Proceedings of the International Conference on Industrial Engineering and Operations Management

Publisher: IEOM Society International, USA
DOI: 10.46254/BA08.20250208
Published: December 20, 2025

A Swarm Intelligence Approach to Multi-Product Vendor Managed Inventory with Nonlinear Demand and Budget Constraints

Arvin Bahreini
Lundquist College of Business
University of Oregon
Eugene, Oregon, USA
arvinb@uoregon.edu

Mohammad Hossein Jalali
Graduate School of Management and Economics
Sharif University of Technology
Tehran, Tehran, Iran
mh_jalali@gsme.sharif.edu

Abstract

This paper presents a novel vendor-managed inventory (VMI) optimization model that incorporates nonlinear inverse demand functions and a global budget constraint across multiple buyers and products. Unlike traditional VMI models that rely on linear demand assumptions and unconstrained financial planning, the proposed framework captures real-world market dynamics by modeling price sensitivity using a quadratic demand structure. The objective is to maximize the total profit for the vendor by determining the optimal order quantities and replenishment cycles while respecting cost limitations. We utilize Particle Swarm Optimization (PSO) to efficiently solve the nonlinear, constrained optimization problem. Numerical experiments demonstrate the model's ability to allocate inventory profitably across four buyers and four products under a shared budget. All buyers received optimal replenishment allocations with positive profit contributions, and the total system profit exceeded $21,000. A validation study using a simplified linear model revealed a significant reduction in profit, highlighting the necessity of modeling nonlinearity in demand. Graphical and cost breakdown analyses further illustrated the robustness and interpretability of the solution. The results confirm the effectiveness of integrating nonlinear modeling with intelligent optimization and suggest promising directions for future research in constrained supply chain design.

Keywords

Vendor Managed Inventory (VMI), Joint Replenishment Policy, Particle Swarm Optimization (PSO), Nonlinear Demand Function and Supply Chain Optimization

1. Introduction

Vendor Managed Inventory (VMI) has established itself as a cornerstone of modern supply chain coordination. By authorizing suppliers to manage inventory levels directly at the buyer's location, VMI optimizes cost structures and service levels across the entire chain (Dong & Xu, 2002; Waller et al., 1999). The fundamental shift here is the transfer of replenishment responsibility from the buyer to the vendor. This strategic realignment unlocks superior demand visibility, minimizes stockouts, and allows for synchronized order scheduling (Achabal et al., 2000). Effectively, VMI acts as a bridge, merging the inventory management focus of operations research with the relationship analysis found in industrial organization.

While earlier research heavily favored deterministic inventory models and joint replenishment strategies within two-echelon systems—often grounded in the foundational Economic Order Quantity (EOQ) framework (Harris, 1913)—these approaches typically relied on fixed demand assumptions. However, real-world markets are rarely static. To capture true market dynamics, more recent studies have begun employing inverse demand functions. in these models, price is not fixed but is dependent on the ordered quantity, creating a complex interplay between pricing strategy and customer demand (Xiao & Xu, 2013; Almehdawe & Mantin, 2010).

The literature increasingly advocates for integrated approaches to supply chain optimization. Research generally falls into two camps: local optimization, where each entity minimizes its own costs independently, and global integration, where the supply chain acts as a unified system to maximize collective profit (Arora et al., 2010; Yao et al., 2007). VMI has proven particularly effective for the latter. For instance, empirical studies suggest that VMI coordination can significantly improve service levels, while others highlight its critical role in dampening the "bullwhip effect"—the phenomenon where demand fluctuations amplify as they move up the supply chain (Kristianto et al., 2012; Disney & Towill, 2003).

Furthermore, the mathematical methods used to solve VMI models have evolved from traditional linear programming to highly sophisticated meta-heuristics. Researchers have successfully applied techniques such as Genetic Algorithms (Pasandideh et al., 2011; Nachiappan & Jawahar, 2007), Ant Colony Optimization (Nia et al., 2014), and other tuned meta-heuristics (Sadeghi et al., 2013) to solve these complex problems. While multi-objective formulations have been proposed to balance conflicting goals like cost versus service (Cárdenas-Barrón et al., 2012), many existing models still suffer from unrealistic assumptions, such as linear demand curves, static costs, or unlimited budgets.

This paper aims to overcome these limitations. We introduce a novel model that integrates nonlinear demand functions and a global budget constraint within a VMI joint replenishment framework. Building on the structural foundations laid by previous research (Khajehnezhad, 2018; Sajadifar & Pourghannad, 2010), we extend the methodology by utilizing Particle Swarm Optimization (PSO). This solution engine provides the necessary flexibility to navigate nonconvex, constrained optimization landscapes. Our goal is to explore how to optimize resource allocation and buyer-level profits under realistic constraints, offering both methodological innovation and actionable managerial insights.

1.1 Objectives

The primary objective of this study is to develop an integrated supply chain model that maximizes total profit under a VMI structure, specifically accounting for nonlinear inverse demand functions and global inventory budget constraints. Unlike traditional models that rely on deterministic linear demand and assume unlimited resources, our approach captures the diminishing marginal revenue associated with increased order quantities. This offers a far more accurate representation of actual buyer markets.

Specifically, we aim to formulate a joint replenishment policy where the vendor determines order quantities and cycles for multiple buyers and products (Darwish & Odah, 2010). This decision-making process must account for the intricate relationships between pricing, demand, and inventory holding behavior. By introducing a budget constraint, we force the vendor to make value-based decisions, prioritizing product allocations to buyers with the highest profitability potential. To solve this complex, nonlinear problem, we employ Particle Swarm Optimization (PSO)—a population-based metaheuristic ideally suited for managing constraint violations and navigating nonconvex solution spaces.

Through this framework, we seek to:

1. Optimize profit allocation under realistic market constraints.
2. Demonstrate how nonlinearity and budget caps impact service strategies at the buyer level.
3. Provide numerical evidence to guide decision-makers in adopting flexible, algorithm-driven inventory practices within VMI partnerships.

2. Literature Review

The study of inventory control and coordination mechanisms has long been a central pillar of operations research. The Economic Order Quantity (EOQ) framework established the initial groundwork, balancing ordering and holding costs

under deterministic conditions (Harris, 1913). While EOQ models remain a pedagogical foundation, they often struggle to capture the complexity of multi-echelon systems or dynamic market responses (Hariga et al., 2013).
Vendor Managed Inventory (VMI) has garnered significant attention as a strategy that reassigns inventory management duties to the supplier. Early work by Waller et al. (1999) and Achabal et al. (2000) demonstrated the efficiency gains of VMI, particularly in reducing the bullwhip effect and improving supply chain visibility. Disney and Towill (2003) further documented how VMI adoption leads to tangible improvements in service levels and information sharing. Additionally, coordination mechanisms involving rebates and consignment have been shown to further enhance the effectiveness of VMI relationships (Chen et al., 2010; Wong et al., 2009).

Recent contributions have expanded these traditional models to incorporate demand responsiveness and price-sensitive markets. For example, research has moved toward two-echelon supply chain models with inverse demand functions, where price adjusts based on the ordered quantity (Almehdawe & Mantin, 2010). This evolution captures consumer behavior more accurately, shifting the focus from simple cost minimization to comprehensive profit maximization (Kim & Park, 2010).

In parallel, optimization techniques have advanced to meet the demands of these high-dimensional models. Traditional linear programming often falls short when faced with the nonconvexities introduced by nonlinear demand or discrete decision variables. Consequently, metaheuristic methods have become the standard for solving such problems. Significant success has been reported using Genetic Algorithms (Pasandideh et al., 2011; Nachiappan & Jawahar, 2007), Ant Colony Optimization (Nia et al., 2014), and hybrid algorithms utilizing tuned parameters (Sadeghi et al., 2014; Sadeghi et al., 2013).

Despite these advances, a gap remains in integrating realistic market features—such as diminishing returns, buyer prioritization, and strict financial limitations—within VMI models. The majority of existing research either assumes unlimited resources or focuses narrowly on cost minimization. Our study fills this gap by modeling a constrained profit-maximization problem under a nonlinear pricing scheme and solving it with PSO. In doing so, we contribute to both the theoretical development of integrated inventory models and the practical application of modern optimization techniques (Khajehnezhad, 2018).

3. Methods

This section presents the mathematical formulation and solution methodology for the proposed Vendor Managed Inventory (VMI) model with nonlinear demand and budget constraints. The formulation is designed to capture realistic features of buyer behavior and inventory cost dynamics in a two-echelon supply chain. First, we describe the model structure and define the objective function. Next, we outline the operational and financial constraints governing the system. Finally, we detail the Particle Swarm Optimization (PSO) algorithm used to solve the nonlinear optimization problem efficiently.

3. 1 Model Structure

We consider a two-echelon supply chain consisting of a single vendor and multiple buyers (j = 1, 2, ..., N), each demanding a set of M different products (i = 1, 2, ..., M). The vendor is responsible for determining the order quantity y_{ij} and replenishment cycle T_j for each buyer. The model incorporates a nonlinear inverse demand function and a global budget constraint on inventory-related costs.

The inverse demand function faced by each buyer is quadratic and given by:

$$P_{ij}(y_{ij}) = a_{ij} - b_{ij}y_{ij} - c_{ij}y_{ij}^2$$

where a_{ij}, b_{ij}, c_{ij} are demand parameters specific to buyer j and item i.

The vendor's goal is to maximize the total profit, defined as:

$$\max_{y_{ij},T_j} Z = \sum_{j=1}^{N}\sum_{i=1}^{M}\left[P_{ij}(y_{ij})y_{ij} - \delta_i y_{ij} - h_i\left(\frac{y_{ij}T_j}{2}\right)\right] - \sum_{j=1}^{N}\frac{K_j}{T_j}$$

where:
- δ: unit production cost of item i

- h_i: unit holding cost of item i
- k_j: fixed ordering cost for buyer j

3.2 Constraints

The proposed model is subject to operational and financial limitations, including bounds on order quantities and cycle times, as well as a global inventory budget. These constraints ensure that the solution remains feasible and reflects practical limitations in real-world VMI settings.

1. Order quantities must be non-negative and bounded:

$$0 \leq y_{ij} \leq y_{ij}^{max}$$

2. Replenishment cycles are bounded:

$$0.01 \leq T_j \leq 1.0$$

3. Inventory budget constraint:

$$\sum_{j=1}^{N} \left(\frac{K_j}{T_j} + \sum_{i=1}^{M} h_i \left(\frac{y_{ij} T_j}{2} \right) \right) \leq B$$

where B is the available budget for inventory-related costs.

3.3 Solution Method

To solve the resulting nonlinear and constrained optimization problem, we apply Particle Swarm Optimization (PSO), a metaheuristic algorithm that efficiently explores the solution space. PSO is particularly suited to problems where traditional gradient-based methods are impractical due to nonconvexity or discontinuities.

Here, Particle Swarm Optimization is employed to solve the nonlinear constrained profit maximization problem.

Each particle represents a candidate solution defined as:

$$\mathbf{x} = [y_{11}, \dots, y_{MN}, T_1, \dots, T_N]$$

The fitness of each particle is computed using the total profit function with a penalty term added for any budget violation.

The velocity and position update rules for each particle are given by:

$$v_{id}(t+1) = w v_{id}(t) + c_1 r_1 (p_{id} - x_{id}) + c_2 r_2 (g_d - x_{id})$$
$$x_{id}(t+1) = x_{id}(t) + v_{id}(t+1)$$

Where w is inertia weight, c_1 and c_2 are cognitive and social coefficients respectively, r_1 and r_2 are random numbers between 0 and 1, p_{id} is personal best position, and finally g_d is the global best position.
To handle constraints, we apply a penalty method where any violation of the budget constraint adds a large penalty λ to the fitness function:

$$\text{Fitness } = -Z + \lambda \cdot max(0, \text{inventory cost } - B)$$

The algorithm iteratively updates particle positions and velocities until a stopping criterion (e.g., maximum iterations) is met. The global best particle is reported as the optimal solution.
The next section presents the numerical setup and results of applying this model to a sample supply chain scenario with four buyers and four products.

4. Numerical Study

To validate the proposed model, we conducted a numerical analysis using synthetically generated data. This dataset was carefully designed to mirror typical industrial parameters while providing a controlled environment for rigorously testing the model's performance. The experimental setup features a two-echelon supply chain comprising four distinct buyers and four products, a structure often seen in multi-product VMI literature (Sadeghi et al., 2013).

We calibrated the demand function parameters to specifically exhibit nonlinear price responses and distinguishable market behaviors across the different buyers. To ensure that each buyer demonstrated unique sensitivity to the purchased quantity—capturing the "inverse demand" dynamics discussed by Almehdawe and Mantin (2010)—we generated coefficients within specific, reasonable bounds. For instance, the primary demand intercept ranged from 100 to 140, while the price sensitivity coefficients were set between 1.0 and 1.5, and the nonlinear curvature parameters varied from 0.01 to 0.035.

Regarding cost structures, we defined fixed ordering costs between 300 and 330, unit holding costs between 1.5 and 2.0, and unit production costs ranging from 9 to 11 across the different product lines. Consistent with multi-constraint frameworks (Cárdenas-Barrón et al., 2012), we imposed a strict global inventory budget of 2000. This constraint effectively limits the permissible combinations of ordering and holding expenditures, forcing the algorithm to make trade-off decisions.

We implemented the computational framework using Python 3. Following the established trend of applying meta-heuristics to complex, non-convex VMI problems (Nia et al., 2014; Sadeghi et al., 2014), we developed a custom Particle Swarm Optimization (PSO) algorithm from the ground up rather than relying on pre-packaged commercial solvers. The algorithm incorporates standard tuning parameters, including inertia weight, as well as cognitive and social coefficients, to guide the search process. For data visualization and high-performance vectorized calculations, we utilized the Matplotlib and NumPy libraries, respectively. All experiments were executed within a Jupyter Notebook environment, providing a flexible and interactive platform for parameter tuning and future replication of these results.

5. Results and Discussion

This section presents and analyzes the results of the proposed model, including numerical outputs, graphical visualizations, proposed enhancements, and validation methodology. Each subsection provides insights into the effectiveness and practicality of the integrated nonlinear VMI model solved via Particle Swarm Optimization.

5.1 Numerical Results

The optimal solution derived from the PSO implementation demonstrates strong performance across all buyers. Each buyer receives a positive quantity of at least three products, satisfying demand while adhering to the global budget constraint of 2000 units. The total revenue generated across all buyers is $27,441.11, with a production cost of $3,887.94, ordering cost of $1,260.00, and holding cost of $329.58. The total inventory cost remained within budget at $1,589.58. Consequently, the total profit amounts to $21,963.59.
At the individual level, all buyers contributed positively to the profit:

- Buyer 1 Profit: $5,992.46
- Buyer 2 Profit: $5,841.61
- Buyer 3 Profit: $4,299.43
- Buyer 4 Profit: $5,830.09

This indicates a well-balanced allocation strategy where all market zones were serviced profitably.

5.2 Graphical Results

Graphical analyses are used to provide additional insight into the allocation decisions generated by the proposed nonlinear VMI model. Table 1 summarizes the optimal order quantities for each buyer–product combination, while Figures 1–4 visualize these allocations for individual buyers. In addition, Figure 5 illustrates the overall cost structure of the optimized solution.

Figure 1 shows the order quantities for Buyer 1. This buyer receives relatively high and consistent allocations across all four products, with Item 4 exhibiting the largest quantity. This outcome reflects Buyer 1's favorable demand characteristics and strong profitability under the nonlinear pricing structure. Figure 2 presents the order quantities for Buyer 2. In contrast to Buyer 1, Buyer 2 displays a more balanced allocation pattern across all products, with moderate quantities assigned to each item. This suggests a relatively uniform marginal profitability across products for this buyer. Figure 3 illustrates the order quantities for Buyer 3. Notably, no quantity is allocated to Item 3 for this buyer. This result indicates that, under the nonlinear demand function and global budget constraint, supplying Item 3 to Buyer 3 would yield insufficient marginal profit relative to its cost. The model therefore excludes this allocation in favor of more profitable alternatives. Figure 4 shows the order quantities for Buyer 4. The allocations exhibit a stable and gradually increasing trend across the four products, reflecting consistent demand sensitivity and positive profit contributions for all items supplied to this buyer.

In addition to quantity allocations, Figure 5 presents the cost breakdown of the optimized solution. Production cost constitutes the largest share of total cost at approximately 71%, followed by ordering cost at 23% and holding cost at 6%. This distribution highlights the dominant role of production and ordering decisions in overall cost management, emphasizing the importance of coordinated replenishment and efficient production planning in vendor-managed inventory systems.

Table 1. **Order quantities for Buyers**

Buyer	**Item 1 Qty**	**Item 2 Qty**	**Item 3 Qty**	**Item 4 Qty**
Buyer 1	27.09	28.79	28.36	30.57
Buyer 2	25.41	26.37	26.86	27.44
Buyer 3	24.05	24.74	**0.00**	25.39
Buyer 4	22.99	23.47	24.37	25.26

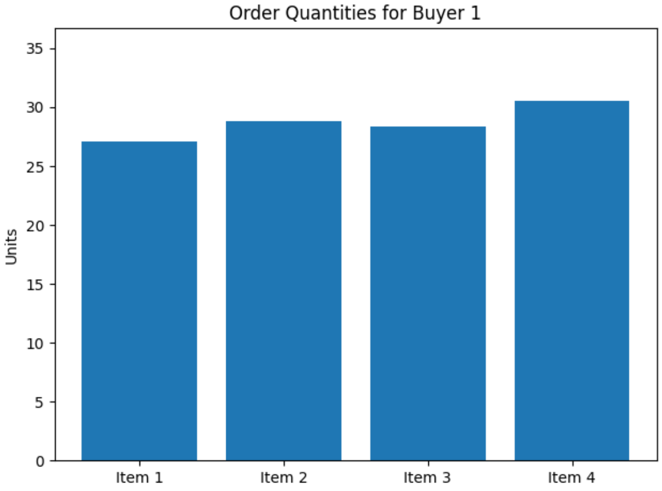

Figure 1. Order quantities for Buyer 1

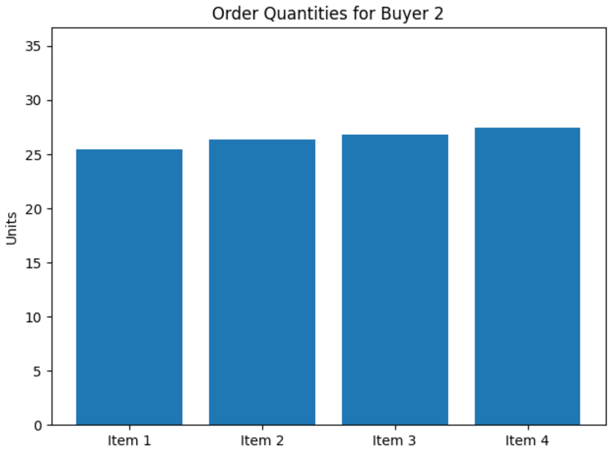

Figure 2. Order quantities for Buyer 2

Figure 3. Order quantities for Buyer 3

Figure 4. Order quantities for Buyer 4

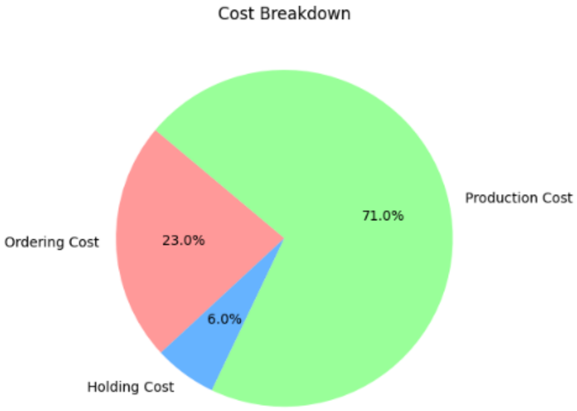

Figure 5. Cost Breakdown

5.3 Proposed Improvements

The results highlight the effectiveness of using nonlinear inverse demand functions in modeling real-world pricing dynamics. Compared to linear or piecewise models, the quadratic pricing structure better captures diminishing returns in buyer markets. Furthermore, the use of a global inventory budget creates a more realistic optimization scenario for vendors facing capital constraints.

The implementation of PSO proved advantageous due to its adaptability in exploring nonconvex regions. Traditional optimization methods may struggle with convergence or constraint handling, while the PSO approach easily incorporates penalty functions for budget adherence. Further improvements could involve hybrid PSO-GA methods or constraint-handling techniques to explore trade-offs between convergence speed and solution quality.

5.4 Validation

To assess the effectiveness and structural necessity of including nonlinearity in the demand function, we conducted a validation experiment by re-running the optimization model under a simplified assumption. In this variant, the quadratic term c_{ij} in the inverse demand function was set to zero, effectively linearizing the pricing response across all buyers and products. This approach enabled a controlled comparison between the original nonlinear model and a corresponding linear baseline.

The original nonlinear model achieved a best fitness value (i.e., negative total profit) of -21963.59, stabilizing by iteration 90 of the PSO process. In contrast, the linearized model produced a significantly worse best fitness value of -28331.96, indicating a substantial reduction in optimal profit. The linear model's convergence was also slower, with meaningful changes in the fitness function occurring well beyond iteration 40, whereas the nonlinear model reached its best performance earlier and maintained stability.

This performance gap confirms that the nonlinear demand formulation better captures the true revenue potential associated with buyer behavior. The quadratic form allows for marginal price sensitivity to be reflected more accurately, particularly in markets with diminishing returns or elastic pricing. As a result, the nonlinear model allocates quantities more strategically across buyers and items, optimizing profit while maintaining budget feasibility.

This validation demonstrates that incorporating nonlinearity into the demand model is not only theoretically sound but practically essential for achieving higher profitability and more realistic decision outcomes in vendor-managed inventory systems.

6. Conclusion

This study proposed and validated a novel Vendor Managed Inventory (VMI) framework that integrates nonlinear inverse demand functions with a strict global inventory budget across multiple buyers and products. Through this formulation, we addressed a critical limitation found in traditional inventory literature, which often relies on the simplifying assumptions of linear price-demand relationships and unlimited financial resources (Hariga et al., 2013; Harris, 1913). By explicitly embedding a quadratic pricing structure, our model captures the diminishing marginal returns typical of real-world buyer markets, offering a representation that is far more reflective of actual operational dynamics than previous deterministic approaches (Almehdawe & Mantin, 2010).

To solve the resulting nonlinear, constrained optimization problem, we employed Particle Swarm Optimization (PSO). Consistent with recent advancements in applying meta-heuristics to complex supply chain problems (Nia et al., 2014; Sadeghi et al., 2013), PSO proved highly effective in navigating the solution space. It achieved rapid convergence to optimal or near-optimal solutions while rigorously accommodating hard constraints, such as the multi-product inventory budget (Cárdenas-Barrón et al., 2012). We implemented the model in Python, leveraging open-source libraries to ensuring both reproducibility and extensibility for future research.

Our numerical results demonstrate that the proposed model efficiently allocates inventory across buyers while ensuring that every participant contributes positively to the total profit—a key requirement for sustainable supply chain coordination (Kim & Park, 2010; Chen et al., 2010). Each buyer received a tailored quantity profile, and the global inventory budget was respected without sacrificing overall profitability. The final scenario yielded a total profit exceeding \$21,000, with individual buyer profits ranging from \$4,200 to nearly \$6,000, indicating a robust and fair allocation mechanism.

Further graphical analysis provided granular insights into individual buyer behaviors and system-level cost distributions. For instance, Buyer 1 consistently demonstrated strong product uptake, while the cost breakdown revealed that production was the dominant expense. This finding underscores the importance of strategic production planning over marginal inventory tweaks, offering actionable managerial takeaways for decision-makers considering VMI adoption (Achabal et al., 2000; Waller et al., 1999).

Validation experiments comparing our approach against a simplified linear demand model confirmed the critical importance of accounting for nonlinearity. The linear variant yielded nearly \$6,000 less in total profit and suffered from slower convergence. These results not only affirm our methodological direction but also highlight the significant risks associated with oversimplification in supply chain modeling (Disney & Towill, 2003).

In summary, this work contributes to the field by bridging the gap between advanced demand modeling and practical optimization under resource constraints. The findings advocate for the broader adoption of nonlinear, constraint-aware models in VMI systems and demonstrate the applicability of swarm intelligence techniques like PSO in achieving scalable solutions.

7. Future Work

Future work can be expanded to capture the nuances of dynamic environments. For instance, incorporating stochastic demand (Sajadifar & Pourghannad, 2010) and dynamic pricing (Xiao & Xu, 2013) would enhance the model's ability to reflect market volatility. From an algorithmic perspective, hybrid approaches like PSO-GA combinations warrant investigation, as they often improve convergence in larger problem sets (Sadeghi et al., 2014). Future research should also look toward multi-period planning and empirical validation using industry datasets. Furthermore, applying this framework to decentralized supply chains (Kim & Park, 2010) and performing comprehensive robustness analyses would help verify the model's stability under uncertain parameters.

References

Achabal, D. D., S. H. McIntyre, S. A. Smith, and K. Kalyanam, *A Decision Support System for Vendor Managed Inventory,* Journal of Retailing, vol. 76, no. 4, pp. 430–454, 2000.

Almehdawe, E., and B. Mantin, *Vendor managed inventory with a capacitated manufacturer and multiple retailers: Retailer versus manufacturer leadership,* International Journal of Production Economics, vol. 128, no. 1, pp. 292–302, 2010.

Arora, V., F. T. S. Chan, and M. K. Tiwari, *An integrated approach for logistic and vendor managed inventory in supply chain,* Expert Systems with Applications, vol. 37, no. 1, pp. 39–44, 2010.

Cárdenas-Barrón, L. E., G. Treviño-Garza, and H. M. Wee, *A simple and better algorithm to solve the vendor managed inventory control system of multi-product multi-constraint economic order quantity model,* Expert Systems with Applications, vol. 39, no. 3, pp. 3888–3895, 2012.

Chen, J. M., I. C. Lin, and H. L. Cheng, *Channel coordination under consignment and vendor-managed inventory in a distribution system,* Transportation Research Part E: Logistics and Transportation Review, vol. 46, no. 6, pp. 831–843, 2010.

Darwish, M. A., and O. M. Odah, *Vendor managed inventory model for single-vendor multi-retailer supply chains,* European Journal of Operational Research, vol. 204, no. 3, pp. 473–484, 2010.

Disney, S. M., and D. R. Towill, *Vendor-managed inventory and bullwhip reduction in a two-level supply chain,* International Journal of Operations & Production Management, vol. 23, no. 6, pp. 625–651, 2003.

Dong, Y., and K. Xu, *A supply chain model of vendor managed inventory,* Transportation Research Part E: Logistics and Transportation Review, vol. 38, no. 2, pp. 75–95, 2002.

Hariga, M., M. Gumus, A. Daghfous, and S. K. Goyal, *A vendor managed inventory model under contractual storage agreement,* Computers & Operations Research, vol. 40, no. 8, pp. 2138–2144, 2013.

Harris, F. W., *How many parts to make at once,* Factory: The Magazine of Management, vol. 10, no. 2, pp. 135–136, 1913.

Kim, B., and C. Park, *Coordinating decisions by supply chain partners in a vendor-managed inventory relationship,* Journal of Manufacturing Systems, vol. 29, no. 2, pp. 71–80, 2010.

Kristianto, Y., P. Helo, J. R. Jiao, and M. Sandhu, *Adaptive fuzzy vendor managed inventory control for mitigating the bullwhip effect in supply chains,* European Journal of Operational Research, vol. 216, no. 2, pp. 346–355, 2012.

Khajehnezhad, M., *Integrating a supply chain with vendor managed inventory and joint replenishment policies,* Proceedings of the International Conference on Industrial Engineering and Operations Management, vol. 8, no. 1, pp. 145–152, 2018.

Nachiappan, S. P., and N. Jawahar, *A genetic algorithm for optimal operating parameters of VMI system in a two-echelon supply chain,* European Journal of Operational Research, vol. 182, no. 3, pp. 1433–1452, 2007.

Nia, A. R., M. H. Far, and S. T. A. Niaki, *A fuzzy vendor managed inventory of multi-item economic order quantity model under shortage: An ant colony optimization algorithm,* International Journal of Production Economics, vol. 155, pp. 259–271, 2014.

Pasandideh, S. H. R., S. T. A. Niaki, and A. R. Nia, *A genetic algorithm for vendor managed inventory control system of multi-product multi-constraint economic order quantity model,* Expert Systems with Applications, vol. 38, no. 3, pp. 2708–2716, 2011.

Sadeghi, J., S. M. Mousavi, S. T. A. Niaki, and S. Sadeghi, *Optimizing a multi-vendor multi-retailer vendor managed inventory problem: Two tuned meta-heuristic algorithms,* Knowledge-Based Systems, vol. 50, pp. 159–170, 2013.

Sadeghi, J., S. Sadeghi, and S. T. A. Niaki, *A hybrid vendor managed inventory and redundancy allocation optimization problem in supply chain management: An NSGA-II with tuned parameters,* Computers & Operations Research, vol. 41, pp. 53–64, 2014.

Sajadifar, S. M., and B. Pourghannad, *An integrated supply chain model for the supply uncertainty problem,* Proceedings of the International Conference on Industrial Engineering and Operations Management, vol. 11, no. 1, pp. 211–218, 2010.

Waller, M. A., M. E. Johnson, and T. Davis, *Vendor-managed inventory in the retail supply chain,* Journal of Business Logistics, vol. 20, no. 1, pp. 183–203, 1999.

Wong, W. K., J. Qi, and S. Y. S. Leung, *Coordinating supply chains with sales rebate contracts and vendor-managed inventory,* International Journal of Production Economics, vol. 120, no. 1, pp. 151–161, 2009.

Xiao, T., and T. Xu, *Coordinating price and service level decisions for a supply chain with deteriorating item under vendor managed inventory,* International Journal of Production Economics, vol. 145, no. 2, pp. 743–752, 2013.

Yao, Y., P. T. Evers, and M. E. Dresner, *Supply chain integration in vendor-managed inventory,* Decision Support Systems, vol. 43, no. 2, pp. 663–674, 2007.

Appendix A: Summary of Notations

The following notations are used to formulate the VMI joint replenishment problem:

Indices and Sets:

- j: Index for buyers ($j = 1, 2, \ldots, N$)
- i: Index for products/items ($i = 1, 2, \ldots, M$)

Decision Variables:

- T: Common replenishment cycle time (years)
- Q_{ij}: Order quantity of product *i* for buyer *j*

Parameters:

- α_{ij}: Maximum price intercept (market potential) for product *i* at buyer *j*
- β_{ij}: Linear price sensitivity coefficient
- γ_{ij}: Nonlinear (quadratic) demand curvature coefficient
- $C_{p,i}$: Unit production cost for product *i*
- $C_{h,i}$: Unit inventory holding cost for product *i* per year
- A_j: Fixed ordering cost for buyer *j* per cycle
- B: Total available budget for inventory-related costs
- $P_{ij}(Q_{ij})$: Inverse demand function (Selling Price)

Biographies

Arvin Bahreini is a doctoral student in Operations and Business Analytics at the University of Oregon's Lundquist College of Business. He holds a Bachelor of Science in Electrical Engineering from Sharif University of Technology and a Master of Science in Electrical Engineering from the University of Rochester. With a foundation in engineering and strong analytical capabilities, Arvin's academic journey bridges technical problem-solving with strategic business decision-making. His research interests span supply chain optimization, risk management, and behavioral economics, with a growing focus on integrating machine learning into business analytics. Throughout his studies, he has engaged in a wide range of research projects—from demand modeling and trade analysis to medical imaging and renewable energy systems—demonstrating a deep commitment to interdisciplinary investigation and applied innovation. He has also been a teaching assistant in diverse subjects such as engineering mathematics, electrical circuits, and programming, reflecting his passion for both learning and pedagogy. With his broad academic foundation and cross-functional skills in programming, modeling, and analytics, he aspires to contribute to impactful, data-driven solutions in complex organizational systems.

Mohammad Hossein Jalali is an emerging researcher and quantitative analyst with a distinct academic profile that bridges engineering precision with financial strategy. He holds a Master of Science in Business Administration with a specialization in Finance and a Bachelor of Science in Electrical Engineering from Sharif University of Technology, where he also achieved a notable 25th rank in the National Universities Entrance Exam. With a robust foundation in quantitative methods, Mohammad Hossein's research interests lie at the intersection of Applied Econometrics, Behavioral and Experimental Economics, and Quantitative Risk Management. His academic journey is defined by a commitment to interdisciplinary investigation; for instance, his master's thesis utilized machine learning and econometric models to analyze operational performance in blockchain-based crowdfunding platforms. Professionally, Mohammad Hossein has applied his analytical skills as a Quantitative Researcher at the Persian Gulf Investment Bank, where he leveraged statistical modeling to identify market trends and improve portfolio performance. He also served as a Quantitative Researcher at the Risk Lab, developing open-source libraries in Julia and Python for financial analysis. Beyond research, he is deeply engaged in academia as a Teaching Assistant for courses such as Corporate Finance, Risk Management, and Advanced Programming. Technically proficient in Python, R, Julia, and MATLAB, Mohammad Hossein combines strong programming capabilities with deep economic insight. Driven by a passion for solving complex organizational problems, he aspires to contribute to data-driven solutions in the financial sector while pursuing advanced academic research in economics and decision sciences.