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Comparative Analysis of Weighted-Sum Scalarization and Compromise Programming for SAW Flux Optimization

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Abstract

Efficient welding flux design is crucial for achieving optimal weld-metal quality and operational efficiency. This often involves balancing conflicting criteria. While many multicriteria optimization methods are commonly employed in welding flux performance optimization, the Weighted-sum Scalarization (WSS) method remains underexplored. This study applied and compared WSS and Compromise Programming (CP) for submerged arc welding flux design, aiming to minimize oxygen content $\left(f_{OC}(x_j)\right)$ and maximize silicon transfer $\left(f_{SiT}(x_j)\right)$. Criteria values obtained for $\left[f_{OC}(x_j),\ f_{SiT}(x_j)\right]$, when criteria were equally weighted, were (282.67, 0.1735) for WSS, (284.43, 0.1740) for Manhattan norm and (314.57 ppm, 0.1780 %) for both Euclidean and Chebyshev norms. Results revealed an efficient frontier with no solutions dominating any other, indicating that WSS provides solutions of comparable quality to that of CP. The WSS offers advantages in modelling simplicity and computational efficiency, while CP excels in deviation control and trade-off analysis. The choice of which method to use depends on the specific welding flux design context and the preferences of the welding flux formulator.

Keywords
Compromise solution, Multi-criteria optimization, Pareto front, Welding flux, Weld-metal quality.

1. Introduction

Weld-metal quality and the productivity of the welding process depend on the process parameters and consumables. Welding flux design is one of the areas of arc welding technology that has received much attention because of its enormous contribution to achieving desired weld-metal quality and the efficiency of the welding process. An overview of the contributions of welding flux has been presented in the literature (Adeyeye and Oyawale, 2010a; Adeyeye, 2021). For instance, weld-metal quality requirements such as the chemical compositions, microstructure, mechanical properties and weld bead are determined by the type and proportions of the ingredients in the welding flux (Cho *et al*, 2022; Garg *et al*, 2022; Coetsee *et al,* 2023; and Fan, 2024). Researchers have also focused on the effects of welding flux formulation on operational requirements, such as arc stability, penetration control, spatter, slag control, and deposition control. Environmental and health concerns, such as the effects of flux constituents and their relative proportions on fume generation rate, particle mass distribution, and toxic materials have received the attention of researchers (Jenkins, 2005; Sivapirakasam, 2015; Sivapirakasam, 2018; Vishnu, 2018; Madhusoodhanan, 2022).

Welding flux design is not a trivial problem. Apart from the complex way the base metal, welding wire constituents, welding flux ingredients, and process parameters react and interact, numerous quality requirements, which are often conflicting, must be considered. The flux constituents must be carefully chosen and in the correct proportions such that the flux is able to meet the operational, environmental and weld-metal quality requirements (Adeyeye and Osinubi, 2021). Welding flux formulation is a multi-criteria design problem because there are often many quality criteria the flux is expected to achieve simultaneously. Due to the conflict among the quality criteria, there is no unique or common optimum for all the criteria. The so-called optimum flux formulation is actually the one that gives the best balance or compromise among the criteria over the flux design space.

Researchers have proposed numerous methods to determine the flux formulation that achieves the optimal compromise among the criteria. Prominent among them are the Desirability Function (DF) approach, Preemptive Goal Programming (PGP), Non-preemptive Goal Programming (NGP), Compromise Programming (CP), and Weighted-sum Scalarization (WSS) method (Adeyeye and Oyawale, 2010a). Since the suggestion of these methods for welding flux design, the DF has gained popularity among welding flux researchers. While the PGP, NGP, and CP have also been explored, the WSS has not been tested in welding flux optimization, probably due to a lack of evidence on its efficacy. One common aspect of these methods is their use of a distance metric as a key element in modelling the flux optimization problem. The WSS method stands out as the only one that does not utilize a distance metric. Testing the WSS method is necessary to determine its suitability for welding flux formulation. The relative merits of these methods in terms of their application in welding flux design are sparse in the literature. To be guided when making a choice on the right method for a given situation, welding flux designers need to know their relative merits.

1.1 Objectives

This study aims to: (i) explore the suitability of the weighted-sum scalarization method for submerged arc welding flux system design, especially, as far as we know, it has not been tested in this context (ii) apply and evaluate compromise programming method as a representative of the distance-based methods for welding flux design (iii) conduct a comparative analysis of WSS and CP in terms of their effectiveness and practicality of achieving optimal balance welding flux quality criteria (iv) provide practical insights and guidance for welding flux designers. Review of the literature is presented in the next section followed by the description of the WSS and CP models. Next, the models are applied and followed by the results and discussion. Finally, the conclusion is presented.

2. Literature Review

Welding flux performs several functions such as protecting the molten metal, refinement of the weld metal and many more (Omiogbemi *et al*, 2022, Zhang *et al*, 2023 and Gupta *et al*, 2024). Adeyeye and Oyawale, (2010a) and Adeyeye, (2021) classified these functions into bundles such as weld-metal properties, operational, and health and environmental requirements. The weld-metal quality requirement bundle consists of weld-metal composition, microstructure, mechanical properties, and weld bead morphology, while the operational requirement bundle consists of quality characteristics such as arc stability, spatter, penetration control, slag control and detachment. Health and environmental requirement bundle consists of characteristics such as fume generation rate, particle number, mass distribution, toxicity, and odor. Other bundles of functions include thermophysical and physicochemical properties which consist of the following welding flux requirements; thermal diffusivity, thermal conductivity, wettability, specific heat, contact angle, spread area, surface tension, viscosity, and change in enthalpy (Sharma and Chhibber 2020 and Yuan *et al*, 2025). These functions are the quality characteristics upon which the performance of the welding flux is based.

The effects of flux composition in terms of ingredient types and proportion on the quality characteristics have been an active area of research. For instance, Pandey *et al*, (1994), Kanjilal *et al*, (2004), Singh *et al*, (2018), Coetsee *et al*, (2021) and Coetsee and Bruin, (2023) studied the effects of flux ingredients on weld-metal composition such as the O, Mn, Si, S, Ni, C, P, and N content. The contributions of flux ingredient types and their relative proportions to the transfer of O, Si, Mn, S, P, and C to the weld metal were investigated using $CaO\text{-}SiO_2\text{-}MnO$, $Al_2O_3\text{-}TiO_2\text{-}ZrO_2$ and $Al_2O_3\text{-}TiO_2\text{-}SiO_2$ flux systems respectively by Fan *et al*, (2024), Bang *et al*, (2009), and Omiogbemi *et al.* (2021). Jindal *et al*, (2013) studied the effects of welding flux on the nucleation and grain size of acicular ferrite, grain-boundary ferrite, and bainite as well as on Ultimate Tensile Strength (UTS) and microhardness. Bang *et al*, (2009) investigated the contribution of flux to UTS and toughness while its effects on bead geometry such as reinforcement, bead width, penetration and shape factor were studied by Kumar (2011), Singh *et al*, (2013), Garg *et al*, (2022), Han *et al*, (2024) and Xie *et al*, 2025 along with the metallurgical features such as inclusion size, inclusion density and volume fraction as well as operational factors such as deposition rate and slag detachability. The effect of welding flux ingredient types and their relative proportions on fume generation rate, mass distribution and toxicity such as the hexavalent Cr and Mn were studied by Jenkins *et al*, (2005), Yoon and Kim (2006), Sivapirakasam *et al*, (2015) and Sivapirakasam *et al*, (2018).

Welding flux design involves the consideration of many relevant quality attributes as performance criteria and are more often than not conflicting over the design space. Apart from the numerous conflicting criteria, the base metal, electrode wire, welding flux, and process parameters react and interact in a complex way. Until the last two decades, welding flux formulation was by series of try-and-test experiments resulting in long lead time and high cost. Also, the flux is not guaranteed to be optimal (Adeyeye and Oyawale, 2009 and Adeyeye, 2021). It has been observed that these drawbacks were due to the lack of prediction and optimization tools in welding flux development. To address this challenge, Kanjilal *et al*, (2004, 2005, 2006, 2007) introduced the use of mixture experiment and developed Scheffe's quadratic canonical regression equations for the prediction of weld-metal chemical content, mechanical properties and microstructural constituents in the weld metal. However, the equations could not adequately predict C, P and N content. Apart from prediction, their approach was also able to identify the direction and magnitude of the main effects of the individual flux ingredients and the binary interactions among them. While this approach could not identify ternary interactions, it provided more insight into how the flux ingredients interact to determine the values of the individual quality characteristics. Shamar and Chhibber (2019, 2020), Mahajan and Chhibber (2020), Mahajan *et al*, (2020) and Kumar and Chhibber, (2024) developed Scheffe's cubic and special cubic equations for the prediction of density, weight loss, specific heat, thermal conductivity, change in enthalpy, and thermal diffusivity as a function of flux ingredients. Beyond individual and binary interactions, their prediction equations were able to identify the magnitude and direction of ternary effects. Adeyeye *et al*, (2022), extended these equations by incorporating edge effects which was able to adequately predict phosphorus content while Kumar *et al* (2024) developed artificial neural network models for the prediction of microhardness, microstructure and element transfer.

Adeyeye and Oyawale (2010a) extended the work of Kanjilal *et al,* (2004) beyond prediction. They suggested various multi-criteria optimization methods such as the DF, CP, GP and WSS. These methods have received varying degrees of attention. For instance, Mahajan and Chhibber (2020), Mahajan *et al*, (2020) and Sharma and Chhibber (2020) used DF approach to develop welding fluxes that optimized the physicochemical and thermophysical properties. Adeyeye and Oyawale (2010b) used NGP to optimize weld-metal chemistry as a function of flux ingredients, while Adeyeye and Allu (2017) used CP to achieve desired values of weld-metal microstructure and mechanical properties. Adeyeye and Oyawale (2021) used PGP for flux formulation situation where the desired weld-metal properties were in different hierarchical levels. Singh et al, (2020) and Adeyeye and Osinubi, (2021) incorporated the relative importance of the various quality characteristics using principal component analysis and analytical hierarchy process respectively.

While other multi-criteria methods proposed by Adeyeye and Oyawale (2010a), have received attention, the WSS method is yet to be tested in welding flux formulation, probably because its applicability and efficacy are not evident. One common thing to those methods that have been explored by researchers is the use of one form of distance metric or the other as a key element in modelling the flux design problem. For instance, CP uses traditional distance function such as the Manhattan and Euclidean distance from a certain target point in the flux design space which is a Utopian point usually infeasible which corresponds to the ideal values of each criterion (Adeyeye and Allu 2017). The GP uses deviation from a predefined target point, which represents the desired performance levels for the various flux quality criteria rather than the traditional distance measures. Meanwhile, the DF uses the desirability indices metric to quantify how close a given solution is to the best possible value (Adeyeye and Oyawale 2010a).

3. Methods

3.1 Flux Formulation Problem

Consider welding flux formulation situation in which there is a set (I) quality criteria to be satisfied. The set (I) has two subsets, (B)) and (C). The subset B consists of all the beneficial quality criteria for which larger values represent better performance, while C consists of non-beneficial criteria for which smaller values indicate better performance. There also exists a set J of relevant flux ingredients with their respective lower (LL_j) and upper (UL_j) limits in terms of their proportions in the flux. The flux design problem is to determine the proportion (x_j) of each $j \in J$ that gives the optimum compromise among the flux quality criteria. Develop the mathematical relationship $f_i(x_j)$ for each quality criterion as a function of flux ingredient. The procedure for developing mathematical relationships has been discussed elsewhere and is beyond the scope of the present study (Kanjila, *et al.* 2004, 2005). Identify all the constraints of the problem which usually includes the lower and upper limits of the ingredients based on the technology of the problem. For criteria, i the problem may be stated as in Equation (1) below.

Maximize, $f_i(x_j)$

Subject to: (1)

$$LL_J \leq x_j \leq UL_J$$

3.2 Weighted-sum Scalarization (WSS)

The procedural steps of the WSS method are briefly presented here. The details abound in the literature (Adeyeye and Oyawale, 2010a).

Step 1: Convert all the flux quality criteria functions $f_i(x_j)$ to either maximizing or minimizing form.

Step 2: Convert the criteria to their normal forms, $f_i^N(x_j)$ using Equation (2).

$$f_i^N(x_j) = \frac{w_i}{\sqrt{\sum_k^K c_{ik}^2}} f_i(x_j);\ for\ each\ i \in I \tag{2}$$

Where, w_i is the weight of criterion i, and c_{ik} is the coefficient of the k^{th} term in $f_i(x_j)$

Step 3: Combine the normal forms of the equations into one and add the constraints as presented in Equation (3) below.

$$Maximize,\ F = \sum f_i^N(x_j)$$

Subject to: (3)

$$LL_J \leq x_j \leq UL_J;\ for\ each\ j \in J$$

The solution of Equation (3) gives the optimum compromise among the quality criteria.

3.3 Compromise Programming (CP)

The details of the CP method are available in the literature (Adeyeye and Oyawale, 2010a and Ballestero and Garcia-Bernabeu, 2015). A brief description is presented below.

Step 1: Solve $f_i(x_j)$ for each $i \in I$ and determine their respective ideal/anchor values, $f_i^{\cdot}(x_j)$.

Step 2: Identify the anti-ideal/nadir value, $f_i^{\cdot\cdot}(x_j)$ for each $i \in I$. The nadir values represent the least favourable value for each criterion within the flux design space.

Step 3: Develop the normalized distance representing the closeness of each criterion as given in Equation (4).

$$D_i = \left| \frac{f_i^{\cdot}(x_j) - f_i(x_j)}{f_i^{\cdot}(x_j) - f_i^{\cdot\cdot}(x_j)} \right|;\ for\ each\ i \in I \tag{4}$$

Step 4: Combine the normalized distances as shown in Equation (5) to obtain the overall distance function.

$$DL_p = \left(\sum \left(\frac{w_i\left(f_i^{\cdot}(x_j) - f_i(x_j)\right)}{f_i^{\cdot}(x_j) - f_i^{\cdot\cdot}(x_j)} \right)^p \right)^{\frac{1}{p}} \tag{5}$$

where,

$p = 1,$ for Manhattan distance (linear compromise) which implies all deviations from the ideal point are of equal importance.

$p = 2,$ for Euclidean distance. This is quadratic compromise which implies each deviation is weighed in proportion to its magnitude.

$p = \infty,$ for Chebyshev distance. This is the case where the worst deviation dominates. That is, only the largest deviation counts, and the problem becomes a mini-max problem

Finally, minimize the overall distance subject to the structural constraints for the $p = 1$ and $p = 2$, while for the $p = \infty$ norm, the minimax constraint is added to the structural constraints (see Equation (6)).

$$\frac{w_i\left(f_i^{\cdot}(x_j) - f_i(x_j)\right)}{f_i^{\cdot}(x_j) - f_i^{\cdot\cdot}(x_j)} \leq D_\infty;\ for\ each\ i \in I \tag{6}$$

4. Model Application

4.1 The Flux Formulation Problem

We considered SAW flux situation where the flux is expected to minimize oxygen content and maximize silicon transfer to the weld metal using the work of Kanjilal *et al*, (2004, 2007). Kanjilal, *et al*, (2004, 2007) developed prediction models using experimental data from mixture design. They identified four flux ingredients with their respective lower and upper bounds based on the technology of the flux design problem (see Table 1). The four ingredients constitute 80% of the flux while the remaining 20% were made up of ingredients of constant composition, namely: SiO_2 (10%), $Fe-Mn$ (4%), $Fe-Si$ (3%), Ni (1%) and $Bentonite$ (2%).

Table 1. Flux Ingredients with their Composition Limits

Ingredient	Lower Limit (%)	Upper Limit (%)
CaO	15	35
MgO	10	32.40
CaF_2	10	40
Al_2O_3	8	40

Source: Kanjilal et al, (2004 & 2007)

The prediction equations for oxygen content (f_{OC}) and silicon transfer (f_{SiT}) they developed and confirmed are used as the criteria of the flux formulation problem (See Equations 7 & 8). The problem is to find the proportions of CaO (x_{CaO}), MgO (x_{MgO}), CaF_2 (x_{CaF_2}) and Al_2O_3 ($x_{Al_2O_3}$) such that the best compromise between f_{OC} and f_{SiT} is achieved. In this study, oxygen content and silicon transfer are presumed to be of equal importance. Hence, $w_{OC} = w_{SiT}$, where, w_{OC} and w_{SiT} are the weights reflecting the relative importance of oxygen content, $f_{OC}(x_j)$ and silicon transfer, $f_{SiT}(x_j)$ respectively.

$$\begin{aligned} Minimize,\ f_{OC} = {} & 63..305x_{CaO} - 12.42\,x_{MgO} + 6.457\,x_{CaF_2} + 16.775\,x_{Al_2O_3} \\ & -0.945x_{CaO}x_{MgO} - 1.557\,x_{CaO}x_{CaF_2} - 2.061x_{CaO}x_{Al_2O_3} \\ & +0.835x_{MgO}x_{CaF_2} + 0.767x_{MgO}x_{Al_2O_3} + 0.378\,x_{CaF_2}x_{Al_2O_3} \end{aligned} \tag{7}$$

$$\begin{aligned} Maximize,\ f_{SiT} = {} & 0.012176x_{CaO} - 0.055635x_{MgO} + 0.006303x_{CaF_2} + 0.013559x_{Al_2O_3} \\ & -0.001364x_{CaO}x_{MgO} - 0.000063x_{CaO}x_{CaF_2} - 0.00019x_{CaO}x_{Al_2O_3} \\ & -0.001332x_{MgO}x_{CaF_2} - 0.001429x_{MgO}x_{Al_2O_3} - 0.00022x_{CaF_2}x_{Al_2O_3} \end{aligned} \tag{8}$$

The flux ingredient limits define the flux design space and are the constraints that must be satisfied. The constraints are presented in Equation (9-12).

$$15 \le x_{CaO} \le 35 \tag{9}$$
$$10 \le x_{MgO} \le 32.40 \tag{10}$$
$$10 \le x_{CaF_2} \le 40 \tag{11}$$
$$8 \le x_{Al_2O_3} \le 40 \tag{12}$$

4.2 Weighted-sum Scalarization Model

The procedure described in Section 3.2 was followed to develop the problem as a WSS model. First, Equation (7) was converted to its maximizing form. Next, Equations (8) and (9) were normalized and combined into one objective. The WSS model is presented in Equation (13) below.

$$\begin{aligned} f_{WSS} = {} & -0.36886x_{CaO} + 0.564672x_{MgO} + 0.005317\,x_{CaF_2} + 0.01008x_{Al_2O_3} \\ & -0.00452x_{CaO}\,x_{MgO} + 0.011079x_{CaO}x_{CaF_2} + 0.01376x_{CaO}x_{Al_2O_3} \\ & -0.01753x_{MgO}x_{CaF_2} - 0.01785x_{MgO}x_{Al_2O_3} - 0.00095x_{CaF_2}x_{Al_2O_3} \end{aligned} \tag{13}$$

Subject to:

Eqs. (9) - (12)

4.3 Compromise Programming Model

According to the procedure described in Section 3.3, the CP model for cases $p = 1, 2,$ and $p = \infty$ respectively in Equations (14 - 16).

$for\, p = 1$ (Manhattan distance norm)

$$Minimize\ DL_p = \left(\frac{w_{OC}\left(249.1658 - f_{OC}(x_j)\right)}{249.1658 - 597.45285}\right) + \left(\frac{w_{SiT}\left(0.231314 - f_{SiT}(x_j)\right)}{0.231314 - 0.154046}\right)$$

Subject to: (14)

Eqs. (9) - (12)

where, w_{OC} and w_{SiT} are the weights reflecting the relative importance of f_{OC} and f_{SiT} respectively.

$for\, p = 2$ (Euclidean distance norm)

$$Minimize, DL_2 = \left(\left(\frac{w_{OC}\left(249.1658 - f_{OC}(x_j)\right)}{249.1658 - 597.45285}\right)^2 + \left(\frac{w_{SiT}\left(0.231314 - f_{SiT}(x_j)\right)}{0.231314 - 0.154046}\right)^2\right)^{\frac{1}{p}}$$

Subject to: (15)

Eqs. (9) - (12)

$for\, p = \infty$ (Chebyshev norm): the Minimax constraints are added to the model (see Equation (16)).

$$Minimize,\ DL_P = D_\infty$$

Subject to:

Eqs. (9)-(12)

$$\frac{w_{OC}\left(249.1658 - f_{OC}(x_j)\right)}{249.1658 - 597.45285} \leq D_\infty \qquad (16)$$

$$\frac{w_{SiT}\left(0.231314 - f_{OC}(x_j)\right)}{0.231314 - 0.154046} \leq D_\infty$$

5. Results and Discussion

5.1 Flux Composition

The results of the WSS and CP models are presented in Table 2 below, showing the corresponding flux compositions. It is seen that the proportions of MgO and CaF_2 remained constant for all the methods (Table 2). This is the advantage of optimization as against the traditional approach, which relied solely on experiment to achieve the required balances among the flux quality criteria. In the traditional flux formulation approach, the flux designer draws upon the principles of physics, chemistry, metallurgy, and accumulated experience to make guesses on which flux ingredient(s) proportion to change. The flux designer then makes the flux and uses it to weld. Next, the weld deposit is tested to see if the desired balances are achieved. The guess-weld-test procedure is done iteratively until an acceptable flux is achieved. For the case under study, the flux formulator may not know that the proportions of MgO and CaF_2 should remain untouched and that only the proportions of CaO and Al_2O_3 need to be varied. The traditional approach is technically and economically inefficient because it consumes a lot of man-hours, materials and energy.

Table 2. Solutions for WSS and CP

Model Type	Flux Quality Criteria				Flux Composition (%)			
	Oxygen Content		Silicon Transfer		CaO	MgO	CaF_2	Al_2O_3
	$f_{OC}(x_j)$ ppm	Deviation from Ideal (%)	$f_{SiT}(x_j)$ (%)	Absolute Deviation from Ideal (%)				
WSS	282.67	13.45	0.1735	25.00	25.8	32.4	10	11.8
$p = 1$	284.43	14.15	0.1740	24.78	26.6	32.4	10	11.0
$p = 2$	314.57	26.25	0.1780	23.00	29.6	32.4	10	8.0
$p = \infty$	314.57	26.25	0.1780	23.00	29.6	32.4	10	8.0
Ideal Values	***249.1658***		***0.231314***					

5.2 Flux Quality Criteria

From Table 2, the ideal value for the oxygen content minimizing problem was $\left(f_{OC}^{\cdot}(x_j) = 249.1658\, ppm\right)$ when solved independently, while that of silica transfer was $\left(f_{SiT}^{\cdot}(x_j) = 0.231314\%\right)$. The $\left[f_{OC}^{\cdot}(x_j),\, f_{SiT}^{\cdot}(x_j)\right]$ values represent the Utopian flux that the flux formulator aims for, but it is not attainable because the oxygen content minimizing and silicon transfer maximizing criteria conflict with one another over the welding flux design space. The closest to the ideal oxygen content value was 282.67 ppm, while that of silica transfer was 0.1780%. Hence, all the solutions obtained are compromises. It is interesting to note that the $p = 2$, and $p = \infty$ norms gave identical solutions. This does not mean that the $p = 2$, and $p = \infty$ norms always give the same solution. Generally, CP is norm sensitive. The $p = 2$ norm is sensitive to large deviations because of the square terms in the distance measure, while the $p = \infty$ norm focuses only on the largest deviation, making it robust against the maximum distance. The addition of minimax constraints to the $p = \infty$ norm changes the shape of the flux design feasible space (i. e. feasible region) and creates new vertices in addition to some of the vertices of the original structural constraint set. This often leads to different solutions. However, the $p = 2$, and $p = \infty$ norms yield identical solutions when the solution lies on one of the remaining vertices of the original structural constraint set (i. e. the flux ingredients composition limits) which is the case in the present study. At such a vertex, the optimization under both the p = 2 and p = ∞ norms coincide because both norms effectively reduce to selecting the same extreme feasible point. This geometric property explains why the solutions matched in the current work, while highlighting that divergence between the two norms should be expected under different feasible regions or non-vertex optima.

None of the solutions dominates any other (see Table 2). While the $p = 2$ and $p = \infty$ norms are the worst in terms of the oxygen content (26.25% deviation), they are the best in terms of silicon transfer (23% deviation). Similarly, while the WSS solution is the best in terms of closeness to the ideal value of oxygen content (13.45% deviation), it is farthest from the ideal value of silicon transfer (25% deviation). For the $p = 1$ norm, the solution is better than the Euclidean and Chebyshev norms in terms of the oxygen content (14.15%) but worse when compared that of WSS. On the other hand, its solution is better than that of WSS on silicon transfer (24.78%) but worse than that of Euclidean and Chebyshev norms. This indicates that the criteria values belong to the Pareto frontier. Because none of the solutions dominates another, we cannot conclude that any of the methods is superior to the other in terms of solution quality. Since the WSS method generated a compromise welding flux formulation not dominated by the formulations prescribed by the CP variants, WSS is also a suitable tool for welding flux design. Another observation is that the solutions of WSS and the Manhattan norm are close to each other on the Pareto front. This is so because both methods prioritize a balanced compromise between criteria.

The choice of which one to use will depend on the nature of the welding flux design situation. The designer needs to consider the following (i) deviation control (ii) trade-off analysis, and (iii) complexity of modeling. For instance, if the welding flux designer has a clear understanding of the relative importance of the flux quality criteria and they can be accurately represented by weight and there is no need to control the deviation of quality characteristics from the Utopian, then WSS is recommended. However, if the desire of the flux formulator is to control the deviation of individual quality criteria from their ideal values, then CP is recommended. Trade-off exploration can influence the choice of the method. For instance, if flux formulator is not satisfied with the solution prescribed with the weight structure, s/he may want to explore the trade-off options available to him. For WSS, the flux designer may use different weight structures to generate other feasible solutions. The CP is superior because it can explore the Pareto frontier better because of the use of weight structure and the norm parameters. As an example, if deviations are of equal concern, s/he can choose the $p = 1$ norm and vary the weight structure to explore the Pareto frontier for more available options. The same can be done with the $p = 2$ norm if larger deviations are of more concern and with $p = \infty$ if only the largest deviation is of concern. The CP is superior if there is uncertainty in the determination of the precise weight structure and flux formulator wants to explore a range of solutions that balance the flux quality criteria. In terms of complexity, WSS is more suitable because of its simplicity in modeling and computational efficiency compared to CP which is more complex. However, CP is more flexible in handling nonlinear criteria and constraints, as the distance metric can accommodate complex relationships. While WSS and CP were applied to submerged arc welding flux in this study, they can be applied to other welding methods.

This study establishes WSS as a valuable and computationally efficient addition to submerge arc welding flux design and optimization toolkit and provides a clear, comparative benchmark against the CP method. The findings confirm that the optimal choice of method is context-dependent, providing a methodological guide for welding flux designers to enhance the efficiency and precision of welding flux formulation. The implications of the findings of this study are both practical and foundational. For industrial practitioners, they provide actionable guidance that can significantly reduce reliance on costly, iterative trial-and-error experimentation, thereby shortening development cycles and improving the consistency of weld quality. More broadly, the findings offer generalizable insights for selecting appropriate multi-criteria decision-making tools in other welding flux design contexts.

6. Conclusion

This paper presents a comparative analysis of weighted-sum scalarization and compromise programming for submerged arc welding flux optimization. Their strengths and practical utility in welding flux formulation were evaluated by minimizing oxygen content and maximizing silicon transfer. Both methods are suitable for efficient welding flux formulation and generate non-dominated flux formulations. The weighted-sum scalarization method offers advantages in model simplicity and computational efficiency. Compromise programming performs better in deviation control and trade-off assessment. The sensitivity of compromise programming solutions to the selected norm shapes the final flux formulation. Findings provide actionable guidance to reduce reliance on trial-and-error experimentation, thereby compressing welding flux development cycles. Welding flux designers can strategically select a method that aligns with their design goal. The study establishes Weighted-sum scalarization method as valuable addition to welding flux optimization toolkit. The optimal choice of method is context dependent.

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