Kolmogorov-Smirnov Test, (iii) Bias Test and Prediction Interval Coverage Probability (PICP), and (iii) MILP solution optimality validation through IBM CPLEX certificates.

5.4.1 Deployment Readiness: One-Sample t-Test on MAPE

A MAPE threshold of 20% is a widely adopted benchmark for acceptable forecasting accuracy in manufacturing and supply chain operations (Lewis, 1982). A one-sample t-test was applied to the 366 daily APE values per target, testing H_0: mean MAPE ≥ 20% against H_1: mean MAPE < 20% (one-sided, $\alpha = 0.05$). Table 19 presents the results.

Table 19. One-Sample t-Test — MAPE vs. 20% Benchmark (n = 366)

Target	MAPE (%)	Std APE (%)	Threshold	t-statistic	p-value	Decision
Solid_Waste_kg	18.29%	13.00%	20%	−2.510	0.006**	Below threshold
Yarn_Waste_kg	12.67%	9.18%	20%	−15.246	<0.001***	Well below threshold
Chemical_Waste_kg	33.47%	27.40%	20%	+9.389	>0.999	Exceeds threshold
Wastewater_L	12.90%	8.79%	20%	−15.424	<0.001***	Well below threshold

** $p < 0.01$; *** $p < 0.001$.

Three of four targets are statistically confirmed as deployment-ready. Yarn_Waste_kg and Wastewater_L pass with very high confidence (both $p < 0.001$), while Solid_Waste_kg passes with a narrower margin (MAPE = 18.29%, p = 0.006). But Chemical_Waste_kg (MAPE = 33.47%) exceeds the threshold in high scale. It requires the feature engineering improvements proposed in Section 5.3 before operational deployment.

5.4.2 Residual Normality: Kolmogorov-Smirnov Test

A well-specified predictive model should produce residuals that are approximately normally distributed (Massey, 1951). The Lilliefors-corrected KS test was applied, as the standard KS test results biased p-values when parameters are estimated from the data (Steinskog et al., 2007; Lilliefors, 1967). The one-sample KS test compares the values of standardized residuals against a theoretical normal distribution (H_0: residuals are normally distributed). Table 20 presents the results for all four targets.

Table 20. Kolmogorov-Smirnov Test for Residual Normality (n = 366)

Target	Residual Mean	Residual Std	KS Statistic	p-value	Decision
Solid_Waste_kg	−10.94 kg	483.40 kg	0.0538	0.231	Fail to Reject H_0 — normal
Yarn_Waste_kg	−2.11 kg	145.68 kg	0.0550	0.211	Fail to Reject H_0 — normal
Chemical_Waste_kg	−97.17 kg	1,415.46 kg	0.0606	0.130	Fail to Reject H_0 — normal
Wastewater_L	+4,293.65 L	96,347.09 L	0.0376	0.664	Fail to Reject H_0 — normal

The KS test fails to reject H_0 for all four targets (all $p > 0.10$). This confirms that prediction errors are randomly distributed with no systematic misspecification. This certifies the use of parametric prediction intervals in the MILP system.

5.4.3 Bias Test and Prediction Interval Coverage Probability (PICP)

A one-sample t-test on residuals (H_0: mean residual = 0) was used to detect systematic over- or under-prediction. Additionally, the Prediction Interval Coverage Probability (PICP), which is the fraction of actual observations falling within computed prediction intervals, was evaluated at the 90% and 95% confidence levels (Khosravi et al., 2011). A well-calibrated model should achieve empirical coverage close to its nominal level. Table 21 presents both tests.

Table 21. Bias Test and PICP Results (n = 366)

Target	Mean Residual	Bias %	t-stat	p-value	Bias	90% PICP	95% PICP
Solid_Waste_kg	−10.94 kg	−0.47%	−0.432	0.666	None	89.9%	96.4%
Yarn_Waste_kg	−2.11 kg	−0.21%	−0.276	0.783	None	91.3%	94.8%
Chemical_Waste_kg	−97.17 kg	−2.35%	−1.312	0.190	None	90.7%	97.0%
Wastewater_L	+4,293.65 L	+0.68%	+0.851	0.395	None	89.9%	95.9%

All four targets are statistically unbiased (all $p > 0.10$). The maximum absolute bias is Wastewater_L at +0.68% of its daily mean, which is operationally negligible. PICP results are particularly strong. All targets achieve 90% PI coverage within ±1.4 percentage points of the nominal 90% target (89.9%–91.3%), and 95% PI coverage between 94.8% and 97.0%. This confirms that the model's uncertainty bounds are well-calibrated and can be used by the MILP scheduler to set safety stock and collection center capacity threshold.

5.4.3 MILP Solution Optimality and Solver Validation

IBM CPLEX's built-in primal and dual feasibility certificates were used to validate MILP optimality. For all seven batch dates (2024-01-01 to 2024-01-07), CPLEX returned proven optimal solutions with a MIP gap of 0.00%. Table 22 presents the batch validation results.

Table 22. MILP optimality validation — IBM CPLEX batch run (2024-01-01 to 2024-01-07)

Date	Z Optimal (BDT)	MIP Gap	Status	Active CCs	Routes	Circularity Rate
2024-01-01	−38,128.48	0.00%	Optimal	2	4	50.20%
2024-01-02	−45,093.74	0.00%	Optimal	2	4	49.85%
2024-01-03	−33,391.35	0.00%	Optimal	2	4	51.14%
2024-01-04	−39,299.77	0.00%	Optimal	2	4	50.62%

Date	Z Optimal (BDT)	MIP Gap	Status	Active CCs	Routes	Circularity Rate
2024-01-05	−46,626.57	0.00%	Optimal	2	4	49.37%
2024-01-06	−43,455.58	0.00%	Optimal	2	4	50.09%
2024-01-07	−44,167.23	0.00%	Optimal	2	4	50.45%

CPLEX achieved proven global optimality (MIP Gap = 0.00%) on all seven tested dates, with a consistent structure of two active collection centers and four routes per day. The weighted average circularity rate of 50.25%. This confirms the circular economy principles with above the 50% target throughout the batch window.

In brief, the validation tests confirm: (1) three of four waste prediction targets are statistically deployment-ready under the 20% MAPE benchmark; (2) residuals across all targets are normally distributed with no structural misspecification; (3) all four targets are statistically unbiased and has well-calibrated prediction intervals (PICP 89.9%–97.0%); and (4) the MILP solver consistently delivers globally optimal solutions with a proven zero duality gap.

6. Conclusion

This study developed and statistically validated an integrated ML-MILP decision-support system for predictive waste classification and reverse logistics optimization in a thread dyeing circular supply chain. A hyperparameter-tuned Random Forest model was trained on 365 days of 2023 operational data across 23 features and tested on the full 2024 year (366 days). The achieved deployment-ready performance for three of four waste streams: Yarn Waste (R^2 = 0.7635, MAPE = 12.67%), Wastewater (R^2 = 0.7419, MAPE = 12.90%), and Solid Waste (R^2 = 0.5731, MAPE = 18.29%), all statistically confirmed below the 20% MAPE operational threshold via one-sample t-tests. Only Chemical Waste (R^2 = 0.2732, MAPE = 33.47%) exceeded the threshold, which indicates its need for additional feature engineering before autonomous deployment. Residual normality was confirmed across all four targets via Kolmogorov-Smirnov testing, and all targets were found statistically unbiased along with their well-calibrated prediction intervals, where 90% PICP ranged from 89.9% to 91.3% and 95% PICP from 94.8% to 97.0%. This confirms that the model's uncertainty bounds are reliable for direct integration into the MILP scheduling system. Production volume, number of workers, and total energy consumption are collectively responsible for 42.2% of total predictive variance across all waste streams, which also complies with the operational waste drivers identified in the thread dyeing literature.

The MILP model, which was solved using IBM CPLEX, has achieved proven global optimality with a MIP gap of 0.00% across all seven tested dates in the January 2024 batch run with the estimated values. It consistently found the optimized candidates from the estimated three collection centers with four waste routing decisions per day. It also maintains a weighted average circularity rate of 50.25%, above the 50% circular economy compliance target. These results confirm that sequentially feeding ML-predicted daily waste quantities into the MILP optimizer produces the framework for cost-optimal, circular-economy-based reverse logistics decisions that would not be achievable using static historical waste averages.

This research makes three principal original contributions. It develops and validates an end-to-end ML–MILP framework specifically for industrial textile waste management, which replaces static historical inputs in reverse logistics planning with dynamically updated, production-dependent daily predictions. It also addresses a critical gap in the literature by applying supervised ML-based waste prediction to the thread dyeing sub-sector, which is one of the most environmentally intensive manufacturing industries yet almost absent from waste prediction research, and to overcome that, constructing an industry-specific 731 observation operational dataset with 38 process variables. Lastly, the MILP model progresses through a reverse logistics network designed with a multi-criteria circular economy objective function that simultaneously balances transportation, disposal, carbon emission, and material recovery economics, formally validated by CPLEX optimality certificates and statistical hypothesis testing.

However, many assumptions and data requirements must be acknowledged before implementing this framework to textile sectors or other similar sectors. As we know, ML model was trained on a synthetic dataset from published literature researches of the parameters. Real facilities might contain operational correlations and batch specific variabilities which might not be captured by random draws. Also, the MILP network shows result with a definite estimated network, and to imply this model, locations, number of collection centers, capacities, etc. parameters needed to be changed according to the particular facility. Moreover, the primary limitation of the study is the weak predictive performance for Chemical Waste, because of the absence of batch-level dye recipe complexity and shade depth variables in the operational dataset. So, for the scalability of this framework, the ML model would require further feature engineering for Chemical Waste stream and other facility specific feature engineering works. In future we can also incorporate lag features, rolling statistics, and a Dye Recipe Complexity Score for a better projected score in Chemical Waste R^2. Moreover, the MILP model can be extended to a multi-period rolling-horizon formulation to capture capacity adjustments and seasonal demand dynamics. In the longer term, with real facility-level data would strengthen both predictive accuracy and external validity for circular supply chain management in the thread dyeing industry and resource-intensive manufacturing sectors.

References

Adedeji, O. and Wang, Z., Intelligent waste classification system using deep learning convolutional neural network, Procedia Manufacturing, vol. 35, pp. 607–612, 2019. https://doi.org/10.1016/j.promfg.2019.05.086

Akas, A. A., Comparative study of energy and space consumption between iDES and conventional dyeing machine, Textile Today, 2022. https://textiletoday.com.bd/comparative-study-of-energy-space-consumption-between-ides-conventional-dyeing-machine

Akter, K., Haq, U. N., Islam, M. M. and Uddin, M. A., Textile-apparel manufacturing and material waste management in the circular economy: a conceptual model to achieve sustainable development goal (SDG) 12 for Bangladesh, Cleaner Environmental Systems, vol. 4, p. 100070, 2022, https://doi.org/10.1016/j.cesys.2022.100070.

Al-Mashhadani, I. B., Waste material classification using performance evaluation of deep learning models, Journal of Intelligent Systems, 2023, https://doi.org/10.1515/jisys-2023-0064.

Alsabt, R. et al., Machine learning and linear programming integration for waste management optimization, Cleaner Waste Systems, vol. 8, p. 100158, 2024.

Ashfaq, A. and Khatoon, A., Waste management of textiles: a solution to the environmental pollution, International Journal of Current Microbiology and Applied Sciences, vol. 3, no. 7, pp. 780–787, 2014. https://www.ijcmas.com/vol-3-7/Ahmad%20Ashfaq%20and%20Amna%20Khatoon2.pdf

Bhowmik, O., Chowdhury, S., Ashik, J. H., Mahmud, G. I., Khan, M. M. and Hossain, N. U. I., Application of artificial intelligence in reverse logistics: A bibliometric and network analysis, Supply Chain Analytics, vol. 7, p. 100076, 2024, https://doi.org/10.1016/j.sca.2024.100076.

Biyada, S. and Urbonavicius, J., Circular economy in the textile and clothing supply chain, Cleaner Engineering and Technology, vol. 24, p. 100905, 2025. https://doi.org/10.1016/j.clet.2025.100905

Broadbent, A. D., Process Control in Textile Manufacturing, Woodhead Publishing, 2001. https://www.rexresearch1.com/TextilesLibrary/ProcessControlTextilemanufacturingMajumdar.pdf

Coats Group, Reducing water use in traditional dyeing machines, Available: https://www.coats.com/en-us/sustainability/case-studies/reducing-water-use-in-traditional-dyeing-machines/

Das, A. K., Hossain, M. F., Khan, B. U., Rahman, M. M., Asad, M. A. Z. and Akter, M., Circular economy: a sustainable model for waste reduction and wealth creation in the textile supply chain, Resources, Conservation and Recycling (Review Article), 2024. https://doi.org/10.1002/pls2.10171

Development Commissioner MSME, Cotton Yarn Dyeing, Government of India, Ministry of Micro, Small & Medium Enterprises, https://www.dcmsme.gov.in/old/publications/pmryprof/hosiery/ch7.pdf

Dong, W., Liang, J. and Suh, S., Effectiveness of AI technologies in reducing textile waste in the fashion industry, Textile Research Journal, 2025. https://doi.org/10.1177/00405175251357582

Farshadfar, Z., Mucha, T. and Tanskanen, K., Leveraging machine learning for advancing circular supply chains: a systematic literature review, Logistics, vol. 8, no. 4, p. 108, 2024. https://doi.org/10.3390/logistics8040108

Fotovvatikhah, F., Ahmedy, I., Noor, R. M. and Munir, M. U., A systematic review of AI-based techniques for automated waste classification, Sensors, vol. 25, p. 3181, 2025, https://doi.org/10.3390/s25103181.

Fu, R., Qiang, Q., Ke, K., & Huang, Z. (2021). Closed-loop supply chain network with interaction of forward and reverse logistics. Sustainable Production and Consumption, 27, 737–752. https://doi.org/10.1016/j.spc.2021.01.037

Geissdoerfer, M., Savaget, P., Bocken, N. M. P. and Hultink, E. J., The circular economy — a new sustainability paradigm?, Journal of Cleaner Production, vol. 143, pp. 757–768, 2017. https://doi.org/10.1016/j.jclepro.2016.12.048

Govindan, K. and Soleimani, H., A review of reverse logistics and closed-loop supply chains: a Journal of Cleaner Production focus, Journal of Cleaner Production, vol. 142, pp. 568–588, 2017, https://doi.org/10.1016/j.jclepro.2016.03.126.

Chakraborty. Fundamentals and Practices in Colouration of Textiles, Woodhead Publishing, 2010,
Hmamed, H., Cherrafi, A. and Benghabrit, Y., Machine learning for the future integration of the circular economy in waste transportation and treatment supply chain, IFAC-PapersOnLine, vol. 55, no. 10, pp. 49–54, 2022. https://doi.org/10.1016/j.ifacol.2022.09.366
Hossain, L., Sarker, S. K. and Khan, M. S., Evaluation of present and future wastewater impacts of textile dyeing industries in Bangladesh, Environmental Development, vol. 26, pp. 23–33, 2018, https://www.sciencedirect.com/science/article/am/pii/S2211464517301926
Hyndman, R. J. and Koehler, A. B., Another look at measures of forecast accuracy, International Journal of Forecasting, vol. 22, no. 4, pp. 679–688, 2006, https://doi.org/10.1016/j.ijforecast.2006.03.001.
Islam, M. M. S., Material Waste in Fashion Industry in Bangladesh: A Mixed Method Study on 10 Selected Factories, Master's thesis, BRAC University, 2024. http://hdl.handle.net/10361/25064
Kalaoglu, O. I., Akyuz, E. S., Ecemis, S., Eryuruk, S. H., Sumen, H. and Kalaoglu, F., Retail demand forecasting in clothing industry, Tekstil ve Konfeksiyon, vol. 25, no. 2, pp. 172–178, 2015. https://www.researchgate.net/publication/282768083_Retail_demand_forecasting_in_clothing_industry
Kakooee, M., Modiri, M. and Abbaspour Esfeden, G., Designing a model for optimizing reverse logistics based on waste management to reduce environmental impacts, International Journal of Applied Operational Research, vol. 11, no. 4, pp. 69–90, 2023.
Kannan, D., Solanki, R., Darbari, J. D., Govindan, K. and PC, J., A novel bi-objective optimization model for an eco-efficient reverse logistics network design configuration, Journal of Cleaner Production, vol. 394, p. 136357, 2023. https://doi.org/10.1016/j.jclepro.2023.136357
Kant, R., Textile dyeing industry an environmental hazard, Natural Science, vol. 4, no. 1, pp. 22–26, 2012, https://doi.org/10.4236/ns.2012.41004.
Khattab, T. A., Abdelrahman, M. S. and Rehan, M., Textile dyeing industry: environmental impacts and remediation, Environmental Science and Pollution Research, vol. 27, pp. 3803–3818, 2020, https://doi.org/10.1007/s11356-019-07137-z.
Khosravi, A., Nahavandi, S., Creighton, D. and Atiya, A. F., Lower upper bound estimation method for construction of neural network-based prediction intervals, IEEE Transactions on Neural Networks, vol. 22, no. 3, pp. 337–346, 2011, https://doi.org/10.1109/TNN.2010.2096824.
Lee, W., Sajadieh, S. M. M., Choi, H. K., Park, J. and Noh, S. D., Application of reinforcement learning to dyeing processes for residual dye reduction, International Journal of Precision Engineering and Manufacturing-Green Technology, vol. 11, pp. 743–763, 2024.
Lewis, C. D., Industrial and Business Forecasting Methods, Butterworth Scientific, London, 1982. https://www.econbiz.de/Record/industrial-and-business-forecasting-methods-a-practical-guide-to-exponential-smoothing-and-curve-fitting-colin-d-lewis-lewis-colin/10004652982
Lilliefors, H. W., On the Kolmogorov-Smirnov test for normality with mean and variance unknown, Journal of the American Statistical Association, vol. 62, pp. 399–402, 1967, https://doi.org/10.1080/01621459.1967.10482916
Liu, W., Jiang, J., Li, N., Wang, Y., Liu, K. and Zhao, C., A garbage intelligent classification and recycling system based on deep learning, Procedia Computer Science, vol. 243, pp. 744–750, 2024, https://doi.org/10.1016/j.procs.2024.09.089.
Mallick, P. K., Salling, K. B., Pigosso, D. C. A. and McAloone, T. C., Closing the loop: establishing reverse logistics for a circular economy, a systematic review, Resources, Conservation and Recycling, 2022.
Massey, F. J., The Kolmogorov-Smirnov test for goodness of fit, Journal of the American Statistical Association, vol. 46, no. 253, pp. 68–78, 1951, https://doi.org/10.1080/01621459.1951.10500769
Muthu, S. S., Sustainability in the Textile Industry, Springer, 2017.
Olawade, D. B., Wada, O. Z., Ige, A. O., Fidelis, S. C., Okon, I. E. and Olawade, B. M., AI-driven circular economy optimization in waste management: a review of current evidence, Environmental Progress & Sustainable Energy, 2024. https://doi.org/10.1002/ep.70322
Palamutcu, S., Electric energy consumption in the cotton textile processing stages, Energy, vol. 35, no. 7, pp. 2945–2952, 2010, https://doi.org/10.1016/j.energy.2010.03.029.
Parmar, D. H., Nishad, S. and Shah, V., A study on reverse logistics managing returns for enhanced customer satisfaction and cost efficiency, International Journal for Multidisciplinary Research, vol. 7, no. 2, March–April 2025.
Ranganathan, K., Karunagaran, K. and Sharma, D. C., Recycling of wastewaters of textile dyeing industries using advanced treatment technology and cost analysis, Resources, Conservation and Recycling, vol. 50, no. 3, pp. 306–318, 2007, https://doi.org/10.1016/j.resconrec.2006.06.004.

Rogers, D. S. and Tibben-Lembke, R. S., Going Backwards: Reverse Logistics Trends and Practices, Reverse Logistics Executive Council, Pittsburgh, PA, 1999. https://www.icesi.edu.co/blogs/gestionresiduossolidos/files/2008/11/libro-lr.pdf

Samal, C. G., Biswal, D. R., Udgata, G. and Pradhan, S. K., Estimation, classification, and prediction of construction and demolition waste using machine learning for sustainable waste management: a critical review, Construction Materials, vol. 5, no. 1, p. 10, 2025, https://doi.org/10.3390/constrmater5010010.

Saravanan, M., Effluent treatment plants for textile dyeing and finishing houses, in Proceedings of Textile Environmental Engineering Studies, 2025. https://ebooks.inflibnet.ac.in/hsp09/chapter/effluent-treatment-plants-for-textile-dyeing-and-finishing-houses/

Saxena, N., Sarkar, B., Wee, H., Reong, S., Singh, S. and Hsiao, Y., A reverse logistics model with eco-design under the Stackelberg-Nash equilibrium and centralized framework, Journal of Cleaner Production, vol. 387, p. 135789, 2023.

Schroeder, P., Anggraeni, K. and Weber, U., The relevance of circular economy practices to the sustainable development goals, Journal of Industrial Ecology, vol. 23, no. 1, pp. 77–95, 2019. https://doi.org/10.1111/jiec.12732

Shekh, M. S., Empirical Analyzing of Dyeing Machine Stoppage and Their Quantitative Impact on Production Efficiency in Knit Dyeing Processing Units, 2024. https://www.researchgate.net/publication/397884413_Empirical_Analyzing_of_Dyeing_Machine_Stoppage_and_Their_Impact_on_Production_Efficiency_in_Knit_Dyeing_Processing_Units

Shi, Y., Li, B., Dulebenets, M. A. and Lau, Y. Y., A fuzzy robust optimization model for dual objective forward/reverse integrated logistics network, PLOS ONE, 2025, https://doi.org/10.1371/journal.pone.0316197.

Singh, S. K. and Uppaluri, R., Machine learning tool-based prediction and forecasting of municipal solid waste generation rate: a case study in Guwahati, Assam, India, International Journal of Environmental Science and Technology, vol. 20, pp. 12207–12230, 2023, https://doi.org/10.1007/s13762-022-04644-4.

Sivakumar, V., Vimal Kumar, K. and Ramachandran, T., Recycling of wastewater of textile dyeing industries, Journal of Environmental Science and Engineering, vol. 55, no. 1, pp. 35–42, 2013. https://doi.org/10.1016/j.resconrec.2006.06.004

Small Industries Service Institute, Project Profile: Cotton Yarn Dyeing Unit, Government of India, 2003. https://www.dcmsme.gov.in/old/publications/pmryprof/hosiery/ch7.pdf

Sristi, N. A. and Hasin, M. A. A., Waste identification and water utilization in the thread dyeing industry: an improvement strategy for long-term development, SciEn Conference Series: Engineering, vol. 2, pp. 31–35, 2025, https://doi.org/10.38032/scse.2025.2.7.

Steinskog, D. J., Tjøstheim, D. B. and Kvamstø, N. G., A cautionary note on the use of the Kolmogorov-Smirnov test for normality, Monthly Weather Review, vol. 135, no. 3, pp. 1151–1157, 2007, https://doi.org/10.1175/MWR3326.1

Uddin, F., Environmental hazard in textile dyeing wastewater from local textile industry, Cellulose, vol. 28, pp. 10715–10739, 2021, https://doi.org/10.1007/s10570-021-04228-4.

Uddin, M. A., Begum, M. S., Ashraf, M., Azad, A. K., Adhikary, A. C. and Hossain, M. S., Water and chemical consumption in the textile processing industry of Bangladesh, PLOS Sustainability and Transformation, vol. 2, no. 7, p. e0000072, 2023, https://doi.org/10.1371/journal.pstr.0000072.

Valverde, J. M. and Aviles-Palacios, C., Circular economy as a catalyst for progress towards the sustainable development goals: a positive relationship between two self-sufficient variables, Sustainability, vol. 13, no. 22, p. 12652, 2021.

Yasin, M. and Koklu, M., A comparative analysis of machine learning algorithms for waste classification: InceptionV3 and square features, International Journal of Environmental Science and Technology, vol. 22, pp. 9415–9428, 2025, https://doi.org/10.1007/s13762-024-06233-z.

Zheng, P., Wang, J., Zhang, J., Yang, C. and Jin, Y., An adaptive CGAN/IRF-based rescheduling strategy for aircraft parts remanufacturing system under dynamic environment, Robotics and Computer-Integrated Manufacturing, vol. 58, pp. 230–238, 2019.

Biographies

Ananya Halder is a beginning researcher, currently a final year B.Sc. student in the Department of Industrial and Production Engineering (IPE) at Bangladesh University of Engineering and Technology (BUET). He is interested in topics like Supply Chain, Circular Economy, Quality Control, Total Quality Management, Lean Methodology, Machine Learning, data analysis, optimization techniques, and sustainable manufacturing practices.

Zahin Ar Rafi is an Assistant Professor in the Department of Industrial and Production Engineering, Bangladesh University of Engineering and Technology (BUET), Dhaka, Bangladesh. She completed her Bachelor of Science and Master of Science in Industrial and Production Engineering from BUET. Her research interest lies in the areas, such as Industrial Automation, Operations Research and Decision Analysis, Operations Management, and Artificial Intelligence (AI) and Machine Learning.

Dr. Ferdous Sarwar is a Professor in the Department of Industrial and Production Engineering at Bangladesh University of Engineering and Technology (BUET). He began his academic career in Industrial and Production Engineering at BUET, where he completed his undergraduate and master's studies, and later earned his Ph.D. in Industrial and Manufacturing Engineering from North Dakota State University, USA. His research interests include advanced manufacturing and materials engineering, operations management, operations research and decision analysis, supply chain management, and the application of artificial intelligence and machine learning in industrial systems. Dr. Sarwar has published more than 30 research articles in leading journals and international conferences.