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A Hybrid Learning-Interference Model for the Dual-Resource Constrained Flexible Job Shop Scheduling Problem

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Abstract

In Dual-Resource Constrained Flexible Job Shop Scheduling (DRCFJSP), the common assumption of fixed processing times overlooks the significant impact of worker experience on productivity. This paper introduces a hybrid learning–interference model that replaces simple repetition counts with a more realistic effective experience measure. This measure is novel in several key aspects: it couples position- and duration-based learning, weights experience transfer using both task and machine similarity to capture the human-machine-task fit, models skill decay as a continuous interference process, and incorporates worker heterogeneity. By embedding these dynamics directly into the time formulation, our model provides a theoretically more accurate estimation of processing times. Simulation results indicate that this increased fidelity leads to more effective and robust scheduling solutions for human-centric manufacturing environments.

Keywords
Dual-Resource Constrained Flexible Job Shop Scheduling (DRCFJSP), Learning Effect, Interference Effect, Task Similarity, Combining Position and Sum of Processing Time-Based Learning Effect (PST-LE).

1. Introduction
Most scheduling studies still treat processing times as fixed inputs; even when learning is considered, it is typically modeled along a single axis—either position-based or cumulative-time–based. In DRCFJSP settings, this misses three realities: experience must transfer across frequent task switches, its usability varies with the machine context, and its influence decays continuously over time. The consequence is under-modeling of human factors, biased time estimates, and weaker downstream optimization.

This paper targets the following gaps: (1) the lack of a learning mechanism that couples position-based and cumulative-time effects; (2) limited treatment of similarity that stops at job/operation similarity and omits machine similarity, which is necessary to quantify human–machine–task transfer and fit; (3) reliance on "forgetting as interruption" while overlooking continuous, interference-driven decay; and (4) insufficient integration of worker heterogeneity in skill levels and learning rates into time estimation. To close these gaps, we propose a hybrid learning–interference model for DRCFJSP and embed it directly into the processing-time formulation to provide higher-fidelity estimations for simulation-based optimization.

Problem Statements:

- Static-time misspecification: Processing time is often treated as a fixed input, ignoring its data-generating process in manual operations where time evolves with accumulated experience—leading to plan–shop floor mismatch.

- Uncoupled learning dimensions: Existing models typically adopt either position-based or cumulative-time learning in isolation, lacking a unified mechanism to explain efficiency evolution under frequent task switching.
- Incomplete similarity modeling: Prior work focuses on part/operation similarity while overlooking machine similarity and the human–machine–task triadic fit, yielding under-identified estimates of transferable experience.
- Mischaracterized decay dynamics: Experience attenuation is commonly modeled as interruption-driven, discrete "forgetting," ignoring continuous, context-dependent interference that governs time-based decay and weakens predictive robustness.
- Omitted worker heterogeneity: Differences in worker skill levels and learning rates are not systematically embedded in processing-time models, limiting the fidelity and usefulness of human-centric scheduling.

1.1 Objectives

- ➢ Develop a coupled learning formulation: integrate position-based and cumulative-time learning under a DeJong-style plateau to represent processing time.
- ➢ Define an effective-experience measure: replace repetition counts with a transferable experience metric weighted by operation- and machine-level similarity (human–machine–task fit).
- ➢ Model continuous interference: capture experience attenuation via time-continuous decay with contextual interference.
- ➢ Encode worker heterogeneity: incorporate differences in skill levels and learning rates into processing-time estimation.
- ➢ Embed in DRCFJSP: Integrate the proposed hybrid learning–interference model into DRCFJSP processing-time computation to produce higher-fidelity time estimations and enable subsequent simulation-based scheduling and evaluation.

2. Literature Review

Transitioning from fixed to dynamic processing times is widely regarded as a decisive step toward more realistic scheduling models. In labor-intensive settings, worker productivity typically improves with repeated exposure to similar tasks, a well-documented phenomenon known as the learning effect. To quantify this effect, the literature has developed a range of formulations. Early reviews, exemplified by Azzouz et al. (2018), organized these formulations into two principal families—position-based and sum-of-processing-time-based—and identified substantial gaps in complex environments such as the Flexible Job Shop (FJSP). Building on this foundation, Paredes-Astudillo et al. (2022) refined the taxonomy into position-based learning (P-LE), sum-of-processing-time learning (ST-LE), hybrid position–sum learning (PST-LE), and DeJong-type learning (DJ-LE), the last of which explicitly incorporates an incompressibility or plateau component.

In the Flexible Job Shop (FJSP) literature, Tayebi Araghi et al. (2014) are among the earliest to incorporate learning, adopting Biskup's (1999) position-based model in a setting where learning acts only on sequence-dependent setup times (SDST), while the processing times themselves are described by a linear time-dependent deterioration function. Subsequently, Renna (2019) studied dynamic FJSP, combining Wright's (1936) learning curve with the Carlson and Rowe (1976) forgetting curve within a multi-agent framework to enable context-responsive, real-time updates. For green FJSP (GFJSP), Li and Chen (2023) explicitly used the DeJong position-based model, leveraging its incompressibility (plateau) component to bound the time lower limit and quantify impacts on makespan and carbon emissions. In dual-resource constrained FJSP (DRCFJSP), Wu et al. (2018) employed a truncated sum-of-processing-time learning model (ST-LE), imposing an upper bound to reflect realistic limits on efficiency gains. There are also approaches that do not rely directly on learning curves; for example, Du et al. (2025) in flexible assembly job shops (FAJSP) represent learning as a static skill-proficiency coefficient that adjusts baseline processing times.

To move beyond single-mechanism learning, Cheng et al. (2008) introduced a hybrid formulation that multiplies a sum-of-processing-time–based factor with a position-based factor, allowing both learning mechanisms to act concurrently. In a single-machine setting, Lu et al. (2015) proposed a more elaborate hybrid that combines an exponential factor of the sum of log processing times with a position-based term, and imposes a truncation to enforce a practical lower bound on learning. Collectively, these studies exemplify a "product + truncation" strategy that integrates multiple learning drivers to improve robustness across operating conditions; however, systematic deployment and validation in FJSP/DRCFJSP contexts remain limited.

As attention to experience transfer has grown, studies have begun to incorporate similarity into learning measures. Lou et al. (2022) were early to introduce adjacent-task similarity within a learning–forgetting framework, enabling experience accumulation from tasks that are similar but not identical, and they incorporated the DeJong plateau to guarantee a realistic lower bound on time. In green FJSP, (Peng et al., 2022) made a substantive modification to the DeJong model by replacing the simple repetition count r with a six-dimensional composite similarity aggregate and by applying truncation to impose a saturation limit on learning, which better reflects small-lot, high-mix environments. Going further, Cai et al. (2024) proposed an integrated learning–forgetting–fatigue–recovery (LFFRM) framework for FJSP. The learning component is sum-of-processing-time–based and uses a weighted composite similarity.

In the area of forgetting and skill dynamics, Han and Gong (2025) couple DeJong learning with the Carlson & Rowe forgetting model, not to recompute processing time directly, but to dynamically update a worker's proficiency state, thereby indirectly influencing efficiency and time estimation. Xu et al. (2025) advance this line with a hybrid learning–forgetting formulation: on the learning side, they incorporate both position effects and worker-specific cumulative processing time; on the forgetting side, they employ a full-matrix similarity mechanism, under which the similarity between the current task and all other task types jointly determines the forgetting rate. They also impose upper and lower bounds on proficiency/learning intensity to preserve realistic skill dynamics. Taken together, these studies indicate that forgetting is not merely a discrete, interruption-time phenomenon, but is coupled with continuous, time-based decay and context-dependent (similarity-driven) interference.

Beyond the above modeling advances, Paredes-Astudillo et al. (2025) conducted an empirical comparative study in a flow-shop case, using real operational data to evaluate the goodness-of-fit of three classical learning curves—P-LE, ST-LE, and DJ-LE—thereby enabling data-driven model identification and parameter calibration.

The literature has progressed from single-mechanism learning toward hybrid formulations, incorporating key elements such as similarity, the DeJong plateau (incompressible component), truncation, and skill dynamics, with diverse applications across FJSP/DRCFJSP/FAJSP. Yet three central gaps persist. First, there is no systematic coupling of position-based and sum-of-processing-time learning, which limits the joint representation of sequence progression and temporal accumulation. Second, similarity modeling remains task-centric; machine similarity and the joint task–machine characterization are underdeveloped, leaving a coarse quantification of the human–machine–task fit. Third, forgetting/decay is often treated as a discrete, interruption-time event, with no unified, computable treatment of continuous time-based decay and context-dependent (similarity-driven) interference.

Beyond learning-effect studies, the choice of mathematical formulation for FJSP has a major impact on solvability. Demir and Kürşat İşleyen (2013) compiled five representative MILP formulations grouped by their binary-variable type—sequence-position, precedence, and time-indexed—and compared them on the F-data benchmarks. The results indicate that precedence-variable formulations dominate both sequence-position and time-indexed variants; in particular, Model M2 (a Manne-type precedence formulation) consistently achieved the lowest CPU times, while the time-indexed model suffered from large variable/constraint counts and discretization-induced performance loss on larger instances.

Building on this line, Roshanaei et al. (2013) evaluated five MILPs side-by-side: MILP-1 (Fattahi; sequence-position), MILP-2 (Özgüven; precedence), MILP-3 (Roshanaei 2012; precedence), and two new models—MILP-4 (sequence-position) and MILP-5 (precedence). Their study shows that precedence-based models are generally superior to position-based ones, and among all candidates MILP-5 delivers the strongest overall performance (solution quality and computational efficiency), while MILP-2 serves as a strong precedence-based baseline from prior literature.
The existing literature shows that most studies model learning with a single mechanism, either position-based (P-LE) or sum-of-processing-time (ST-LE), thereby neglecting the joint influence of repetition order and cumulative duration. Transfer is typically driven only by task similarity, overlooking machine differences and limiting performance under frequent changeovers. Forgetting is also commonly treated as a discrete, interruption-based loss, which does not account for continuous time decay or contextual dissimilarity. In contrast, interference represents negative transfer as continuous decay modulated by task×machine dissimilarity, providing a closer fit to observed patterns and avoiding the parameter inconsistencies associated with discrete forgetting.

To address these gaps, we develop a hybrid learning–interference model that (i) retains the DeJong plateau as a realistic lower bound, (ii) defines effective experience that couples positional progression with cumulative duration, (iii) weights transfer by task×machine similarity, and (iv) uses continuous interference (time decay × contextual dissimilarity) to unify learning loss. For the scheduling formulation, we follow benchmark evidence that precedence-variable approaches outperform sequence-position and time-indexed formulations, and we embed the time model within this precedence framework. The resulting setup offers a more realistic processing-time representation while retaining practical computational performance for DRCFJSP.

3. Problem Description

3.1 FJSP Model Description

In this study, we consider the Flexible Job Shop Scheduling Problem (FJSP) with worker flexibility. There are n products J_i (i = 1, …, n). Each product J_i consists of N_i operations $O_{i,j}$ (j = 1, …, N_i), where N_i denotes the number of operations of J_i. The system has m machines (let F={1,…,m}) and w workers (let H={1,…,w}). For each operation $O_{i,j}$, the set of eligible machines is $M_{i,j} \subseteq F$; for each machine $f \in F$, the set of eligible workers is $H_f \subseteq H$. The baseline processing time of an operation is determined by the selected machine–worker pair (Figure 1).

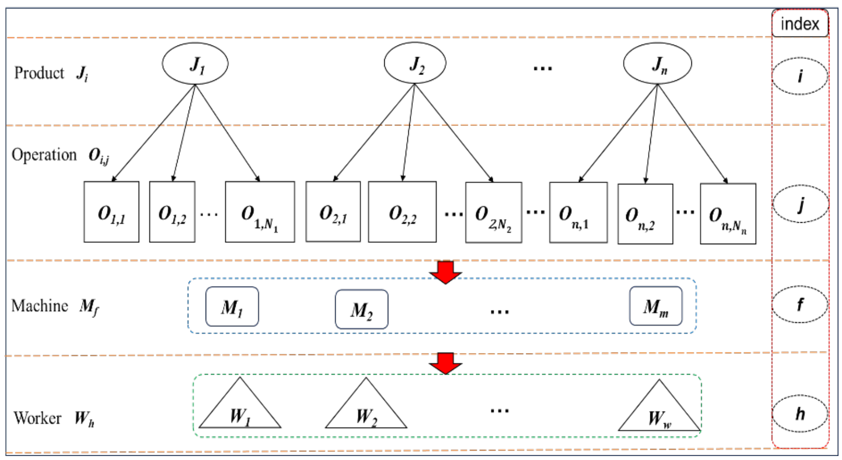

Figure 1. Schematic diagram of the model

Sub-problems

The scheduling problem is divided into three sub-problems:
(1) sequencing the operations.
(2) assigning machines to corresponding operations.
(3) assigning workers to corresponding machines.

3.2 Combined Position–Sum-of-Processing-Time Learning-Effect with Interference (PST-LE-I) Model

3.2.1 Actual processing-time model with skill heterogeneity and De Jong learning

In labor-involved manufacturing, processing times are inherently variable due to human factors, a reality that is especially pronounced in high-mix, low-volume settings and other contexts that rely heavily on manual work. Worker proficiency is a primary driver of throughput, and one of its key determinants is the learning effect. The most widely used formulation, the Wright Learning Curve (WLC)(Wright, 1936), models performance improvement as an exponential function of accumulated experience (or cumulative work hours) and the worker's effective learning capacity. A common specification is as follows:

$$T_n = T_1 \cdot n^{-b} \tag{1}$$

Let T_n denote the actual time to process the n-th unit of a homogeneous task, and let T_1 be the baseline time for the first unit. The learning exponent is $b = -\log(\theta) / \log(2)$, where θ is the learning rate (LR in percent divided by 100). A common range is $0 \le b \le 1$, corresponding to LR between 100% and 50%. The Wright learning curve (WLC) tends to underestimate early-unit times and overestimate long-run times; moreover, as the batch size grows large, $T_n \to 0$. To address these issues, De Jong (1957) introduced an incompressible (plateau) fraction δ in [0, 1), yielding a curve that captures "rapid initial learning followed by stabilization":

$$T_n = T_1[\delta + (1-\delta)n^{-b}] \tag{2}$$

In contrast to learning, production also exhibits proficiency loss driven by informational interference. In repetitive work, each execution leaves a "memory trace." When attention shifts or other tasks interrupt, these traces decay over time, so by the n-th execution the callable effective experience is smaller than the nominal repetition count n. This is the effect of interference. Following Jaber et al. (2021), the effective experience at time t can be defined as the decayed sum of all past traces. A commonly used exponential form is as follows:

$$E_n = \sum_{i=1}^{n} e^{-\alpha(t-t_i)},\ \alpha \geq 0 \tag{3}$$

Let t_i denote the time of the i-th execution and let t be the current time (corresponding to the n-th execution). The parameter α (> 0) governs the exponential decay of each memory trace as a function of the interval $(t - t_i)$.
In real manufacturing, workers differ markedly in both skill and learning speed. So we propose a DeJong-based processing-time model that replaces the positional count n with an effective experience $E_{i,j,f,h}$ to capture the learning effect more precisely. Worker heterogeneity is introduced in two complementary terms: a skill coefficient SK_h that shapes the baseline time, and a learning exponent α_h that governs the rate of improvement. This yields an actual processing-time model that combines worker skill and learning-speed differences, interference-adjusted effective experience, and a plateau-type learning structure:

$$T_{i,j,f,h} = SK_h \cdot t_{i,j,f,h} \cdot \left(\delta + (1-\delta) \cdot \left(1 + E_{i,j,f,h}\right)^{-\alpha_h}\right) \tag{4}$$

where:

- $T_{i,j,f,h}$: The actual processing time required when worker h performs operation $O_{i,j}$ on machine f. (after adjustments for effective learning–interference and skill).
- $t_{i,j,f,h}$: Baseline processing time. Time required when worker h performs operation $O_{i,j}$ on machine f without learning/interference effects
- δ: Non-compressible factor, representing the portion of task time that cannot be improved through learning, $0 \leq \delta \leq 1$.
- $E_{i,j,f,h}$: Effective experience. When the current operation $O_{i,j}$ is processed on machine f by worker h, $E_{i,j,f,h}$ denotes the amount of relevant accumulated experience that contributes to reducing the actual processing time $T_{i,j,f,h}$.
- $\alpha_h > 0$: Learning exponent of worker h.
- SK_h: Inherent skill coefficient of worker h, reflecting the worker's basic ability.

3.2.2 Effective Experience under Dual Job–Machine Similarity, Sum-of-Processing-Time-Based Learning Effect and Interference

To construct the effective experience $E_{i,j,f,h}$ used in Eq. (4), we propose an integrated model based on operation duration – dual similarity – interference, as specified in Eqs. (5)–(6). Its key features are:

(1) **Experience accumulation by operation duration**
We use the logarithm of processing time, $\ln t$, as the workload scale to capture diminishing returns in experiential gains (Liang et al., 2019), and embed it at the operation × machine × worker granularity. This design lets past processing time directly drive experience accumulation and couples the position-based learning effect (P-LE) with the sum-of-processing-time learning effect (ST-LE).

(2) **Extending transfer from "task" to "task × machine" dual similarity**
While only a few studies model task/operation similarity (e.g., Cai et al., 2024; Peng et al., 2022), we jointly incorporate task similarity and machine similarity into the experience measure to better reflect transfer across the human–machine–task triad.

(3) **Dissimilarity-driven continuous interference decay**
Departing from discrete, one-off forgetting, we introduce an exponential, continuous interference factor whose decay strengthens with time gaps and dissimilarity, and is modulated by a worker-specific interference parameter α_h^m. Consequently, older or less-similar past operations retain less effective experience.

$$E_{i,j,f,h} = \sum_{i'=1}^{n} \sum_{j'=1}^{N_{i'}} \sum_{f' \in M_{i',j'}} \left(\theta_{(i,j),(i',j')} \cdot \theta_{f,f'} \cdot \ln t_{i',j',f',h} \cdot I_{(i',j',f')(i,j,f)}^{(h)} \right) \quad (i',j') \neq (i,j) \tag{5}$$

where:

$$I_{(i',j',f')(i,j,f)}^{(h)} = e^{\left(-\alpha_h^m \left(1-\theta_{(i,j),(i',j')}\right)\left(1-\theta_{f,f'}\right)\left(S_{i,j}-D_{i',j'}\right)\right)} \cdot x_{i',j',f',h} \cdot x_{i,j,f,h} \tag{6}$$

- n: Total number of products.
- N_i: Number of operations of product J_i.
- m: Total number of machines.
- $\theta_{(i,j),(i',j')} \in [0,1]$: Similarity between operation $O_{i,j}$ and $O_{i',j'}$
- $\theta_{f,f'} \in [0,1]$: Similarity between machines f and f'.
- $I_{(i',j',f')(i,j,f)}^{(h)}$: Interference decay factor. For the same worker h, it quantifies the residual carry-over (e.g., distraction, fatigue, switching cost) from any previously processed operation $O_{i',j'}$ on machine f' to the current operation $O_{i,j}$ on machine f.
- $x_{i,j,f,h}$: Assignment variable. Equals 1 if operation $O_{i,j}$ is processed on machine f by worker h; 0 otherwise.
- $x_{i',j',f',h}$: Assignment variable. Equals 1 if operation $O_{i',j'}$ is processed on machine f' by worker h; 0 otherwise.
- $t_{i',j',f',h}$: Baseline processing time. Time required when worker h performs operation $O_{i',j'}$ on machine f' without learning/interference effects.
- $S_{i,j}$: Start time of operation $O_{i,j}$.
- $D_{i',j'}$: Completion time of operation $O_{i',j'}$.
- $\alpha_h^m > 0$: Memory-decay parameter of worker h.

4. Mathematical model of the FJSP- PST-LE-I

Assumptions

(1) Each machine can process at most one operation at any given time.
(2) Each worker can operate at most one machine at any given time.
(3) Each operation is processed exactly once on a single machine.
(4) Each operation is executed exactly once by a single worker.
(5) No preemption: once an operation starts, it cannot be interrupted.
(6) Operations within the same job must follow the specified technological precedence order.
(7) Baseline processing times for each operation–machine–worker combination are known in advance.
(8) Actual processing times are influenced by learning effects and interference effects.

Symbol Explanation:

w: Total number of workers.
H: set of all workers.
$M_{i,j}$: Feasible set of machines for operation $O_{i,j}$
H_f: Feasible set of workers for the machines f.
M: A sufficiently large constant (used for M constraints).

Decision Variables

Binary variables

$x_{i,j,f,h} \in \{0,1\}$: Assignment variable. Equals 1 if operation $O_{i,j}$ is processed on machine f by worker h; 0 otherwise.
$Z_{(i',j'),(i,j)} \in \{0,1\}$: Equals 1 if operation $O_{i',j'}$ is completed before operation $O_{i,j}$; 0 otherwise.

Continuous variables

$S_{i,j} \geq 0$: Start time of operation $O_{i,j}$.
$D_{i,j} \geq 0$: Completion time of operation $O_{i,j}$.

Auxiliary variables
$T_{i,j,f,h} \geq 0$: The actual processing time required when worker h performs operation $O_{i,j}$ on machine f. (after adjustments for effective learning–interference and skill).
$E_{i,j,f,h} \geq 0$: Effective experience. When the current operation $O_{i,j}$ is processed on machine f by worker h, $E_{i,j,f,h}$ denotes the amount of relevant accumulated experience that contributes to reducing the actual processing time $T_{i,j,f,h}$.
$D_{final} \geq 0$:makespan

Objective Function

$$\text{Minimize } D_{final}$$

Constraints

(1) Operation Sequencing

$$S_{i,j} \geq D_{i,j-1} \qquad \forall i,j; i = 1, \ldots, n;\ j = 2, \ldots, N_i \tag{7}$$

(2) Linking Makespan to Operation Completion Times

$$D_{final} \geq D_{i,j} \quad \forall i,j; i = 1, \ldots, n;\ j = 1, \ldots, N_i \tag{8}$$

Start and End Time Relationship

$$D_{i,j} = S_{i,j} + \sum_{f \in M_{i,j}} \sum_{h \in H_f} \left(x_{i,j,f,h} \cdot T_{i,j,f,h} \right) \qquad \forall i,j; i = 1, \ldots, n;\ j = 1, \ldots, N_i \tag{9}$$

(3) Consistency of Variable Z with Time

$$S_{i,j} \geq D_{i',j'} - M \left(1 - Z_{(i',j'),(i,j)} \right) \qquad \forall i,j,i',j';\ (i',j') \neq (i,j) \tag{10}$$

(4) Preventing circular dependencies

$$Z_{(i',j'),(i,j)} + Z_{(i,j),(i',j')} \leq 1 \qquad \forall i,j,i',j';\ (i',j') \neq (i,j) \tag{11}$$

(5) Machine Availability

$$Z_{(i',j'),(i,j)} + Z_{(i,j),(i',j')} \geq \sum_{h' \in H_f} x_{i',j',f,h'} \cdot \sum_{h \in H_f} x_{i,j,f,h}$$
$$\forall i,j,i',j'; f \in M_{i,j} \cap M_{i',j'}; (i',j') \neq (i,j) \tag{12}$$

$$S_{i,j} \geq D_{i',j'} - M \times \left[3 - Z_{(i',j'),(i,j)} - \sum_{h' \in H_f} x_{i',j',f,h'} - \sum_{h \in H_f} x_{i,j,f,h} \right]$$
$$\forall i,j,i',j'; f \in M_{i,j} \cap M_{i',j'}; (i',j') \neq (i,j) \tag{13}$$

$$S_{i',j'} \geq D_{i,j} - M \times \left[3 - Z_{(i,j),(i',j')} - \sum_{h' \in H_f} x_{i',j',f,h'} - \sum_{h \in H_f} x_{i,j,f,h} \right]$$
$$\forall i,j,i',j'; f \in M_{i,j} \cap M_{i',j'}; (i',j') \neq (i,j) \tag{14}$$

(6) Worker Availability

$$Z_{(i',j'),(i,j)} + Z_{(i,j),(i',j')} \geq \sum_{f \in M_{i,j}: h \in H_f} x_{i',j',f',h} + \sum_{f \in M_{i,j}: h \in H_f} x_{i,j,f,h} - 1$$
$$\forall i,j,i',j'; h \in H, (i',j') \neq (i,j) \tag{15}$$

$$S_{i,j} \geq D_{i',j'} - M \times \left[3 - Z_{(i',j'),(i,j)} - \sum_{f' \in M_{i',j'}:h \in H_{f'}} x_{i',j',f',h} - \sum_{f \in M_{i,j}:h \in H_f} x_{i,j,f,h}\right] \quad \forall i,j,i',j';\ h \in H; (i',j') \neq (i,j) \tag{16}$$

$$S_{i',j'} \geq D_{i,j} - M \times \left[3 - Z_{(i,j),(i',j')} - \sum_{f' \in M_{i',j'}:h \in H_{f'}} x_{i',j',f',h} - \sum_{f \in M_{i,j}:h \in H_f} x_{i,j,f,h}\right] \quad \forall i,j,i',j';\ h \in H; (i',j') \neq (i,j) \tag{17}$$

(7) Unique Assignment

$$\sum_{f \in M_{i,j}} \sum_{h \in H_f} x_{i,j,f,h} = 1 \qquad \forall i,j; i = 1,\dots,n;\ j = 1,\dots,N_i \tag{18}$$

(8) Calculation of Actual Machining Time

$$T_{i,j,f,h} = SK_h \cdot t_{i,j,f,h} \cdot \left[\delta + (1-\delta) \cdot \left(1 + E_{i,j,f,h}\right)^{-\alpha_h}\right] \qquad \forall i,j,f \in M_{i,j}, h \in H_f \tag{19}$$

(9) Effective Experience Calculation

$$E_{i,j,f,h} = \sum_{i'=1}^{n} \sum_{j'=1}^{N_{i'}} \sum_{f' \in M_{i',j'}} \left(\theta_{(i,j),(i',j')} \cdot \theta_{f,f'} \cdot \ln t_{i',j',f',h} \cdot e^{\left(-\alpha_h^m \left(1-\theta_{(i,j),(i',j')}\right)\left(1-\theta_{f,f'}\right)\left(S_{i,j} - D_{i',j'}\right)\right)} \cdot x_{i',j',f',h} \cdot x_{i,j,f,h} \cdot Z_{(i',j'),(i,j)}\right) \qquad \forall i,j,f \in M_{i,j}, h \in H_f, (i',j') \neq (i,j) \tag{20}$$

(10) Feasibility Constraints

$$x_{i,j,f,h} = 0 \qquad \forall i,j,f \notin M_{i,j}, h \tag{21}$$

$$x_{i,j,f,h} = 0 \qquad \forall i,j,f,h \notin H_f \tag{22}$$

5. Results and Discussion

This section evaluates the effectiveness and practical value of the proposed hybrid learning–interference model (PST-LE-I) for the dual-resource constrained flexible job shop scheduling problem (DRCFJSP). By benchmarking against the classical DeJong learning formulation and a no-learning baseline, we demonstrate through simulation experiments the advantages of embedding a higher-fidelity human-factors representation within scheduling optimization, highlighting measurable gains over conventional, purely position-based or learning-agnostic approaches.

5.1 Experimental Setup and Computing Environment

All computational experiments were conducted in MATLAB R2023b on a 64-bit Windows 10 platform. The workstation is equipped with a 6-core/12-thread CPU and an NVIDIA GeForce GTX 1660 Ti Max-Q GPU. Benchmarking was performed on a medium-scale DRCFJSP instance comprising 5 products, 18 operations, 6 machines, and 4 workers. The key parameters were set as follows. The incompressibility factor δ=0.40 reflects moderate automation levels in modern human-machine collaborative environments. Worker skill coefficients $SK_h \in [0.70, 1.20]$ capture typical efficiency variations in heterogeneous workforces. Learning rates of 81%–84% correspond to moderate manual proficiency improvement in assembly tasks. The memory decay parameter $\alpha \in [0.25, 0.35]$ captures continuous interference in high-mix settings. Task similarity ranges from 0.10–0.40 (cross-job) to 0.70–0.80 (adjacent operations), and machine similarity from 0.20–0.55 to 0.60–0.75, reflecting variability in flexible job shop routing.

5.2 Comparative Results and Analysis

Figure 2 depicts the schedule optimized under the proposed PST-LE-I framework. The resulting plan attains a makespan of 46.90 in a computational time of 264.84 seconds. This result serves as the primary validation for the mathematical model presented in Chapter 4, demonstrating its solvability and effectiveness. It proves that by incorporating the complex dynamics of learning, dual-similarity transfer, and interference, the model is capable of generating a concrete, high-quality schedule within a reasonable computation time.

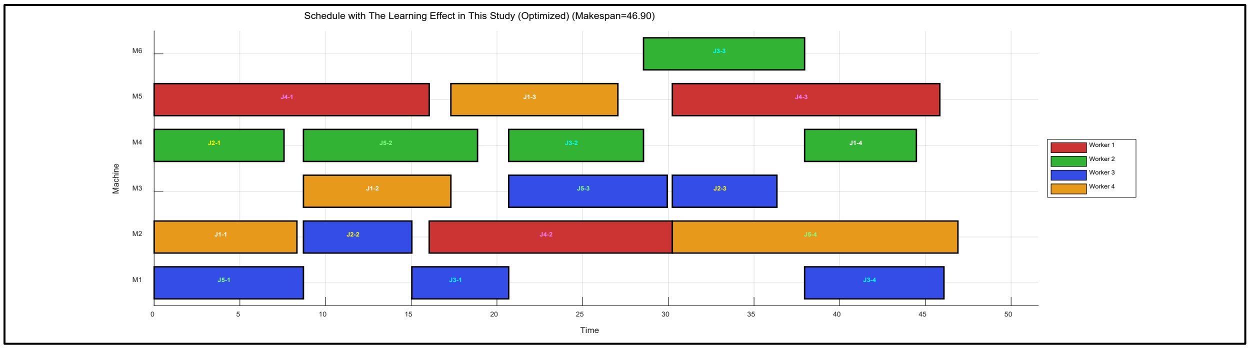

Figure 2. Optimized schedule of the proposed hybrid learning-interference (PST-LE-I) model

To gauge the practical value of PST-LE-I, we first benchmark it against the static scheduling paradigm still common in industry, which assumes fixed processing times. As shown in Figure 3, optimizing under this no-learning specification yields an expected schedule with a makespan of 50.42. This outcome rests on the premise that processing time is invariant—irrespective of the worker, task type, or repetition count.

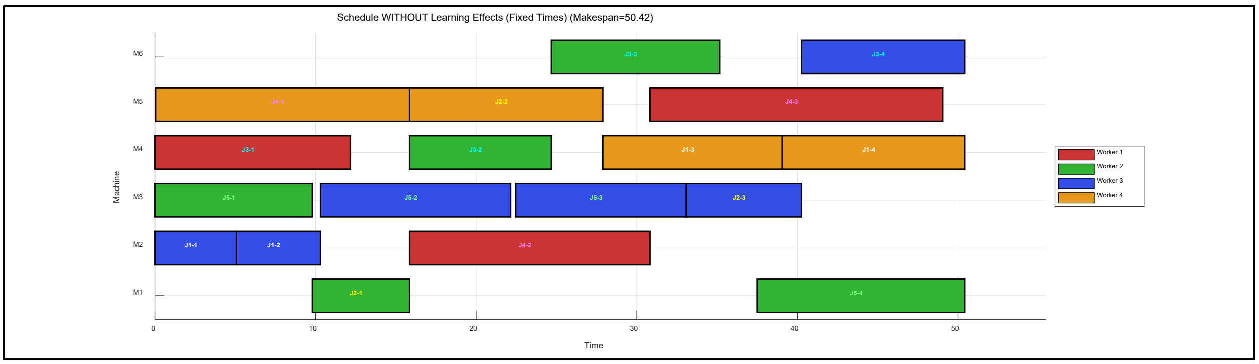

Figure 3. Optimized schedule based on fixed processing times

To assess how the static plan would perform under the shop-floor dynamics, we retained the dispatching decisions from Figure 3 and re-evaluated the simulated processing times using the proposed PST-LE-I model as the evaluation baseline. The simulated execution outcome is reported in Figure 4.

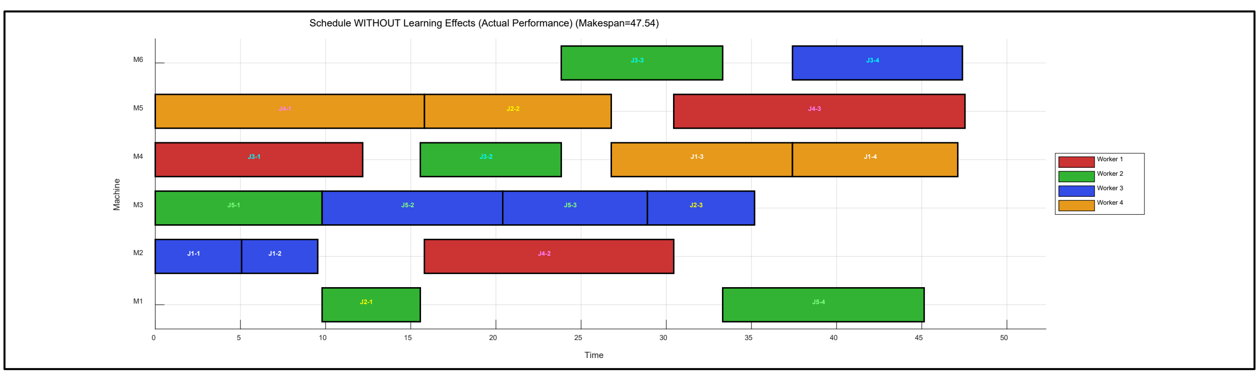

Figure 4. Simulated actual performance of the static schedule

Figure 4 shows a makespan of 47.54, while superior to the static forecast (50.42), still lags the proposed model's optimal solution (46.90) by 1.36%. This performance gap stems from the static decisions' inability to recognize and leverage the learning potential from worker skills and task clustering. For instance, in the assignment for Job J1, the static model, aiming to reduce machine switching, assigns the first two operations ($O_{1,1}$, $O_{1,2}$) to the least skilled worker, W3 (SK_3=0.70). In contrast, the proposed model assigns the first three operations of J1 ($O_{1,1}$, $O_{1,2}$ and $O_{1,3}$) as a high-similarity (=0.75) task chain entirely to the more skilled worker, W4 (SK_4=1.15). This strategy allows W4's initial skill advantage (15% faster) to be compounded by the rapid accumulation of experience (E= 1.5), reducing the actual processing time of subsequent operations to below 85% of the baseline. The static model's decision not only increases the base time by using an inefficient worker but also severs this learning chain, leading to an extended overall processing cycle for product J1 and ultimately causing the deficit in the total makespan.

The static formulation disregards learning altogether, whereas classical learning-curve models (e.g., DeJong) acknowledge learning but do so in a simplified manner. To assess the proposed model's ability to capture richer learning dynamics—including heterogeneity across tasks and transfer/interference effects—we next benchmark it against the canonical DeJong specification.

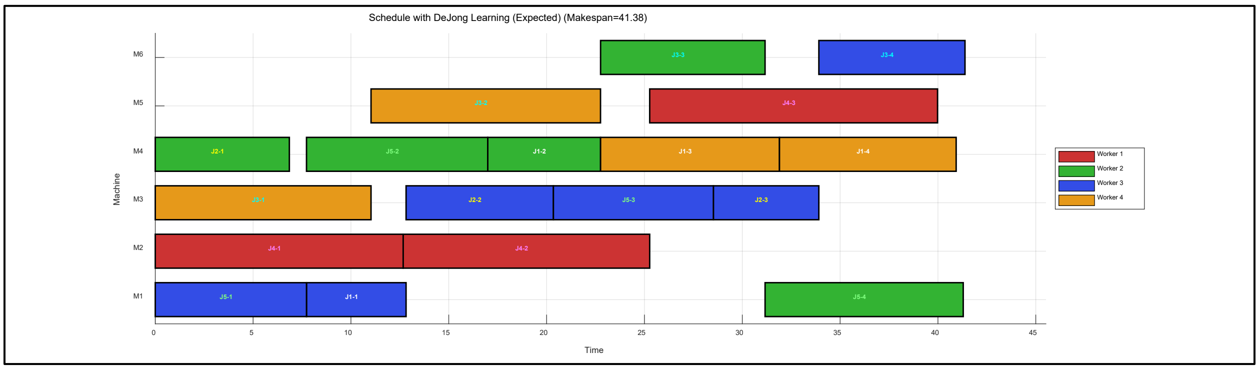

Figure 5. Optimized schedule based on the DeJong learning model

Figure 5 reports the schedule obtained when optimization is performed under the DeJong learning curve. The predicted makespan is 41.38, notably lower than the 46.90 achieved by our PST-LE-I optimization. This apparent advantage is an artifact of DeJong's coarse representation of experience: learning is modeled purely as a function of repetition (position-based), with unqualified, frictionless transfer of experience across tasks. Such assumptions systematically overstate learning benefits. They ignore heterogeneity in task–machine similarity and omit temporal decay and context switching effects (interference), thereby yielding an overly optimistic forecast that is misaligned with human-in-the-loop production realities.

To assess the simulated performance of the DeJong-optimized plan under the assumed shop-floor dynamics, we fixed the assignment and sequencing decisions from Figure 5 and re-evaluated the resulting operation times using the proposed PST-LE-I model as the evaluation baseline. The simulated execution outcome is reported in Figure 6.

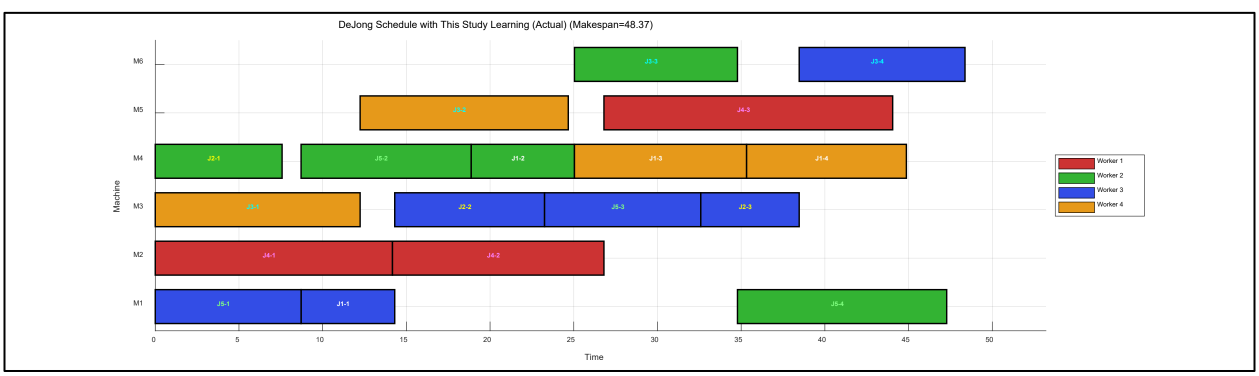

Figure 6. Simulated Actual Performance of the DeJong Schedule

Figure 6 reveals the actual performance of the DeJong schedule (Figure 5), with a makespan reaching 48.37. This result not only exhibits a significant negative deviation of 16.89% from its overly optimistic forecast (41.38) but is also substantially inferior to the proposed model's optimal result (46.90). The reason is that the DeJong optimizer is misled by its own systemically flawed, repetition-count-based learning model, which leads to suboptimal job sequences and assignments. For example, in the task sequence for worker W2, the DeJong model scheduled an inefficient transition from $O_{1,2}$ (on M4) to $O_{3,3}$ (on M6). The model predicted a processing time of just 8.78 units based solely on the repetition count (n=2), while completely ignoring the extreme heterogeneity of this switch—task similarity and machine similarity were as low as 0.20 and 0.25, respectively. According to the proposed model's assumed dynamics, this low-similarity switch triggers a strong interference effect, causing the actual effective experience carried over by W2 to be nearly zero (E =0.04). Ultimately, this extended the actual processing time of $O_{3,3}$ to 9.43 units, creating a 7.4% prediction error on this single step alone. In contrast, the proposed model achieves true efficiency gains by accurately identifying and constructing high-similarity task chains (such as having W2 consecutively perform, $O_{2,1}$, $O_{3,2}$ and $O_{1,4}$ on the same machine, M4), thereby maximizing the accumulation of effective experience. The results from the DeJong model suggest that schedules built on overly optimistic estimations tend to be fragile and inefficient when evaluated under the proposed model's dynamics.

Table 1. Comparative and Ablation Experiment Results

	Method	Predicted	Actual	Deviation	vs PST-LE-I	Computational Time (s)
Comparative Experiments	PST-LE-I	-	46.90	-	-	264.84
	DeJong Learning	41.38	48.37	+16.89%	+3.13%	91.87
	Fixed Time	50.42	47.54	−5.72%	+1.36%	27.28
Ablation Study	Without Task Similarity	44.25	48.70	+10.05%	+3.84%	232.34
	Without Machine Similarity	44.36	47.76	+7.68%	+1.83%	245.20
	Without Interference Decay	46.58	47.30	+1.54%	+0.85%	213.89
	Without Worker Skill Heterogeneity	54.19	63.67	+17.50%	+35.76%	258.31

Table 1 presents the comparative and ablation results, where Computational Time denotes the duration of each optimization run. We employed a two-phase evaluation protocol: schedules were first optimized using each model variant and then re-evaluated under the full PST-LE-I dynamics to establish their performance. The results highlight that worker skill heterogeneity is the dominant factor; ignoring this component leads to a substantial performance deterioration of 35.76%, confirming that neglecting individual differences results in highly inefficient assignments. Task and machine similarity also yield measurable benefits (+3.84% and +1.83%, respectively), whereas the interference effect had a marginal impact (+0.85%) on this specific instance size.

6. Conclusion

This study introduces the PST-LE-I model to overcome the limitations of static and simplified learning assumptions in DRCFJSP. By defining "effective experience" through dual-similarity transfer and continuous interference, the model captures complex shop-floor dynamics. Simulation experiments reveal that static approaches fail to leverage skill evolution, whereas the standard DeJong model produces fragile schedules driven by overly optimistic transfer estimates. In contrast, the proposed method improves robustness by prioritizing high-similarity task chains. Ablation analysis further confirms that while worker heterogeneity is the dominant driver of performance, accounting for task-machine similarity and interference effects is essential for refining scheduling precision.

Future research should focus on calibrating model parameters with real operational data and developing efficient meta-heuristics to extend validation to larger-scale industrial instances. This work demonstrates that in human-centric environments, model fidelity is a decisive factor for scheduling quality.

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