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Min Max Fairness in the Non-Convex Latency-Based Evacuation Model

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Abstract

In this paper, the evacuation process is modeled as a Dynamic Traffic Assignment (DTA) problem with the objective of achieving fair evacuation outcomes. To address the nonconvex nature of the resulting Mixed-Integer Linear Programming (MILP) formulation, an iterative algorithm is proposed to determine the Min-Max Fair (MMF) evacuation times among evacuees. For benchmarking purposes, the proposed algorithm is also applied to a convex Linear Programming (LP) version of the model obtained by fixing the binary decision variables. The proposed framework seeks to bridge the gap between traditional static traffic assignment approaches and more realistic dynamic traffic assignment models by explicitly incorporating temporal traffic dynamics and fairness considerations. The model and solution methodology are first demonstrated on a small illustrative network and subsequently evaluated on a benchmark evacuation network commonly used in the literature. To assess the effectiveness of the proposed approach, several alternative objectives are considered for comparison, including Network Clearance Time (NCT), Average Evacuation Time (AET), Average Evacuation Time subject to Network Clearance Time (AET_{NCT}), and Average Evacuation Time under Equal Demand assumptions (AET_{ED}). Fairness is quantified using the Sum of Absolute Deviations (SAD) of evacuees' evacuation times, enabling a direct comparison between efficiency-oriented and fairness-oriented evacuation strategies.

Keywords
Evacuation planning, disaster management, fairness, mixed integer linear programming.

1. Introduction
In natural and manmade disasters such as hurricanes, nuclear disasters, fire accidents, terrorist attacks, and epidemics, evacuating endangered areas is essential. Due to the sudden surge in evacuation demand, roads become heavily congested, increasing the risk of loss of life and property. This situation is fundamentally different from normal traffic conditions. Hence, numerous modeling approaches and algorithms have been proposed over the last few decades to better allocate resources during the evacuation process. The objective of these models is to develop evacuation plans that mitigate losses and reduce costs. Despite advances in computational resources, evacuation models are still either too complex or fail to reflect the reality of the evacuation process. The goal is to develop a model that captures the key aspects of the evacuation process while remaining computationally tractable for practical implementation. On the

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other hand, the output of the model should conform to the requirements of transportation planners and the users of the transportation network.
This research proposes a novel modeling approach to optimize the evacuation process from the perspectives of both the decision-maker and the users of the transportation network. In this paper, an iterative algorithm is developed to find the min-max fair (MMF) evacuation time using the latency-based model (LBM). The LBM is a mixed-integer linear programming (MILP) model formulated as a latency-based dynamic traffic assignment (LB-DTA) model. The objective of the LBM is to find a fair and efficient solution that balances the minimum average evacuation time (AET), the minimum sum of the absolute deviations (SAD), and the minimum network clearance time (NCT). To the best of our knowledge, no mathematical programming approach has attempted to formulate the evacuation process in a manner similar to the model proposed in this paper, and no dynamic evacuation model provides travel time estimates for each group of evacuees on each route. In the proposed model, the travel time on any road segment is load-dependent as a function of the traffic flow, and the function used is a modified version of the Bureau of Public Roads (1964) function proposed in a recent study by Mtoi and Moses (2014). The proposed model is capable of propagating the delay that occurs downstream in the road network to upstream road segments along the predefined routes.

1.1 Objectives
The primary objective of this research is to develop a fairness-oriented evacuation framework based on a dynamic latency-based traffic assignment model. Specifically, this study aims to: (1) formulate the evacuation problem as a min-max fair (MMF) optimization model that explicitly considers dynamic traffic conditions and travel-time-dependent latency; (2) develop an iterative solution algorithm capable of obtaining MMF evacuation times for the resulting non-convex mixed-integer linear programming (MILP) formulation; (3) investigate the impact of non-convexity on fairness outcomes by comparing the proposed MILP framework with its convex linear programming (LP) counterpart obtained by fixing the binary variables; (4) evaluate the trade-off between fairness and efficiency by comparing the proposed model with commonly used evacuation objectives, including Network Clearance Time (NCT), Average Evacuation Time (AET), Average Evacuation Time subject to Network Clearance Time ($\mathrm{AET_{NCT}}$), and Average Evacuation Time under Equal Demand assumptions ($\mathrm{AET_{ED}}$); and (5) quantify the effectiveness of the proposed approach using the Sum of Absolute Deviations (SAD) of evacuation times as a fairness metric. Through these objectives, the study seeks to advance fairness-oriented evacuation planning and bridge the gap between traditional static traffic assignment models and dynamic latency-based evacuation frameworks.

2. Literature Review
The objectives in most mathematical programming traffic assignment models are the NCT known as system optimal (SO), or the user equilibrium (UE). SO, or the second Wardrop's principle introduced by Wardrop (1952) in the traffic assignment models is the minimum NCT in the optimization models. In SO, the maximum evacuation time on any route is minimized to maximize the evacuation efficiency disregarding the individual travel times. However, SO results in unfair assignment of traffic on the same or different origin-destination pairs. See Tuydes-Yaman and Ziliaskopoulos (2014) for example. Modeling the congestion effect in the problem is a challenging task since it contributes greatly to the model complexity as the load dependent travel time is a non-linear function of the traffic volume. In addition, minimizing the maximum travel time of the traffic in flow networks is an NP-hard problem even with linear latency functions and with a single source and terminal as shown by Correa et al (2007).

UE or the First Wardrop principle by Wardrop (1952) is that the evacuation time of all evacuees is minimized, and no individual evacuee can improve their travel time by changing routes. This objective provides the best solution for each individual. Nash equilibrium is another fair resource allocation approach introduced by Nash (1951) in network games. An n-person game consists of n players, and each player has a finite set of strategies and a payoff function p_i. The Nash equilibrium point solution is when each player maximizes their strategy if the other players' strategies are fixed. Hence, each player has an optimal strategy against other players. Charnes and Cooper (1961) observed the relationship between Wardrop equilibrium and Nash equilibrium. Nash equilibrium converges to Wardrop equilibrium as the number of players increases in a network game with a finite number of players as proved by Haurie and Marcotte (1985). For more illustration, see Correa and Stier-Moses (2011). To find the MMF solution, the water filling or progressive filling algorithm is used introduced by Bertsekas (1987). If the travel time is considered as a resource, the maximum set of travel times is minimized until they cannot be minimized any further, then they are fixed to their minimum and the next set of travel times is minimized. This process is repeated until all travel times are minimized to obtain an optimal MMF solution. In the MMF solution, no individual can improve their travel time without worsening another individual's travel time that is less than or equal to them. In some cases, MMF flow may not use

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routes shorter than the used ones. However, the Nash equilibrium does not leave shorter routes unused. See Correa et al (2007) for more details. A compromised objective between SO and UE is called constrained system optimum (CSO) has been explored by Jahn et al (2005) to balance the system efficiency and the fairness in static traffic assignment networks. Lujak et al (2015) proposed a static traffic assignment modeling approach to find a fair-efficient solution based on Nash Welfare optimization. Finding a fair and efficient solution using CSO is also applicable in facility location. Bayram et al (2015) use the CSO in a non-linear static model to find a compromised system and user equilibrium shelter location. Other objectives have been tested by Bish et al (2014) such as average travel time (ATT) and AET. MMF has been applied to applications with non-convex structures, such as evacuation (Bin Obaid et al., 2024), nurse scheduling (Bin Obaid, 2024), and network restoration (Bin Obaid et al., 2025). The reader is referred to Bin Obaid and Trafalis (2018) for a comprehensive review of fairness.

Nemours algorithms have been proposed to solve the UE static traffic assignment problem. One of the most widely accepted algorithms is Frank-Wolfe (FW). This algorithm finds a UE approximation by solving a set of convex linear programming problems iteratively as an approximation method. See LeBlanc (1975) for more details. Modified versions of the FW method were proposed to improve the efficiency of the FW method such as the conjugate Frank-Wolfe (CFW), and bi-conjugate Frank-Wolfe (BFW). Mitradjieva Lindberg (2013) perform a comparison between these algorithms and show that CFW and BFW outperform the FW method. Path based algorithms such as gradient projection (GP) method developed by Rosen (1960) is widely used in communication networks. Forian et al (2009) introduce an algorithm that decomposes the O-D pairs into subproblem, and each subproblem is solved based on the GP method to find the equilibrium traffic assignment. Some network design models and algorithms are biologically inspired and used for traffic assignment such as genetic algorithm (GA), ant colony routing (ACR) algorithm, and Physarum algorithm. Ceylan and Bell (2005) applied the GA to solve a signalized road network under congestion as the upper level and applied stochastic user equilibrium traffic assignment as the lower level. Cong et al. (2013) implemented an online ACR optimization heuristic to update the traffic distribution on routes by determining the splitting rates on each node. Xu et al (2018) propose a modified version of the Physarum algorithm to solve the user equilibrium traffic assignment model based on the origin nodes. Most models and algorithms assume a static traffic assignment. However, static traffic assignment models have a limitation in capturing the traffic evolution and congestion propagation effect which could lead to erroneous solutions. Another limitation is in capturing the traffic interaction in adjacent links. For example, static traffic assignment model solution may result in expanding two parallel links leading to creating a bottleneck since most of these models' objective is based on the sum of the travel times on all links. Hence, dynamic evacuation models are closer to traffic evolution reality than the static traffic assignment models.

Merchant and Nemhauser (1978a, 1978b) introduced a mathematical program, known as the M-N model, as a discrete-time SO model, and it is considered one of the first attempts to model the dynamic traffic assignment (DTA). The model captures the congestion propagation effect through the link exit functions. The travel time is represented through the link performance function of the traffic volume. The model has been extended and modified with link travel time models and different exit flow function by Carey (1986), Wie et al. (1995), and Carey and McCartney (2002). The relationships among the traffic flow characteristics such as flow, volume, speed on long crowded roads were explored in an early theoretical study by Lighthill and Whitham (1955). The shock waves were introduced to the macroscopic traffic flow model a year later by Richards (1956) to be known as the LWR hydrodynamic model. Based on the LWR hydrodynamic model, the cell transmission model (CTM) was developed by Daganzo (1993) and (1995). The network is preprocessed by dividing the roads into homogenous sections or cells. With the tick of a clock, the evacuees or part of the evacuees advance to the next cell once the capacity allows them to move forward. To model the congestion effect, the capacity of the cells changes with time based on the number of vehicles in the preceding cell, the capacity flow into the cell, and the empty space in the next cell. Ziliaskopoulos (2000) modified the CTM and formulate it as a an LP with a single destination and SO objective known as the NCT, which is the time from the beginning of the evacuation process until the last group of evacuees reaches the safe destination. Ukkusuri and Waller (2008) implemented the UE objective as a DTA using the CTM and compare the results with the SO output.

The motivation of this work is to find the optimal MMF evacuation time using the LBM model. The proposed algorithm in this paper follows a similar approach to the water filling algorithm. However, the proposed algorithm does not rely on the complementary slackness condition to identify the groups whose evacuation time cannot be improved any further. Hence, the algorithm is capable of finding the MMF resources allocation for nonconvex problem structures. On the other hand, the LBM model captures the desirable properties of the static and the dynamic models and fills the gap between them. The static models use the load dependent functions but do not consider the congestion

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propagation. The dynamic models propagate the congestion effect by regulating the entry of traffic to a road segment but do not compute the estimated time each group of evacuees spends in the network. The output of the algorithm is compared with other objectives to show the effectiveness of the developed approach. The two measures used to compare the outputs from different objectives are efficiency and fairness. The AET is considered as a measure of efficiency, and the sum of absolute deviations (SAD) is used to measure fairness.

The organization of this paper is as follows. In section 2, the travel time function piecewise approximation is explained, and the reduced evacuation model is introduced including the set of variables, constraints, and objective functions. In section 3, the model is illustrated on a small example and experimented on a real-world network. Fairness is introduced in section 4 using multicriteria approach with efficiency and fairness objectives. The paper is summed up with computational complexity and concluding remarks in sections 5 and 6 respectively.

3. Evacuation Model

A novel approach to model the evacuation process is presented in this section. To the best of our knowledge, there is no attempt found in the literature to model the evacuation process similar to the one proposed in this paper. The network is preprocessed by identifying all the hub nodes and adding artificial hub nodes before running the model if necessary. Then the set of shortest routes from the endangered nodes to the safe zone nodes are identified since it is unrealistic to send evacuees on longer routes in emergencies. The hub nodes are added so the travel time in free flow speed between each pair of consecutive nodes is constant. Once a group of evacuees pass through a hub node, they enter a new road segment with a hub increment. The constant travel time between each pair of consecutive hub nodes is the system time unit. However, the evacuees may spend a longer time on a road segment if the network is congested. The travel time function (1) used in the model, also known by BPR, is used by the U.S. Department of Commerce Bureau of Public Roads in Bureau (1964). This function is incorporated in the model to simulate the effect of congestion since the travel time load dependent. The travel time function describes non-linear the relationship between traffic volume and travel time. The travel time $T(f)$ with traffic volume f on a road segment is described as follows:

$$T(f) = T^0\left(1 + \alpha\left(\frac{f}{c}\right)^{\beta}\right), \tag{1}$$

where T^0 is the travel time in free-flow speed on a road segment in normal road conditions given the capacity c of the road segment. The tuning parameters α and β describe the road characteristics with $\alpha \geq 0$ and $\beta \geq 0$. Those parameters are set to 0.15 and 4 respectively by the U.S. Department of Commerce Bureau of Public Roads. Since α and β parameters were derived based on data that may not reflect the current road conditions, they are set to 0.2745 and 5.98 respectively based on the recent study conducted by Mtoi and Moses (2014). To avoid non-linear constraints in the model, the travel time function is linearized using a piecewise approximation. Every linear segment is represented by a slope and intercept parameters. The accuracy of the output of the function increases as the number of the linear segments increase.

2.1 Model Definition

Given a graph G = (N, A) with a set of nodes N and a set of directed arcs A, each arc $a \in A$ connects two nodes $i, j \in N$ where $a = (i, j)$. The nodes of the network are composed of a set of source nodes $N_S \subset N$, a set of terminal nodes $N_T \subset N$, and a set of hub nodes $N_H \subset N$ given that $N_S \subset N_H$ and $N_T \subset N_H$ since the evacuees are allowed to enter or exit through a hub node. Given a set of communities K to be evacuated, P_k is the set of the shortest routes that community k can use where R is the set of the shortest routes in the networks from N_S to N_T. The evacuees can enter and exit any route through any hub node. $T = \{1, \dots, t_e\}$ is the set of time periods when evacuees are evacuated given that t_e is the time where the last groups are evacuated, and the evacuation process starts when the time t in T equals 1. The set of hubs nodes $H = \{1, \dots, h_e\}$ are considered as marking points to identify the evacuees' location in time given that h_e is the hub where the last group of evacuees reached their destination. The link (i, j) on a route is linked to the time t of evacuation and hub h through the tuple S given that $(i, j, t, h) \in S$.

2.1.1 Parameters and Sets

K	The set of communities
R	The set of shortest routes from the source nodes to the terminal nodes
Q_k	The population of community k
D	The set of linear segments of the piecewise approximation to the BPR function
m^l	The slope of the linear segment $l \in L$ of the piecewise approximation
b^l	The intercept of the linear segment $l \in L$ of the piecewise approximation

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u The maximum number of evacuees on any arc
μ The minimum number of evacuees to be evacuated in any group
ρ The time in minutes to travel from one hub to the next hub node in free flow speed and the system clock
c_{ij} The time to travel from node *i* to node *j* on the link $(i,j) \in A$
C_r The time to travel on route *r* from source to destination in free-flow speed

2.1.2 Set of Variables:
x_{ijrh}^{kt} The number of evacuees on arc (*i, j)* on route *r* at hub *h* of community *k* evacuated at time *t*
x'_{ijh} The number of evacuees on arc (*i, j)* at hub *h*
f_r^{kt} The number of evacuees of community *k* on route *r* evacuated at time *t*
z_{ijrh}^{kt} Binary variable equals 1 if community *k* evacuated at time *t* is allowed to pass on arc (*i, j)* on route r at hub h
t_{ijh} The travel time on arc (*i, j*) at time *h* based on the piecewise approximated travel time function
τ_{ijh} The latency on arc (*i, j)* at hub *h* transferred to the group of evacuees that follows
$\hat{\tau}_{ijh}$ The slack latency on arc (*i, j)* at hub *h* not transferred to the group of evacuees that follows
τ'^{ktr}_{ijh} The latency on arc (*i, j)* at hub *h* equals to the latency added to slack latency if the group is evacuated, and zero otherwise
s_{ijh}^{ktr} Slack variable to remove the effect of latency of the group of evacuees from community *k* if they do not pass through the arc (*i, j*) at hub *h*
l_r^{kt} The latency of the group of evacuees on route *r* of a community *k* evacuated at time *t*
e_r^{kt} The travel time of community *k* on route *r* evacuated at time *t*
e'^{kt}_r The evacuation time of community *k* on route *r* evacuated at time *t*
E Network clearance time
d_{rq}^{tv} The positive difference of the route *r* of time *t* compared with the route *q* at time *v*

2.2 Constraints

2.2.1 Flow Conservation Constraints

The hub *h* becomes *h+1* if the evacuees pass through a hub node in the flow conservation constraint (2). In the flow conservation constraints (3), evacuees hub *h* on arc (*i, j*) does not change if evacuees do not pass through a hub node. Every community to be evacuated has a population size $of\ Q_k$ as seen in constraint (4).

$$x_{ijr(h+1)}^{kt} - x_{jirh}^{kt} = \begin{cases} f_r^{kt}\ if\ i \in N_S \\ -f_r^{kt}\ if\ i \in N_T\ ,\ if\ i \in N_H \\ 0, otherwise \end{cases} \quad \forall r \in R, (i,j) \in r, k \in K, t, h | (i,j,t,h) \in S \tag{2}$$

$$x_{ijrh}^{kt} - x_{jirh}^{kt} = 0, if\ i \notin N_H \quad \forall r \in R, (i,j) \in r, k \in K, t, h | (i,j,t,h) \in S \tag{3}$$

$$\sum_{t \in T} \sum_{r \in R} f_r^{kt} = Q_k \quad \forall k \in K \tag{4}$$

2.2.2 Networks Link Constraints

Since all evacuees share the same road network, constraint (5) sums all evacuees from different communities *k* started at time *t* using different routes *r* who pass through the arc (*i, j)* at hub *h* to the variable x'_{ijh}.

$$x'_{ijh} = \sum_{r \in R} \sum_{k \in K} \sum_{t \in T} x_{ijrh}^{kt} \quad \forall (i,j) \in A, h \in H \tag{5}$$

2.2.3 Latency Constraints

To find the latency τ_{ijh} on arc (*i, j)* at hub *h* transferred to the group that follows, constraint (6) is used. The latency of an arc (*i, j)* at hub *h* equals the required time for evacuees to travel on the arc based on their volume subtracting the time they would spend in a free flow speed. It can be noticed that $\hat{\tau}_{ijh}$ in (7) is bounded by the time evacuees spend on the arc in free-flow speed, but it can be less in the case that the delay is less than the travel time in free-flow speed or no evacuees are passing on the arc to maintain the constraint feasibility. The initial latency of the arc (*i, j)* equals 0 in constraint (8).

$$\tau_{ijh} = c_{ij} t_{ijh} - \hat{\tau}_{ijh} \quad \forall (i,j) \in A, h \in H \tag{6}$$

$$\hat{\tau}_{ijh} \leq c_{ij} \quad \forall (i,j) \in A, h \in H \tag{7}$$

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$$\tau_{ijh} = 0 \qquad \forall\,(i,j) \in A, h = 0 \tag{8}$$

To find the travel time on a road segment based on the flow volume, the piecewise approximation to the BPR convex function is used. Since the function is convex, the travel time t_{ijh} on arc *(i, j)* at hub *h* can be found as seen in constraint (9).

$$t_{ijh} \geq m^d x'_{ijh} + b^d \qquad \forall\,(i,j) \in A, h \in H, d \in D \tag{9}$$

2.2.4 Delay Propagation Constraints

The congestion effect is transferred to the group that follows if the traffic is slow as seen in constraint (10). The latency increases then added to the latency of the group of evacuees that follows through the preceding link as the link becomes more congested. However, the congestion effect is not fully transferred to the group that follows if there is a gap of time between the leading group and the group that follows. The transferred congestion effect decreases as the gap between the two groups increases as seen in constraint (10).

$$\tau_{ijh} \geq \tau_{jig} - \rho(1 + h - g)(1 - z_{ijrh}^{kt}) \qquad \begin{array}{r}\forall\,(i,j),(j,i) \in A, k \in K,\\ h,g \in H | h \geq g, r \in R, t \in T\\ (i,j,t,h) \in S\end{array} \tag{10}$$

2.2.5 Allowing to Evacuate Constraints

In constraint (11), all evacuees of community *k* at hub *h* evacuated at time *t* on route *r* can pass through the arc (*i, j*); meaning that no part of the group is delayed or outrun the rest of the group. In constraint (12), each group of evacuees is bounded by a minimum number μ decided by the decision-maker. If the variable z_{ijrh}^{kt} equals zero, no evacuees from community *k* on route *r* at hub *h* can pass through arc (*i, j*).

$$x_{ijrh}^{kt} \leq u\, z_{ijrh}^{kt} \qquad \begin{array}{r}\forall\, i \in N_S, (i,j) \in A, r \in R,\\ t,h | (i,j,t,h) \in S, k \in K\end{array} \tag{11}$$

$$x_{ijrh}^{kt} \geq \mu\, z_{ijrh}^{kt} \qquad \begin{array}{r}\forall\, i \in N_S, (i,j) \in A, r \in R,\\ t,h | (i,j,t,h) \in S, k \in K\end{array} \tag{12}$$

2.2.6 Evacuees Latency Constraints

The latency on route *r* of a group evacuated from community *k* at time *t* in constraint (13) is the sum of the latencies on the specific route in addition to the slack latencies that are not transferred to the groups that follow. Since all communities evacuated in various times using different routes share the same network, the latency of a community *k* on route *r* is a subset of the latencies of the network. To prevent the latency effect to add to the latencies of non-evacuated groups due to overlapped routes, the sum of latencies is added if evacuees are using the route as seen in constraints (14)-(16). Hence, the latency of the non-evacuated group is zero.

$$l_r^{kt} = \sum_{(i,j) \in r} \sum_{h \in H} \tau'_{ijh} \qquad \forall\, r \in R, k \in K, t \in T \tag{13}$$

$$\tau'^{ktr}_{ijh} + s_{ijh}^{ktr} = \tau_{ijh} + \hat{\tau}_{ijh} \qquad \forall\,(i,j) \in A, h \in H, k \in K \tag{14}$$

$$\tau'^{ktr}_{ijh} \leq M z_{ijrh}^{kt} \qquad \begin{array}{r}\forall\,(i,j) \in A, k \in K,\\ t,h | (i,j,t,h) \in S\end{array} \tag{15}$$

$$s_{ijh}^{ktr} \leq M(1 - z_{ijrh}^{kt}) \qquad \begin{array}{r}\forall\,(i,j) \in A, k \in K,\\ t,h | (i,j,t,h) \in S\end{array} \tag{16}$$

2.2.7 Evacuation Constraints

The time required to travel for each group of evacuees evacuated at time *t* from community *k* on route *r* to reach their destination is the travel time on route *r* in free-flow speed in addition to the latency as shown in constraint (17). The evacuation time for each group of evacuees evacuated from community *k* on route *r* at time *t* to reach their destination is the sum of the waiting time since the evacuation time, the travel time on route *r* in free-flow speed, and the latency on the route as seen in constraint (18). To minimize the network clearance time, the maximum evacuation time is minimized as shown in constraint (19).

$$e_r^{kt} = C_r z_{ijrh}^{kt} + l_r^{kt} \qquad \begin{array}{r}\forall i \in N_S, r \in R, (i,j) \in r,\\ t,h | (i,j,t,h) \in S\end{array} \tag{17}$$

$$e'^{kt}_r = \rho(t-1) z_{ijrh}^{kt} + C_p z_{ijrh}^{kt} + l_r^{kt} \qquad \begin{array}{r}\forall i \in N_S, r \in R, (i,j) \in r,\\ t,h | (i,j,t,h) \in S\end{array} \tag{18}$$

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$$e'^{kt}_r \leq E \qquad \forall\, k \in K, r \in R, t \in T \qquad (19)$$

2.2.8 Non-negativity Constraints

In addition, the following non-negativity constraints (20) and binary variables constraints (21) are added.

$$x^{kt}_{ijrh}, x'_{ijh}, f^{kt}_r, \tau_{ijh}, \hat{\tau}_{ijh}, \tau'^{kt}_{ijh}, s_{ijh}, l^{kt}_r, e^{kt}_r, e'^{kt}_r, E \geq 0 \qquad \forall\,(i,j) \in A, k \in K, r \in R, t, h|\,(i,j,t,h) \in S \qquad (20)$$

$$z^{kt}_{ijrh} \in \{0,1\} \qquad \forall\,(i,j) \in A, k \in K, r \in R, t, h|(i,j,t,h) \in S \qquad (21)$$

2.3 Objective Function

The objective of the model is to minimize NCT variable *E* where *E* is the total travel time of the last group arriving to the safe destination in addition to the waiting time since the beginning of the evacuation process as seen in the objective (22).

$$Min\ NCT = E \qquad (22)$$

The other objective used in the MMF allocation algorithm is minimizing the positive sum of the mean absolute deviation (PSMAD) as seen in the objective (23). The objective is to minimize the positive difference between the evacuation time e'^{t}_r of the group evacuated on route *r* at time *t* and the preset threshold parameter θ to identify the improvable evacuation times. More details in section 3.

$$Min\ PSMAD = \sum_{r \in R} \sum_{t \in T} d^{+}_{krt} \qquad (23)$$

Subject to

$$e'^{kt}_r - \theta + d^{-}_{krt} - d^{+}_{krt} = 0 \qquad \forall k \in K, r \in R, t \in T \qquad (24)$$

Constraints (2) - (21)

In addition, different objectives are experimented. The AET for each group of evacuees is incorporated in the model in the objectives (25). The sum of z^{kt}_{ijrh} at the source node $i \in N_S$ is the number of groups to be evacuated given that the minimum number of groups to be evacuated is one.

$$Min\ AET = \frac{\sum_{k \in K} \sum_{p \in P} \sum_{t \in T} e'^{kt}_r}{\sum_{k \in K} \sum_{r \in R} \sum_{(i,j) \in r} \sum_{t,h|(i,j,t,h) \in S} z^{kt}_{ijrh}} \qquad (25)$$

3. Fairness in evacuation

In this section, a new algorithm called θ*-progressive-filling* algorithm is introduced to find the optimal MMF evacuation time for the LBM model. This algorithm guarantees convergence and finds the optimal MMF evacuation time. The algorithm starts by setting initial values to the parameters ω_{rt}, γ_{rt}, and θ for the road segment *r* and time of evacuation *t*. The parameter ω_{rt} equals 1 if the objective of minimizing the evacuation time considers the group evacuated on route r at time *t* in the minimization process, and it equals 0 otherwise. The parameter γ_{rt} is the upper bound of the evacuation time. The parameter θ is used as a threshold to identify the improvable evacuation times by minimizing the positive sum of the mean absolute deviations (PSMAD). In the first step, the evacuation time for all groups is considered by setting the parameter ω_{rt} to 1 for the groups evacuated on all routes *r* and time of evacuation *t*. The upper bound parameter γ_{rt} of evacuation time is set to big M given that big M is a sufficiently large number. The initial value of the threshold parameter θ is set to 0 since it has no effect on the first iteration of the algorithm. In step 2, the algorithm stops if all groups evacuation time are excluded from the objective to be minimized. Otherwise, the model is solved to minimize the evacuation time of the groups that are considered for improvement. In the first iteration, the evacuation time for all groups are considered in the objective. In step 3, an upper bound is set for the evacuation times considered in the objective by setting the evacuation time upper bound parameter γ_{rt} for the group evacuated on route *r* at time *t* to equal the minimum E, and setting the threshold parameter θ to equal $E - \varepsilon$ given that ε is a small value. In step 4, the model is solved again with the objective PSMAD to identify the groups of evacuees whose evacuation time can be improved by ε. If the evacuation time e'^{t}_r, in step 5, for the group evacuated on route *r* at time *t* is greater than or equal to the minimum *E* from step 2, it indicates that the evacuation time for that specific group of evacuees cannot be improved any further. Hence, its parameter ω_{rt} is set to 0 to be excluded from the minimization objective in step 2.

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Table 1. $\boldsymbol{\theta}$-progressive filling algorithm

Algorithm:	**Finds MMF resource allocation for general problems**
Step 1	Set $\omega_{rt} = 1, \gamma_{rt} = M$, and $\theta = 0 \ \forall r \in R, t \in T$
Step 2	If $\sum_{r\in R}\sum_{t\in T}\omega_{rt} = 0$, stop. Otherwise find the minimum E by solving the problem $P_{NCT}(\omega, \gamma, \theta)$
Step 3	If $\omega_{rt} = 1$, set $\gamma_{rt} = E$, and $\theta = E - \varepsilon \ \forall r \in R, t \in T$
Step 4	solve the problem $P_{\text{PSAMD}}(\omega, \gamma, \theta)$
Step 5	if $e'^{t}_{r} \geq E$, set $\omega_{rt} = 0 \ \forall r \in R, t \in T$
Step 6	Go to step 2

The computational time of the θ-*progressive-filling* algorithm is very similar to the water filling algorithm if not faster. The water filling algorithm may encounter degeneracy since not all binding constraints are identified due to the unnecessary condition of the complementary slackness. Since this algorithm does not rely on complementary slackness in identifying the blocking constraints, it is capable of finding the optimal MMF resource allocation for a problem of convex or nonconvex structure problems, and it may require less iterations to reach the optimal MMF solution.

Fairness is measured using the sum of absolute deviations (SAD) as seen in the equation (26), to achieve an approximation to the MMF solution as illustrated by Bin Obaid and Trafalis (2018).

$$SAD = \sum_{r,q\in R|r>q} \sum_{t,v\in T|\ t\geq v} \sum_{k,l\in K} \left| e'^{kt}_{r} - e'^{lv}_{q} \right| \tag{26}$$

Assuming that all evacuees are evacuated from one community, d^{tv}_{rq} is the positive difference in evacuation time between every pair of groups evacuated on different routes at various times. The SAD function (26) is converted to the linear constraints shown in (27) and (28).

$$SAD = \sum_{r,q\in R|r>q} \sum_{t,v\in T|t\geq v} d^{tv}_{rq} + d^{vt}_{qr} \tag{27}$$

$$e'^{t}_{r} - e'^{v}_{q} + d^{tv}_{rq} - d^{vt}_{qr} = 0 \qquad \forall r, q \in R, t, v \in T_E, \ r > q, t \geq v \tag{28}$$

4. Illustration and Experimentation

In this section, an example network is used to illustrate the model. In addition, a real world small network is used for illustration and experimentation.

4.1 Illustrative Example

Consider the small network with 7 nodes and 7 arcs as seen in Figure 1. Suppose that the population in the endangered zone in node S0 is to be evacuated to the safe zone in node T6 given that S0 is the source node and node T6 is the terminal node. All nodes are considered as hub nodes, and the travel time in free flow speed from each pair of consecutive hub nodes is 60 minutes. The total travel time from node S0 to node T6 in free flow speed is 240 minutes on either route. Due to the limited capacity of roads, evacuees spend more time in the network as a result of congestion.

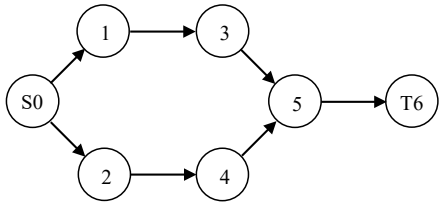


Figure 1. The time paths of the full model and the reduced model

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As shown in Table 1, the travel time is based on the traffic volume on any road segment. If the traffic volume is 0, the latency is 0 indicating that no evacuees are evacuated. This information is used to find the slope and intercept of each linear segment of the piecewise linear approximation of the BPR function. The lane capacity is set to 1750 vehicles/lane/hr., and each road segment in the network is assumed to consist of two lanes in each direction.

Table 2. Latency based on the traffic volume

Number of Vehicles	**Latency (m)**
0	0
2200	1
4350	60
4650	90
4900	120
5200	180

The MMF algorithm is tested on the example network in Figure 1 to find the MMF evacuation time. The output of the algorithm is illustrated in Table 2. Note that the groups of evacuees evacuated at the first time period are delayed by 11.97 minutes on either route, but not causing any delay to the groups following them until they reach the bottleneck road segment (5, 6). At the bottleneck, they are delayed by 180 minutes transferring 120 minutes delay to the groups that follow. The groups evacuated at time 2 are delayed by 7.86 minutes until they are delayed by 120 minutes on the road segments (3, 5) and (4, 5) caused by the groups evacuated at time t additional to the 120 minutes delay on the bottleneck. The group of evacuees evacuated at time 3 are delayed by 7.86 minutes on (0, 1) and (0, 2) road segments, 120 minutes caused by the groups evacuated at time 1, and 60 minutes caused by the groups evacuated at time 2 in addition to the 120 minutes delay at the bottleneck.

Table 3. The results of the example network

	Number of Evacuees (f)		**Latency (τ)** Slack Latency ($\hat{t}$)							**Obj. NCT (m)**
Time	Route1	Route2	(0, 1)	(0, 2)	(1, 3)	(2,4)	(3,5)	(4,5)	(5,6)	
1	2600	2600	0	0						667.86
			11.97	11.97						
2	2450	2450	0	0	0	0				
			7.86	7.86	11.97	11.97				
3	2450	2450	0	0	0	0	0	0		
			7.86	7.86	7.86	7.86	11.97	11.97		
					120	120	120	120	120	
					0	0	0	0	60	
							60	60	60	
							0	0	60	
									60	
									60	

The network in Figure 1 is tested using other objectives as illustrated in Table 3. Note that objective of minimizing the NCT achieves the minimum NCT of 667.86 minutes, but it distributes the evacuees randomly resulting in a relatively high AET and SAD. The objective of minimizing the AET results in a higher NCT of 671.98 minutes and minimum AET of 517.09 minutes among all the tested objectives. Fixing the minimum NCT then minimizing the AET in the AET_{NCT} results in the worst SAD of 769.51 minutes among the tested objectives. Note that the evacuation time of the groups evacuated at time 1 in the MMF is higher compared with the AET_{NCT} objective. The reason of increasing the evacuation time of the groups evacuated at time 1 in MMF compared with AET_{NCT} is to decrease the evacuation time of the groups evacuated at time 2 from 563.95 to 555.72 minutes. In the objective AET_{ED}, the AET is minimized with equal number of evacuees in each group to compare with the other objectives (Table 4).

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Table 4. The evacuation time for each group of evacuees of different objectives tested on the network of example 1 is illustrated in addition to the AET, SAD, and NCT for each objective

	Time	Route	NCT	AET	AET_{NCT}	MMF	AET_{ED}
Evacuation Time	1	**1**	600.00	404.16	404.16	455.93	407.69
		2	421.16	363.00	363.00	455.93	407.69
	2	**1**	660.00	509.4419	563.95	555.72	538.46
		2	540.50	482	563.95	555.72	538.46
	3	**1**	667.86	671.98	667.87	667.86	669.23
		2	667.86	671.98	667.87	667.86	669.23
AET			592.90	517.09	538.47	559.84	538.47
SAD			554.78	699.42	769.51	523.81	653.84
NCT			667.86	671.98	667.86	667.86	669.23

4.2 Experimentation

The model is experimented on the Fort Worth Dallas network studied in the literature by Sbayti and Mahmassani (2006) and Murray-Tuite (2007). The network is composed of 179 nodes and 459 arcs. Some of the existing nodes are set as hub nodes, and artificial nodes are added as hub nodes as seen in Figure 2. After adding the artificial nodes, the network consists of 195 nodes and 475 arcs. The population of the community is to be evacuated from the set of source nodes $N_S = \{108, 155, 178, 194\}$ to the set of terminal nodes $N_T = \{116, 117\}$ assuming the terminal nodes do not have capacities. It is assumed that the population to be evacuated consists of 2,000 vehicles. However, the population is pushed to evacuate in 3 time units to simulate the high demand to evacuate leading to higher congestion.

Table 5. Set of routes of the Fort Worth experiment network

P	Route	Travel Time (minutes)
0	108-107-106-105-104-103-102-101-100-190-116	6
1	108-107-106-105-104-103-98-116	4
2	155-99-176-96-151-97-89-4-22-2-116	4
3	155-99-152-146-145-140-91-90-6-26-25-21-116	4
4	155-153-147-141-92-8-7-85-84-83-82-81-116	6
5	178-177-93-10-9-86-72-71-70-69-68-67-130-81-116	7
6	178-177-93-10-9-86-72-73-74-75-76-77-78-79-117	8
7	178-177-93-10-9-40-11-43-46-13-50-15-56-57-17-61-117	7
8	194-154-95-149-94-12-11-43-44-45-49-180-55-58-62-117	5
9	110-195-111-112-192-113-114-173-115-117	5

The summary of the results and the computational time of different objective is listed in Table 5. When minimizing the NCT, the resulting objective is 62.38 minutes. Minimizing the AET results in the minimum AET among all tested objectives, but it results in significantly higher NCT and SAD. Minimizing the AET subject to minimum NCT results in a similar output to the MMF. However, MMF results in the lowest fairness measure SAD of 3329.25 minutes among the tested objectives (Table 6).

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Table 6. Summary of the results when testing different objectives

Objective (Minimize)	NCT	AET	SAD	Computational time (seconds)
NCT	62.38	49.51	3475.41	0.47
AET	96.78	46.65	5577.94	1.55
AET_{NCT}	62.38	49.34	3389.22	1.37
MMF	62.38	49.33	3329.25	14.71

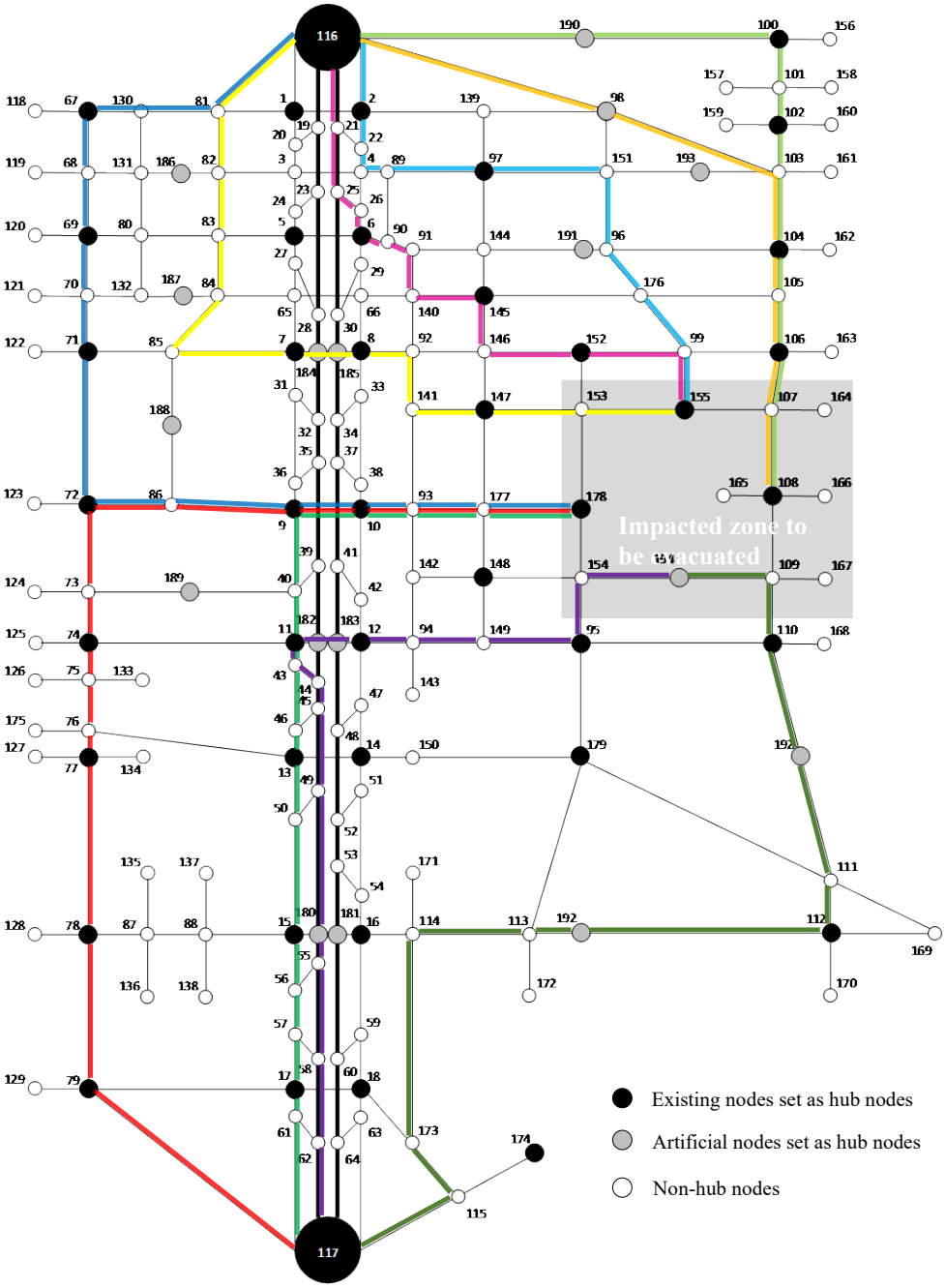


Figure 2. Fort Worth Experiment Network. The affected area is highlighted in grey.

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The algorithm is tested on convex and non-convex structure problems for comparison purposes. For the convex problem, the number of groups of evacuees is fixed assuming that the evacuees use all routes and all times of evacuation. The problem becomes LP as the binary variables are fixed to equal one. Hence, the computational complexity is reduced significantly. For the nonconvex problem, the problem is MILP as there is no restriction on the binary variable. In this section, the difference between these problems is illustrated. The NCT behavior for the convex and nonconvex when ranging the number of evacuees from 200 to 2,000 is illustrated in Figure 3. Note that the NCT is identical when the number of evacuees is greater than 1,000. The evacuation time is higher when the number of evacuees is less than 1,000 since that the evacuees are either forced to evacuate later in time while they have enough space to evacuate earlier, or they are forced to travel on longer routes while the shorter routes are not congested.

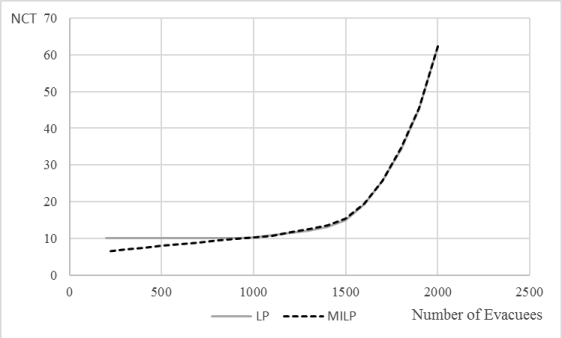


Figure 3. The NCT in minutes for the range of number of evacuees from 200 to 2,000 for the LP and MILP

The AET behavior for the convex and nonconvex problems when ranging the number of evacuees from 200 to 2,000 is illustrated in Figure 4. Note that the AET of the convex is almost identical to the nonconvex as the number of evacuees increases. It indicates that the evacuees are evacuated on all routes and all times of evacuation is ideal for higher demands to evacuate in emergency evacuations.

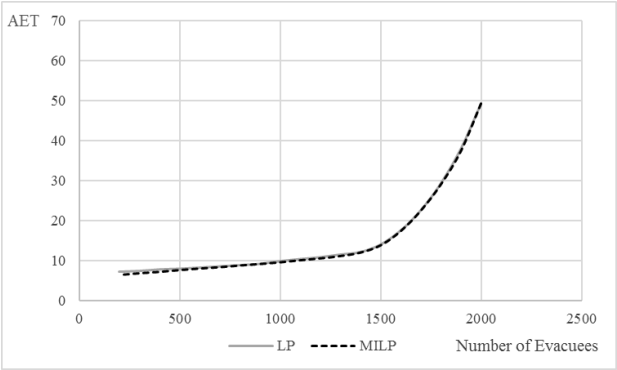


Figure 4. The AET in minutes for the range of number of evacuees from 200 to 2,000 for the LP and MILP

The evacuation time for the convex and nonconvex structure problems for each group of evacuees is illustrated in Figure 5. The evacuation time is illustrated in Figure 5 is for the number of evacuees ranging from 200 to 2,000 evacuees. Darker lines indicate higher demand. Note that the evacuation time decreases as the number of evacuees decrease. In the convex problem, all routes are used in all times of evacuation. However, in the nonconvex problem, the number of routes decrease as the number of evacuees decrease. The number of groups is 30 when the number of evacuees is 2,000 while the number of groups is 7 when the number of evacuees is 200.

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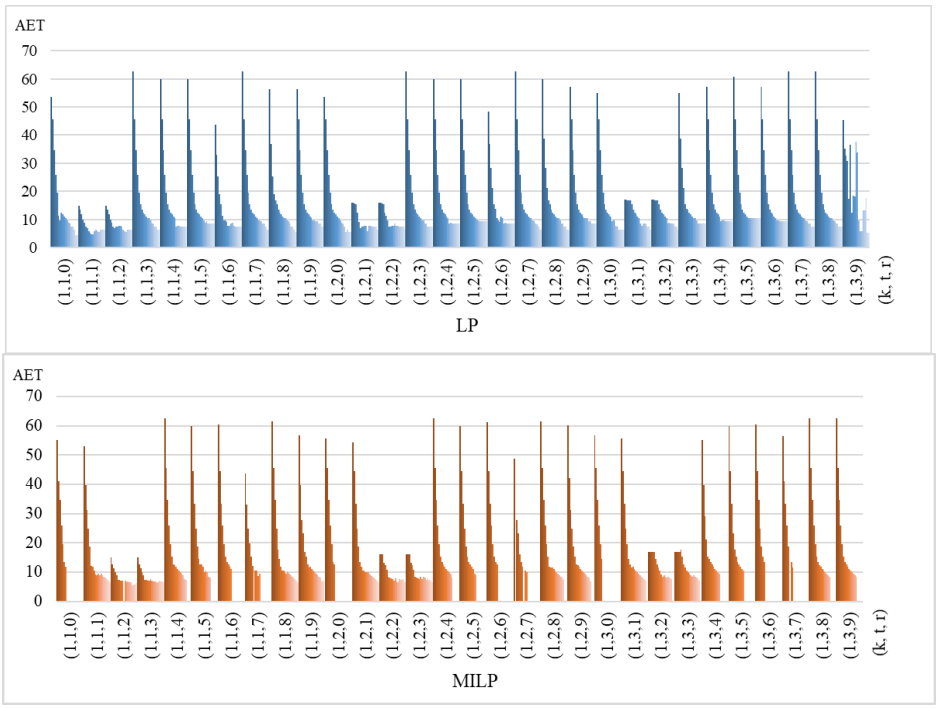


Figure 5. The evacuation time for the convex and nonconvex problems for each group of evacuees. Darker lines indicate higher demands

5. Computational Complexity

The model is tested on Tampa city problem using Gurobi Optimizer (2015). The experiment was conducted on a PC with Intel quad-core 3.4 GH CPU and 32 GB memory. The model can solve small to medium size problems with a limited number of routes. However, the computational time can increase exponentially with the size of the problem. Note that z_{ijrh}^{kt} is the binary variable that indicates whether the evacuees of community k are evacuated at time t. The number of variables z_{ijrh}^{kt} used in the model is decided by the number of routes $|R|$ used in the network and the number of evacuating times $|T|$. The number of hubs does not contribute in the size of the network. The number of variables z_{ijrh}^{kt} in the model is the number of routes r and the number of evacuating times $|T_E|$. The computational complexity of the model is reduced from $O\left(2^{|R|\left(\bar{\gamma}^2+(t-1)\bar{\gamma}\right)}\right)$ to $O\left(2^{|R|\cdot|T|}\right)$ given that $|R|$ is the number of routes, $\bar{\gamma}$ is the average number of hubs in all paths excluding the terminal hub. The two main factors in computational complexity are the number of routes and the evacuating times.

6. Concluding Remarks and Future Work

The modeling approach in this paper is reduced since the number of time paths is significantly reduced resulting in a lower complexity than the original LBM model. The nonlinear BPR function is linearized through piecewise approximation to avoid nonlinear constraints in the model. The model is illustrated through a small network, then tested on a larger network used in the literature. The objectives ATT and AET result in a very similar distribution of evacuees with the same objective values since the evacuees are forced to evacuate in shorter period of time. Pushing the evacuees to evacuate in a shorter period of time lowers the flexibility to variate the number of evacuated groups resulting in same objective values for ATT and AET. Fairness in evacuation time is the main objective of this paper. The measure of fairness used is the SAD to obtain an approximate solution to the MMF as explored in previous studies. The NCT was minimized while maintaining a fair evacuation time for all groups of evacuees. In addition, evacuation time of some of the groups of evacuees was minimized without worsening the evacuation time for groups with equal or higher evacuation time preserving the principle of MMF. In future research, uncertainty will be introduced to the model to test the robustness of the model.

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Dr. Hamoud Bin Obaid is an Assistant Professor in the Department of Industrial Engineering at King Saud University, Riyadh, Saudi Arabia. He received his B.S. in Industrial Engineering from King Saud University, his M.S. in Industrial Engineering from the University of Pittsburgh, and his Ph.D. in Industrial and Systems Engineering from the University of Oklahoma. Dr. Bin Obaid's research spans multiple areas within Operations Research, including mathematical programming, multicriteria optimization, and resource allocation. He is particularly interested in developing methodologies that enhance fairness and efficiency in complex decision-making systems. His recent work explores optimization of flow networks, equitable distribution of resources, and the application of advanced algorithms to practical problems in healthcare, logistics, and engineering systems. In addition to his research, Dr. Bin Obaid has contributed to numerous international conferences and journals, emphasizing the practical implementation of theoretical models. He is committed to advancing knowledge in Operations Research and mentoring the next generation of engineers and researchers in optimization and decision sciences.

Dr. Theodore B. Trafalis is a distinguished professor of Industrial and Systems Engineering at the University of Oklahoma whose research spans operations research, mathematical optimization, artificial intelligence, machine learning, data analytics, and intelligent decision-support systems. He earned his academic degrees in mathematics and industrial engineering from Purdue University and has built an internationally recognized research career focused on integrating optimization and AI to solve complex real-world problems in areas such as healthcare, finance, supply chains, manufacturing, and weather forecasting. Dr. Trafalis founded the Laboratory of Optimization and Intelligent Systems (LOIS) at the University of Oklahoma and has authored more than 200 scholarly publications while supervising numerous graduate students and collaborating with researchers worldwide. His work has significantly contributed to advancing the intersection of optimization and machine learning, making him one of the early pioneers in combining traditional operations research methodologies with modern artificial intelligence techniques.

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