

# **A Sustainable Closed-loop Supply Chain Optimization Model for LPG Gas Cylinder Management**

**Jannatul Ferdous Mim and Md. Mohibul Islam**

Industrial & Production Engineering Department

Rajshahi University of Science and Technology

Rajshahi, Bangladesh

[jannatul.ferdous@ipe.ruet.ac.bd](mailto:jannatul.ferdous@ipe.ruet.ac.bd), [mohibul@ipe.ruet.ac.bd](mailto:mohibul@ipe.ruet.ac.bd)

## **Abstract**

Tremendous growth in the consumption of liquefied petroleum gas (LPG) has resulted in the growing demand to have an effective and sustainable management of refillable LPG cylinders. This research project formulates a complete closed-loop supply chain (CLSC) optimization model to help in the distribution of sustainable cylinder circulation among suppliers, plants, distributors, retailers, customers, and disposal facilities. The model incorporates forward distribution flows with reverse logistics, inspection, repair, filling, and disposal processes considering carbon tax based environmental costing. A deterministic formulation of a linear programming is proposed. The findings suggest that the model provides a sound analytic basis that can be used to make decisions regarding minimizing the cost, minimizing carbon, reverse-flow efficiency, and sustainable resource recovery. The model is the decision-support structure of future empirical research and application to industry.

## **Keywords**

LPG Cylinders, tax, CLSC, optimization, linear programming.

## **1. Introduction**

The world has become more inclined to cleaner household fuel, and this has increased the use of liquid petroleum gas (LPG) particularly in developing nations. The distribution and recovery of cylinders of LPG have been a complicated issue in Bangladesh where millions of LPG cylinders are in active circulation. The lifecycle of an LPG cylinder includes the manufacturing, filling, distribution, consumer use, returning, inspection, potential repair, refilling, and finally disposal. Any stage has inefficiencies or leakages which not only increase the cost of operation but are also a major safety, environmental, and reliability hazard.

Irrespective of the technological and regulatory interventions, there are still multiple challenges associated with LPG cylinder logistics: low empty cylinders return, unsafe or informal repairs, lack of uniform inspections, long routes, and high carbon emission rates due to prolonged transportation routes, as well as illegal storage or disposal by consumers.

The sustainable LPG supply chain requires a holistic approach that recognises these facts. In addition, international climate regulations have brought the concept of carbon tax, which requires industries to absorb any effects on the environment in their decision making. The LPG supply chain research that is already available does not often include environmental taxation, reverse logistics uncertainty, or closed-loop optimization at all supply chain echelons.

To fill these gaps, this research work generates a multi-echelon closed-loop supply chain optimization model with the consideration of the carbon taxation and inspection-based repair-disposal decision. The model is developed in a non-numerical way, which is used as a basis of future empirical processes.

## **1.1 Objectives**

To develop a closed-loop supply chain model aimed at minimizing the total logistics cost for LPG gas cylinder management in Bangladesh.

## **2. Literature Review**

The Closed-loop supply chain (CLSC) modelling has been a booming topic in use where industries are trying to mitigate waste, recover value and internalize environmental costs over product life cycles. Students like Guide and Van Wassenhove (2009) and Govindan, Soleimani and Kannan (2015) have shown that CLSC frameworks provide the opportunity to plan forward and reverse logistics in a coordinated manner based on mathematical optimization and systems modelling. These types of studies are usually directed towards the minimization of costs, other recycling efficiency and reduction of carbon. Nonetheless, the majority of CLSC models presuppose the same rates of returns and rather simplistic inspection or remanufacturing processes, which cannot be related to the high level of safety and regulatory requirements linked to LPG cylinders (Fleischmann et al., 2004). LPG cylinders are a product that demands special pressure tests and structural analysis, unlike other products that are modeled in the current CLSC literature. The characteristics of operation of LPG supply chain alone are very different compared to the normal supply chain. Research on the LPG distribution, including the works by Kabir et al. (2019) and Bercaru (2020) point to issues with variable demand and supply of the product, the absence of services in rural areas, bottlenecks in distribution, and strict safety standards. This research has focused much on transportation routing or last-mile delivery, commonly modeling cylinders as objects whose motion is linear, proceeding directly out of filling plants and to customers. Practically, LPG supply chains are circular: empty cylinders are given back to the customer, and they should be sent through the reverse system to be inspected, repaired, refilled, and redistributed. The available LPG supply chain literature has the tendency of viewing these flows in the sense of isolation of the circulation and therefore partial presentation of the reality of operations (Rahman and Karim, 2021).

The literature on reverse logistics provides a wider understanding of product returns, inspection, remanufacturing and waste management. The modelling of the return rates, inspection stages, and recovery decisions has been highlighted by researchers like Rogers and Tibben-Lembke (2001), Dekker et al. (2013), and Agrawal, Singh and Murtaza (2015) as being important. Nevertheless, these researches usually concern products such as electronics or automotive parts, which have rather high and predictable returns. Instead, LPG industry has tremendously uncertain returns and low returns percentages, as a result of cylinder storage illegally, loss, or even circulation in the secondary markets (Hossain & Rahman, 2020). Very little of the available literature regarding reverse logistics includes the concept of partial return behaviour, although its effect on inventory, the repair capacity planning, and the workload of safety inspections in LPGs is substantial. As the world becomes more concerned about the reduction of carbon and sustainability, environmental supply chain research has grown tremendously. As it is proven in many studies such as by Benjaafar et al. (2013), Chaabane et al. (2012), and Xia et al. (2020), the optimization of transportation paths, production, and recovery can change under the influence of carbon taxes or emission charges included in the supply chain model. The models demonstrate that it is possible to internalize in the optimization frameworks in terms of carbon pricing mechanisms and conversion of emissions factors. Nevertheless, although the process of transporting, filling, checking, and repairing LPG is carbon-intensive, the LPG supply chain literature has not paid the necessary attention to the introduction of carbon tax into optimization planning (Sultana and Rahman, 2022). LPG modelling frameworks cannot properly assess environmental trade-offs or assist in the development of carbon-conscious policy without consideration of emissions. Overall, the theoretical and practical insights on CLSC systems, LPG operations, reverse logistics, and environmental taxation, as presented in the literature, are quite important; however, considerable gaps still exist. The safety-critical nature of inspection and repair of LPG cylinders is rarely addressed in CLSC studies. LPG-specific studies are analytically rich but fail to give the overall mathematical formulations of both forward and reverse flows.

A general overview of the literature available makes it evident that there are several issues that are not clearly answered in the context of LPG-based supply chain modelling. Although the existing literature provides important information about efficient distribution, safety issues, and the optimization of costs, a study that would develop a closed-loop optimization model that would involve the entire lifecycle of LPG cylinder has yet to be developed (Kabir et al., 2019; Sultana and Rahman, 2022). The lack of such a model implies that existing frameworks fail to consider looping of cylinders revolving around the suppliers, filling plants, distributors, customers, and the reverse logistics channels. According to Govindan et al. (2015), to present realistic and implementable planning solutions, CLSC models should manifest the circular nature of product movement - something that has not been achieved by the current research on LPG. Another gap of importance is related to decision making based on inspection. The process of LPG cylinders

goes through strict inspection of safety where leakage, pressure, and structural integrity are addressed. Nevertheless, most of the optimization models in CLSC and LPG circles treat returned products as either reusable or recyclable, but they do not model repairable and disposable shares (Fleischmann et al., 2004). This exclusion does not allow realistic estimation of the repair workloads and disposal needs. The absence of inspection-based modelling is a deviation between the theoretical and practical assumptions in the industry. Another gap is based on unrealistic assumptions of product turnover. The studies conducted in the field of reverse logistics commonly presuppose complete or almost complete recovery of the used products (Rogers and Tibben-Lembke, 2001; Dekker et al., 2013). This is not true in the case of the LPG industry, where the cylinders are usually lost, kept in unsafe places, selling in the black market, or otherwise taken out of circulation (Hossain & Rahman, 2020). The unavailability of partial return modelling results in flawed representations of the availability of the cylinders and the requirements of inspections. Models do not have a return proportion parameter and thus they are unable to reflect on-ground conditions. Another significant weakness is environmental cost integration. Despite the introduction of the issue of carbon taxation into several kinds of supply chain models (Benjaafar et al., 2013; Chaabane et al., 2012), the peculiarities of the impact of cylinder filling, inspection, repair, and transportation on the environment have not been properly studied with LPG. With the ongoing rising of environmental regulations that are being observed globally, the models that ignore the cost of carbon emission are not very helpful when it comes to a policy assessment or the sustainable operation planning. As the pressure on environmental optimization frameworks increases, the LPG sector is still not fully covered by the strategies of environmental optimization. Lastly, most of the current LPG optimization models use either static or single-period modelling (Rahman and Karim, 2021). Nevertheless, the LPG cylinders are not at rest in the system but move dynamically at several periods and the inventories change with time. The nature of models without a multi-period perspective is unable to render the time dynamics of cylinders, elasticity of inspection loads, and inventory changes across the plants. Therefore, a multi-period, closed-loop model of supply chain with the incorporation of inspection results, partial returns, carbon taxation, and multi-tier material movement is evident. This gap is filled by the current paper which will come up with such a model to facilitate the management of LPG cylinders in a sustainable and economically viable way.

### 3. Methodology

This study develops a linear programming (LP) model for an integrated LPG closed-loop supply chain. The methodology focuses solely on model formulation and does not involve real data, computational experiments, or case-based validation (Figure 1).

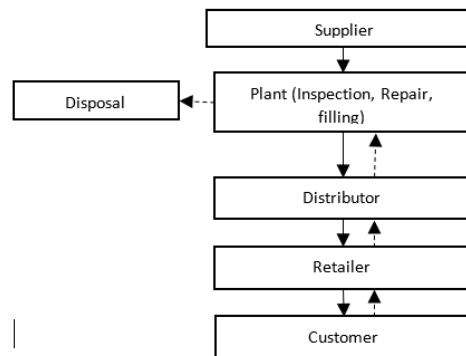


Figure 1. Flow of the stakeholders

#### Assumptions:

1. All the LPG cylinders that are taken into account in the model are assumed to be homogenous in size, nature as well as in handling needs.
2. The returned cylinders received at the plant are classified under predetermined proportions of quality wherein a proportion  $a$  is considered repairable and a proportion  $b$  is to be left to be discarded.
3. Only non-repairable and failed to pass inspection cylinders are disposed of in the facilities.
4. Plants take in the parts of inspection, repairing and filling of the returned cylinders and each of these processes has fixed operational costs and carbon emission levels.

5. The set of inspected cylinders is also assumed to be processed within the same period implying that the inspections, repair and filling activities, are not carried across periods.
6. Cost and carbon emission are supposed to be constant with respect to the inspection, filling, repair and transportation costs and are known beforehand.
7. External suppliers provide cylinders that are believed to be fit to be filled immediately hence do not need inspection on delivery.
8. The demand of customers is supposed to be known and deterministic in each time period during the planning period.
9. The retailers have to meet the demand at all costs, by giving out filled cylinders to their clients; unmet demand will not be afforded in the model.
10. Inventory-balance constraints permit plants, distributors and retailers to have inventories and carry over inventory quantities across periods.
11. The level of initial inventory at every facility is assumed to be known and defined at the start of the planning horizon.
12. There are no assumed material flows that are not operationally feasible in each time period.
13. A homogenous tax on each unit of emission of carbon is imposed on all the activities that produce carbon during the entire supply chain and the environmental effects are internalized by converting all the CO<sub>2</sub> emission to monetary costs in the objective function.
14. Flow, inventory and processing variables are all constrained to be non-negative.
15. The model is developed as a linear programming (LP) model, whereby all the relations between variables are said to be linear.
16. The model is conducted under deterministic circumstances where there is no uncertainty in variables like demand, costs, distances and emission factors.
17. There are no set capacity restrictions on inspection, repair or filling operations at the plant unless it is otherwise reintroduced.
18. Cylinders sent to disposal are deemed to be permanently eliminated out of the system, and they do not find their way back into the closed-loop supply chain.
19. It is assumed that a proportion of all cylinders  $\gamma$  shipped to the customers will go back via the reverse logistics network, and the rest  $1 - \gamma$  is lost, retention, or illegally disposed.

### Indices

Suppliers	:	Set of Suppliers $s=1, 2, \dots, S$
Plant	:	Set of Plants $p=1, 2, \dots, P$
Distributors	:	Set of Distributors $k=1, 2, \dots, K$
Retailers	:	Set of Retailers $r=1, 2, \dots, R$
Disposal	:	Set of Disposal $l=1, 2, \dots, L$
Customers	:	Set of Customers $c=1, 2, \dots, C$
Time	:	Set of Period $t=1, 2, \dots, T$

### Decision Variables

$X_{spt}$	:	Quantity transferred from supplier $s$ to plant $p$ <b>during period <math>t</math></b>
$X_{pkt}$	:	Quantity transferred from plant $p$ to Distributor $k$ <b>during period <math>t</math></b>
$X_{krt}$	:	Quantity transferred from Distributor $k$ to Retailer $r$ <b>during period <math>t</math></b>
$X_{kpt}$	:	Quantity transferred from Distributor $k$ to plant $p$ <b>during period <math>t</math></b>
$X_{rkt}$	:	Quantity transferred from Retailer $r$ Distributor $k$ to <b>during period <math>t</math></b>
$X_{plt}$	:	Quantity transferred from plant $p$ to Disposal $l$ <b>during period <math>t</math></b>

### Parameters

$D_{sp}$	:	Distance between supplier $s$ and plant $p$ in km
$D_{pk}$	:	Distance between plant $p$ and Distributor $k$ in km
$D_{kr}$	:	Distance between Distributor $k$ and Retailer $r$ in km
$D_{pl}$	:	Distance between plant $p$ and Landfill $l$ in km
$T_{sp}$	:	Unit Transportation cost from supplier $s$ to plant $p$ for unit distance
$T_{pk}$	:	Unit Transportation cost from plant $p$ to Distributor $k$ for unit distance

$T_{kr}$	: Unit Transportation cost from Distributor k to Retailer r for unit distance
$T_{pl}$	: Unit Transportation cost from plant p to disposal l for unit distance
$T_{rk}$	: Unit Transportation cost from Retailer r to Distributor k for unit distance
$T_{kp}$	: Unit Transportation cost from Distributor k to plant p for unit distance
$\beta$	: Proportion of cylinders that are disposed after inspection
$\alpha$	: Proportion of cylinders that are repaired after inspection
$\gamma$	: Proportion of cylinders that are returned to the retailer from customer
$CI$	: Unit Cost of inspection
$CF$	: Unit Cost of filling
$CR$	: Unit Cost of repairing
$Y_{ct}$	: Demand of Customer c at time t
$t\chi$	: Tax value per unit of carbon emission
$E_{sp}$	: Carbon emission for travelling a unit from supplier s to plant p for unit distance
$E_{pk}$	: Carbon emission for travelling a unit plant p to Distributor k for unit distance
$E_{kr}$	: Carbon emission for travelling a unit Distributor k to Retailer r for unit distance
$E_{rk}$	: Carbon emission for travelling a unit from Retailer r to Distributor k for unit distance
$E_{kp}$	: Carbon emission for travelling a unit Distributor k to plant p for unit distance
$E_{pl}$	: Carbon emission for travelling a unit from plant p to disposal l for unit distance
$E_i$	: Carbon emission for inspecting a unit
$E_f$	: Carbon emission for filling a unit
$E_r$	: Carbon emission for repairing a unit
$I_{rt-1}$	: Inventory at Retailer r during time t-1
$I_{rt}$	: Inventory at Retailer r during time t
$I_{kt-1}$	: Inventory at Distributor k during time t-1
$I_{kt}$	: Inventory at Distributor k during time t
$I_{pt-1}$	: Inventory at plant p during time t-1
$I_{pt}$	: Inventory at plant p during time t

### Objective Function:

**Minimization =Transportation Cost+ Processing Cost+ Carbon emission tax cost**

$$\text{Transportation Cost} = \sum_s^S \sum_p^P \sum_t^T T_{sp} * X_{spt} + \sum_p^P \sum_k^K \sum_t^T T_{pk} * X_{pkt} + \sum_k^K \sum_r^R \sum_t^T T_{kr} * X_{krt} + \sum_r^R \sum_k^K \sum_t^T T_{rk} * X_{rkt} + \sum_k^K \sum_p^P \sum_t^T T_{kp} * X_{kpt} + \sum_k^K \sum_p^P \sum_t^T T_{pl} * X_{kpt} * \beta$$

$$\text{Processing Cost} = \sum_k^K \sum_p^P \sum_t^T X_{kpt} * CI + \sum_k^K \sum_p^P \sum_t^T X_{kpt} * (1 - \beta) * CF + \sum_s^S \sum_p^P \sum_t^T X_{spt} * CF + \sum_k^K \sum_p^P \sum_t^T X_{kpt} * \alpha * CR$$

$$\text{Carbon emission tax cost} = \sum_s^S \sum_p^P \sum_t^T E_{sp} * X_{spt} * D_{sp} * t\chi + \sum_p^P \sum_k^K \sum_t^T E_{pk} * X_{pkt} * D_{pk} * t\chi + \sum_k^K \sum_r^R \sum_t^T E_{kr} * X_{krt} * D_{kr} * t\chi + \sum_r^R \sum_k^K \sum_t^T E_{rk} * X_{rkt} * D_{kr} * t\chi + \sum_k^K \sum_p^P \sum_t^T E_{kp} * X_{kpt} * D_{kp} * t\chi + \sum_k^K \sum_p^P \sum_t^T E_{pl} * X_{kpt} * \beta * D_{pl} * t\chi + \sum_k^K \sum_p^P \sum_t^T X_{kpt} * E_i * t\chi + \sum_k^K \sum_p^P \sum_t^T X_{kpt} * (1 - \beta) * E_f * t\chi + \sum_s^S \sum_p^P \sum_t^T X_{spt} * E_f * t\chi + \sum_k^K \sum_p^P \sum_t^T X_{kpt} * \alpha * E_r * t\chi$$

### Description of the Objective Function:

$$\sum_s^S \sum_p^P \sum_t^T T_{sp} * X_{spt} \quad 1$$

The calculation of transportation cost between the supplier and plant involves the unit transportation cost between each supplier to each plant added to the quantity of each supplier to each plant in each time period, added up over all of the suppliers, plants, and periods.

$$\sum_p^P \sum_k^K \sum_t^T T_{pk} * X_{pkt} \quad 2$$

Transportation cost between plant and distributor is computed as the product of unit cost of transportation between each of the plants and each of the distributors and the quantity that will be transported at each time period, added over all the plants, distributors and time.

$$\sum_k^K \sum_r^R \sum_t^T T_{kr} * X_{krt} \quad 3$$

Transportation cost between distributor and retailer is computed as the product of unit transportation cost of every distributor to every retailer and the quantity of goods being transported in a given time period added up across all distributors, retailers and periods.

$$\sum_r^R \sum_k^K \sum_t^T T_{rk} * X_{rkt} \quad 4$$

The transportation cost between the retailers and the distributors in the reverse logistics flow is computed as the product of unit transportation cost between each retailer and each distributor and the number of returned cylinders transported at each time period, that is added up across all retailers, distributors, and time periods.

$$\sum_k^K \sum_p^P \sum_t^T T_{kp} * X_{kpt} \quad 5$$

The cost of transportation between the distributor and the plant in the reverse logistics is determined as the product of unit transportation cost between every distributor and every plant and the quantity of products recoiled in each time period, summed along all the distributors, all the plants and all the time periods.

$$\sum_k^K \sum_p^P \sum_t^T T_{pl} * X_{kpt} * \beta \quad 6$$

The calculation of the cost of transportation between the plant and the disposal center is the product of the unit transportation cost of the unit transportation cost of each plant to each logistics center and the quantity shipped in each period which is the rate of  $\beta$  about the amount shipped by the distributor to the plant, the sum of the calculations made in all plants, logistics centers, and time periods.

$$\sum_k^K \sum_p^P \sum_t^T X_{kpt} * CI \quad 7$$

The cost of inspection equalizes to; (unit inspection cost) x (total number of cylinders received by all the distributors in all the plants in a time period) and adding this value to all the distributors, all the plants and all time periods.

$$\sum_k^K \sum_p^P \sum_t^T X_{kpt} * (1 - \beta) * CF + \sum_s^S \sum_p^P \sum_t^T X_{spt} * CF \quad 8$$

The unit filling cost is multiplied by the total number of cylinders received by the distributor that passes through the examination in conjunction with the cylinders received by the supplier in calculating the cost of filing. In each time period the number of inspected multiplied with the inspection cost per cylinder is obtained.

$$\sum_k^K \sum_p^P \sum_t^T X_{kpt} * \alpha * CR \quad 9$$

To determine the cost of repair, the summation of the unit cost to repair at the plant, multiplied by the number of cylinders incurred by the distributors, which failed the inspection, divided by the number of plants and time periods.

$$\sum_s^S \sum_p^P \sum_t^T E_{sp} * X_{spt} * D_{sp} * tx \quad 10$$

Transportation cost of carbon tax comprises of the product of the number of cylinders transported to the plant multiplied by the CO2 emission per unit of the transported cylinder per unit distance, distance between the supplier and the plant and the cost of the carbon tax per unit amount of CO2. This is accumulated across all time, plants and suppliers.

$$\sum_p^P \sum_k^K \sum_t^T E_{pk} * X_{pkt} * D_{pk} * tx \quad 11$$

The transportation cost of the carbon tax is determined through the following formula whereby, the number of cylinders transported is multiplied by the CO2 emission per unit distance, a unit distance between the plant and distributor and the actual amount of the carbon tax per unit of CO2. This is being added together across all plants, distributors and periods.

$$\sum_k^K \sum_r^R \sum_t^T E_{kr} * X_{krt} * D_{kr} * tx \quad 12$$

The carbon tax cost for transportation from distributor to retailer is calculated by multiplying the quantity of cylinders transported by the CO<sub>2</sub> emission per cylinder per unit distance, the distance between the distributor and retailer, and the carbon tax per unit of CO<sub>2</sub>. This is summed over all distributors, retailers, and time periods.

$$\sum_r^R \sum_k^K \sum_t^T E_{rk} * X_{rkt} * D_{kr} * tx \quad 13$$

The cost of the carbon tax that is applied in the reverse logistics between distributor and retailer is determined by multiplying the combined number of the number of the cylinders returned by the CO<sub>2</sub> emission per cylinder per unit distance, the distance between the retailer and the distributor and the carbon tax per unit of the CO<sub>2</sub>. This is added together across all the retailers, distributors and periods.

$$\sum_k^K \sum_p^P \sum_t^T E_{kp} * X_{kpt} * D_{kp} * tx \quad 14$$

Reverse logistical carbon tax price between the distributor and the plant is computed by multiple of the number of returned cylinders by the CO<sub>2</sub> emission per cylinder per unit distance, the space among the distributor and the plants and the carbon tax per unit of CO<sub>2</sub>. This is accumulated across all the distributors, plants and time periods.

$$\sum_k^K \sum_p^P \sum_t^T E_{kp} * X_{kpt} * \beta * D_{pl} * tx \quad 15$$

The calculation of the cost of carbon tax on transportation cost between the plant and the disposal location is done by multiplying the number of the cylinders disposed of by the distance between the plant and the disposal facility and the carbon tax per unit distance of the CO<sub>2</sub>. This is added up across all plants, disposal sites and time.

$$\sum_k^K \sum_p^P \sum_t^T X_{kpt} * E_i * tx \quad 16$$

The price of the carbon tax that is incurred because of the inspection is calculated by multiplying the number of cylinders that are inspected with the CO<sub>2</sub> emission per cylinder as a result of the inspection process and the carbon tax per unit of CO<sub>2</sub>.

$$\sum_k^K \sum_p^P \sum_t^T X_{kpt} * (1 - \beta) * E_f * tx + \sum_s^S \sum_p^P \sum_t^T X_{spt} * E_f * tx \quad 17$$

The cost of carbon tax to fill the cylinders is determined by multiplying the quantity of the cylinders filled by the amount of CO<sub>2</sub> per cylinder being filled during the filling process and the unit price of the carbon tax.

$$\sum_k^K \sum_p^P \sum_t^T X_{kpt} * \alpha * E_r * tx \quad 18$$

The cost of carbon tax when repairing is calculated by multiplying the number of cylinders repaired by the CO<sub>2</sub> emission per cylinder as a result of the repair process and the price of carbon tax per unit of CO<sub>2</sub>.

**Constraints:**

**Customer Demand Fulfillment:**

$$\sum_r^R X_{rct} \geq Y_{ct} \quad 19$$

This limitation will make sure that the number of cylinders supplied to each retailer will be as much as or more than the customer demand during that time period.

**Retailer flow balance:**

$$\sum_k^K X_{krt} + I_{rt-1} = \sum_c^C X_{rct} + \sum_k^K X_{rkt} + I_{rt} \quad 20$$

The retailer flow-balance constraint is a constraint that keeps the retailers in possession of sufficient units that are delivered by distributors or in stock in the initial inventory levels so that the retailers can meet customer demand and pass any empty cyclone returned by any customer upstream in the opposite flow. It also forecasts the ending inventory of the retailer in future periods. Since the customer shipment is directly presented in this balance equation, the retailer constraint entails the location where customer demand is a tangible demand to the system. This limit guarantees that a retailer does not deliver more fill cylinders to customers than it gets therefore keeping the forward and reverse interchange constant between customers and distributors.

**Distributor flow balance:**

$$\sum_p^P X_{pkt} + I_{kt-1} = \sum_r^R X_{krt} + \sum_p^P X_{kpt} + I_{kt} \quad 21$$

The distributor flow-balance constraint connects the plant and retailer level as it sets a minimum of the total number of units shipped by the plants (when combined with initial inventory) to meet all outbound shipments delivered to retailers, all backward shipments delivered back to the plants, and final inventory. This is a constraint that makes the distribution stage physically feasible and excludes the possibility of the distribution stage supplying more cylinders to retailers than they actually have.

**Plant Flow Balance:**

$$\sum_s^S X_{spt} + \sum_k^K X_{kpt} + I_{pt-1} = \sum_k^K X_{pkt} + \sum_l^L X_{plt} + I_{pt} \quad 22$$

The plant flow-balance constraint assures that every plant has enough available supply- consisting of fresh production and purchased units and recaptured empty cylinders that have undergone inspection / repair- and meet all the shipments forwarded to distributors during a specific period. It also records units that have been taken out of the system (e.g., disposal) and reinvents stock on hand in the plants. Simply stated, this limit would ensure that the outbound shipments off the plant may not surpass the total amount of cylinders physically present at the start of the period plus the number of units added throughout the period. It maintains continuity of material and does not allow the demand of the model to be fulfilled by non-existent supply.

**Reverse Logistics Return Proportion Constraint:**

$$\sum_k^K X_{rkt} = \gamma * \sum_k^K X_{pkt} \quad 23$$

This limitation will make sure that only a percentage  $\gamma$  of the number of cylinders shipped to the customers will re-enter the reverse logistics system. The rest of the proportion  $(1-\gamma)$  reflects on cylinders that are stored, kept by customers, lost, or destroyed unlawfully and, therefore, does not rent the supply chain. The inclusion of  $\gamma$  in the model offers a less idealized view of a closed loop system where not every distributed cylinder is recovered.

**4. Result**

The closed-loop supply chain model that was developed was tested on a small and realistically simulated numerical scenario to demonstrate the behavior of the system under normal operating conditions. In the three-period planning horizon, there was an overall demand of 585 cylinders in the total customer demand, and this is divided into two retailers. The model was able to balance both the forward and reverse flows and consider the inspection, repair, disposal, filling, transportation and the activities related to the carbon-tax. Period 1 recorded a total demand of 180 units, and this was met by combination of repaired cylinders and the available stock in the plant. The  $\gamma$  return rate gave out returns that were determined and led to 108 units turning back to the plant. Based on a proportion, 86 units were rated as repairable after inspection and 21 units were disposed of. This enabled the plant to end the period with 84 units in stock. The average expenses incurred during this time and transportation and tax on carbon emissions were BDT 68800.

Period 2 saw the demand rising to 210 units, yet the system was able to meet all the customer demands without requiring the supplier to bring new supplies into the system. It was found that more returns (126 units) were brought about by the high demand, hence 93 repairable units and 25 that were thrown away. The inventory of the plant also declined slightly to 78 units at the end of the second period, which represents a steady consumption of the previous stock. The total cost during this period was estimated at BDT 83259 as the major factors that impacted it were increased activity through transportation and increased burden through more inspection and repair works on increased returns.

Period 3 had 195 units needed to satisfy the demand of the customer. During this time, the available stock of repaired and inventory in the plant was not adequate and the system needed 102 new cylinders with the supplier, the only time there would be a need of external supply. There were 117 units of returns of which 93 units were, the repairable category and 23 units, the disposal category. Although new cylinders had been introduced, the total cost was in close correlation with the previous times though a bit higher as there had been extra transportation and filling works concerning the suppliers. The total cost for Period 3 was 87282.

The total cost of operating the closed-loop system including the effects of carbon-tax over the entire three-period horizon was 2,39346. The findings indicate that the model is an effective way of showing the interaction between the repair capacity, the supply of new cylinders, the disposal activities and the reverse-logistics returns. The trend of growing dependency on repaired cylinders during the initial years reflects the potential cost-saving opportunities of the reverse logistics, whereas the eventual necessity to introduce new supply in Period 3 informs about the impact of the cumulative effect of disposal ratios and the non-returned cylinders. On the whole, the findings affirm that the closed-loop model is realistic and assists in the in-depth evaluation of the trade-offs in the functioning of the LPG cylinders.

## **6. Discussion**

The quantitative example shows that the suggested closed-loop supply chain model is a useful tool to capture the dynamics of the LPG cylinder circulation in the course of several periods. The findings reveal a number of valuable managerial and sustainability implications. The first one, the model indicates that there is a significant dependence on repaired cylinders in the first periods that greatly lower the necessity to have new supply. This observation implies that the cost of the system can be significantly reduced by enhancing the return logistics and providing timely check-up and maintenance. The fact that fixed units made a high contribution during Period 1 and Period 2 proves that reverse logistic is not merely a necessary way of saving the environment but also an economical option of operating LPG. Second, the findings indicate that return rates ( $\gamma$ ) are very important in influencing system performance on a total basis. This trend is indicative of the real-world operational issues in LPG distribution systems where loss of the cylinder, customer hoarding and informal market leakage serve to undermine the circularity of the supply-chain. Enhancing customer incentives, deposit-refund programs, or other digital tracking technologies would make the process of increasing return rates more independent of new cylinder purchases, and would enhance economic and environmental performance. Third, the proportion of disposal ( $\beta$ ) has a cumulative effect on the inventory levels. Though the per-period disposal rate is not big, over time, it effects with time and eventually causes depletion of inventory at the plant. This finding underscores the fact that reverse logistics is not the sole approach towards long term sustainability but also cylinder life and repair efficacy. Disposal can be reduced by a better level of inspection, better repair technologies or by more stringent quality control that will prolong the useful life of cylinders and postpone the purchase of new ones at a high price. Carbon tax is also a significant cost to the overall system. In this case, the unit rates of emission are relatively low, but since the emissions are accumulated in repeated transportation, inspection, and processing processes, a carbon tax would be imposed on nontrivial scales. It implies that the transportation path might be optimized to benefit LPG operators, or they can switch to cleaner technologies within logistics or place facilities in the strategic locations to minimize the emissions. The model allows quantification of such trade-offs between sustainability of operations and environmental responsibility. Altogether, the discussion shows that the given model is rather effective to reveal the operational bottlenecks, sustainability risks, and main cost drivers in the LPG supply chain. It will be a structured framework that can help decision-makers to examine the effect that the returns rate, repair rate, disposal rate, and environmental policy will have on system performance in the long term.

## **7. Conclusion**

This paper has constructed a detailed closed-loop supply chain optimization framework specific to the management of the LPG cylinders. In contrast to traditional models, which pay much attention to the forward logistics, the suggested framework is the one that incorporates the whole lifecycle of the cylinder, comprising the reverse logistics, inspection, repair, filling, disposal, and carbon-emission taxation. The model provides a powerful and realistic model of the real LPG distribution systems by including some realistic operational characteristics, including partial returns, repairable, non-repairable ratios, and multi-period inventory dynamics. The quantitative representation proves the fact that the model is logically sound and yields significant information to managers. The findings indicate that repaired cylinders can significantly lower the need to have a new supply, and the level of disposal and non-return depletes inventory of system over time. Also, the carbon taxation allows noting the economic dimension of the environmentally oriented policies, and proves the necessity of using emission-conscious operational planning. These results suggest that the

operational costs of the company and its environmental impact could be minimized at the same time as they allowed enhancing the efficiency of the reverse logistics, increasing the returns, and minimizing disposal. Overall, the suggested model can be a useful tool of analysis to LPG operators, policymakers, and sustainability planners in their endeavors to balance between economic performance and environmental responsibility. Though the current research is based on formulating a model and a representative numerical case, future research may build up on this work by adding uncertainty and stochasticity, or large-scale optimization with state-of-the-art solutions. These extensions would contribute to the increased applicability of the model and make informed decisions on a safer, more sustainable LPG supply chain and more cost-effective.

## References

- Agrawal, S., Singh, R. K., & Murtaza, Q., *A literature review and future perspectives on reverse logistics*, Resources, Conservation & Recycling, 97, pp. 76–92, 2015, doi: 10.1016/j.resconrec.2015.02.009.
- Benjaafar, S., Li, Y., & Daskin, M., *Carbon footprint and the management of supply chains: Insights from simple models*, IEEE Transactions on Automation Science and Engineering, 10(1), pp. 99–116, 2013, doi: 10.1109/TASE.2012.2203304.
- Bercaru, S., *Optimization challenges in LPG distribution systems: A review*, Energy Policy Journal, 13(4), pp. 42–55, 2020.
- Chaabane, A., Ramudhin, A., & Paquet, M., *Design of sustainable supply chains under the emission trading scheme*, International Journal of Production Economics, 135(1), pp. 37–49, 2012, doi: 10.1016/j.ijpe.2010.10.025.
- Dekker, R., Fleischmann, M., Inderfurth, K., & Van Wassenhove, L. N., *Reverse logistics: Quantitative models for closed-loop supply chains*, Springer, 2013, doi: 10.1007/978-3-540-24803-3.
- Fleischmann, M., Bloemhof-Ruwaard, J., Dekker, R., Van der Laan, E., Van Nunen, J., & Van Wassenhove, L. N., *Quantitative models for reverse logistics: A review*, European Journal of Operational Research, 150(1), pp. 1–27, 2004, doi: 10.1016/S0377-2217(02)00792-0.
- Govindan, K., & Hasanagic, M., *A systematic review of sustainable supply chain management in emerging economies*, Transportation Research Part E: Logistics and Transportation Review, 117, pp. 348–367, 2018, doi: 10.1016/j.tre.2018.08.001.
- Govindan, K., Soleimani, H., & Kannan, D., *Reverse logistics and closed-loop supply chain: A comprehensive review*, Transportation Research Part E: Logistics and Transportation Review, 77, pp. 1–28, 2015, doi: 10.1016/j.tre.2015.02.001.
- Guide, V. D. R., & Van Wassenhove, L. N., *The evolution of closed-loop supply chain research*, Operations Research, 57(1), pp. 10–18, 2009, doi: 10.1287/opre.1080.0628.
- Hossain, A., & Rahman, M., *Challenges of LPG cylinder return and informal market practices in developing economies*, Journal of Energy Logistics, 6(2), pp. 55–68, 2020.
- Kabir, M., Chowdhury, S., & Rahman, M., *Operational assessment of LPG supply chain distribution in Bangladesh*, Asian Journal of Energy Research, 3(1), pp. 44–58, 2019.
- Rahman, S., & Karim, M., *Optimization of LPG distribution networks: Issues, constraints, and opportunities*, International Journal of Energy Economics and Policy, 11(5), pp. 230–239, 2021, doi: 10.32479/ijeeep.11552.
- Rogers, D. S., & Tibben-Lembke, R. S., *An examination of reverse logistics practices*, Journal of Business Logistics, 22(2), pp. 129–148, 2001, doi: 10.1002/j.2158-1592.2001.tb00007.x.
- Sultana, N., & Rahman, F., *Environmental implications of LPG supply chain operations in South Asia*, Energy & Environment, 33(7), pp. 1298–1312, 2022, doi: 10.1177/0958305X211046789.
- Xia, X., Govindan, K., & Zhu, Q., *Carbon emissions reduction strategies using carbon tax and carbon trading regulation*, Journal of Cleaner Production, 242, 118436, 2020, doi: 10.1016/j.jclepro.2019.118436.

## Biographies

**Jannatul Ferdous Mim** currently serving as a lecturer in Department of Industrial and Production Engineering in Rajshahi University of Engineering and Technology. She is pursuing her MSc. in Industrial & Production Engineering from Rajshahi University of Engineering & Technology, Bangladesh. She also completed her BSc in Industrial & Production Engineering from Rajshahi University of Engineering & Technology. She started her professional career as a lecturer in Department of Industrial and Production Engineering in National Institute of Textile Engineering and Research. Jannatul is utilizing her expertise to serve several non-profit organizations. She possesses an unwavering passion for research and a remarkable track record in waste management, aggregate production planning, supply chain management, quality control, and related fields.

**Dr. Md. Mohibul Islam** has completed his Ph.D. degree (2023) in the field of Supply Chain and Logistics Management from Nagoya Institute of Technology (NITech), Japan. He was a MEXT scholar from 2019-2023. He has been awarded the President Publication Award 2022 for his Ph.D. research publication from NITech, Japan. His research theme focuses on the development of mathematical models and algorithms to solve industrial problems, mainly related to the fields of Supply Chain and Logistics Management. He is currently working on developing various logistics model considering an unsteady demand and supply environment. He is also interested in developing different © IEOM Society International 573 Proceedings of the International Conference on Industrial Engineering and Operations Management, integrated multi-criteria group decision-making (MCGDM) models using Advanced Fuzzy Concept to solve decision management problems in real-world industrial settings. He is the author of about 26 International Conference papers and journal articles. Besides, he is a member of the Japan Society of Mechanical Engineers (JSME), Life member of Japanese Universities Alumni Association in Bangladesh (JUAAB-0973) and member of the Institute of Engineers in Bangladesh (IEB). In the earlier, he has also completed his M. Sc. Engineering (2015) and B. Sc. Engineering (2010) degrees in the field of Industrial & Production Engineering (IPE) at Rajshahi University of Engineering & Technology (RUET), Bangladesh. He has been working as a Professor in the Department of IPE at RUET.