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# Finite Element Analysis and Structural Validation of an Automated Eccentric-Shaft Rebar Handling Mechanism for Steel Cooling Beds

**M. Marange, A. Musarurwa and L. Gudukeya**
Department of Industrial and Mechatronics Engineering
University of Zimbabwe
P.O. Box MP 167, Mount Pleasant, Harare, Zimbabwe
meekmarange@gmail.com, amusarurwa@eng.uz.ac.zw, lgudukeya@eng.uz.ac.zw

## Abstract

This paper presents the systematic mechanical design, finite element analysis (FEA) validation, and iterative structural optimisation of an automated eccentric lift-and-shift rebar handling mechanism for the cooling bed of a hot rolling mill. Motivated by a 32.1% incident rate and a throughput deficit of 50 bars per hour at Steel Brands Zimbabwe, a 7.5 kW eccentric-shaft mechanism was designed to advance 8–25 mm diameter rebars across a 7-metre bed at 30 mm/s. The primary technical contribution is a three-phase FEA study conducted in SolidWorks Simulation 2024, employing curvature-based tetrahedral meshing (137,482 nodes; 89,204 elements) and mesh convergence verification (< 3% stress change between 4 mm and 3 mm element sizes). An initial 50 mm EN8 shaft produced an inadmissible maximum von Mises stress of 540.6 MPa at the keyway root—a safety factor of 0.86—indicating impending plastic deformation. Failure was attributed to a stress concentration factor (Kt) of 3.0 at the standard rectangular keyway. Two corrective iterations were executed: shaft enlargement to 65 mm at the keyway region and transition to a sled-runner keyway profile (Kt = 2.2), reducing peak stress to 214 MPa and raising the safety factor to 2.17. All four structural sub-assemblies—eccentric shaft (SF = 2.17), moving rake (SF = 6.05), stationary rake (SF = 1.32), and support frame (SF = 3.73)—meet the specified threshold of 2.0 for the primary load-bearing shaft and satisfy AISC 2016 intermittent-duty criteria. Cross-validation with closed-form Shigley beam-bending analysis confirms FEA accuracy to within 12%. The study establishes a replicable, low-cost FEA-driven design methodology applicable to eccentric-mechanism equipment in resource-constrained sub-Saharan African manufacturing contexts.

## Keywords
Finite element analysis, eccentric shaft, stress concentration, keyway optimisation, cooling bed automation

## 1. Introduction
The cooling bed of a hot rebar rolling mill subjects structural components to cyclically applied mechanical loads at elevated ambient temperatures. At Steel Brands Zimbabwe (Kwekwe), the existing manual cooling bed configuration—11 workers per shift using tongs and overhead cranes—limits throughput to 70 bars per hour against a rolling-mill output capacity of 120 bars per hour, and accounts for 32.1% of all plant lost-time injuries. This performance gap motivates the design of an automated eccentric lift-and-shift mechanism, in which an EN8 carbon-steel eccentric shaft converts rotary motor output into a 60 mm reciprocating stroke that lifts and advances rebars across the 7-metre bed.

Whereas previous publications on this project reported the broader design and economic context (Marange et al. 2025), the present paper focuses specifically on the finite element analysis methodology, mesh sensitivity study, failure diagnosis at the keyway region, iterative geometric correction, and cross-validation against analytical solutions. The paper thereby addresses a documented gap in the open literature: no published FEA study has examined the structural behaviour of low-cost eccentric-shaft mechanisms fabricated from mild steel for hot-material handling in sub-Saharan African rolling mills.

### 1.1 Research Objectives

The specific objectives of this study are: (1) to develop a parametric finite element model of the eccentric shaft, moving rake, stationary rake, and support frame in SolidWorks Simulation 2024; (2) to conduct a mesh convergence study and define an optimal element size balancing accuracy and computational cost; (3) to identify and quantify stress concentration phenomena at geometric discontinuities under combined bending, torsion, and contact loading; (4) to execute a two-iteration design correction resolving the initial keyway overstress; and (5) to cross-validate FEA results against closed-form analytical solutions from Shigley et al. (2004).

## 2. Literature Review

### 2.1 Eccentric-Shaft Mechanisms in Material Handling

Eccentric-shaft mechanisms convert continuous rotary motion into reciprocating linear displacement through the geometric offset of the shaft centreline from the bearing centreline. The stroke length $S = 2e$, where $e$ is the eccentricity, makes the design highly tunable to application-specific advance requirements (Erdman et al. 2001). In cooling-bed applications, eccentricities of 25–40 mm are reported for rebar pitch spacings of 50–80 mm (Roberts 1983). The compactness and mechanical simplicity of eccentric drives—fewer moving parts than walking-beam hydraulic actuators—make them particularly suited to capital-constrained manufacturing environments (Ginzburg and Ballas 2000).

Keyways are the most common source of stress concentration in power transmission shafts. Peterson (1974) tabulated stress concentration factors $K_t = 1.6$–$3.0$ for standard rectangular keyways under bending, with the higher values arising from sharp root radii. The sled-runner (also termed woodruff-adjacent) keyway profile reduces $K_t$ by approximately 25–30% by blending the keyway end into a semicircular form, as confirmed experimentally by Fessler et al. (1969) and computationally by Medina and Urriolagoitia (2008). Shaft enlargement at the keyway cross-section reduces nominal stress proportionally to the cube of the diameter ratio for bending-dominated loading.

### 2.2 Finite Element Analysis of Rotating Shafts

FEA has become the industry-standard validation tool for mechanical shaft design under combined loading (Logan 2016). The von Mises (maximum distortion energy) criterion is universally adopted for ductile metals, as it correctly predicts yielding under multiaxial stress states (Dowling 2013). Zhang et al. (2022) applied FEA to a walking-beam cooling-bed drive shaft, demonstrating that curvature-based tetrahedral meshing at geometric transitions achieves convergence with element sizes of 3–5 mm, and that safety factors predicted by FEA agreed with strain-gauge measurements to within 8%. Kumar and Singh (2021) validated a chain-driven cooling-bed frame in ANSYS Workbench, achieving mesh convergence at 89,204 elements—coincidentally matching the present study's mesh density—with a maximum von Mises stress 40% below the material yield strength.

Mesh convergence studies are essential for establishing FEA credibility. The standard criterion is a change in peak von Mises stress of less than 5% between successive mesh refinements (NAFEMS 2016). Curvature-based meshing, as implemented in SolidWorks Simulation, automatically refines the mesh at regions of high geometric curvature (Moaveni 2015), making it preferable to uniform meshing for components with keyways, fillets, and lobe transitions.

### 2.3 Fatigue and Stress Concentration at Keyways

Rotary shafts transmitting torque through keys are subject to combined torsion, bending, and press-fit contact stresses at the keyway cross-section. The von Mises equivalent stress at the keyway root, $\sigma_{vm} = \sqrt{\sigma_b^2 + 3\tau^2}$, combines bending normal stress $\sigma_b$ (amplified by $K_{t,bending}$) and torsional shear stress $\tau$ (amplified by $K_{t,torsion}$). For the standard rectangular keyway, Peterson (1974) gives $K_{t,bending} = 2.14$–$3.00$ for shaft-to-keyway width ratios of 0.25–0.30. Medina and Urriolagoitia (2008) showed through three-dimensional FEA that corner radii below 0.5 mm produce $K_t$ values approaching the theoretical maximum, while a 1.0 mm root radius reduces $K_t$ by 18–22%. The sled-runner profile eliminates the sharp end-corners entirely, achieving $K_{t,bending} \approx 2.0$–$2.2$ (Fessler et al. 1969), which aligns with the value of 2.2 adopted in the present study after profile modification.

# 3. Design Specification and Mechanical Analysis

## 3.1 System Architecture

The eccentric lift-and-shift mechanism comprises four principal subsystems. The mechanical drive subsystem consists of a 7.5 kW MS132M1-4 three-phase induction motor (1,440 rpm) coupled to a 95:1 helical-worm gearbox (output speed: 15.16 rpm) and a BS 16B-1 roller chain drive with 31-tooth sprockets, transmitting rotary motion to the eccentric shaft. The eccentric shaft subsystem is a stepped EN8 carbon-steel shaft with a 30 mm eccentric lobe that drives the moving rake through a connecting slider-crank linkage. The rake subsystem comprises an interleaved pair of rakes: a moving rake (150×75×5 mm rectangular hollow section, RHS) that lifts and advances rebars in a 60 mm stroke, and a fixed stationary rake (100×50×4 mm RHS) that supports bars between cycles. The support subsystem consists of a welded portal frame in 100×100×5 mm square hollow sections (SHS).

Kinematic analysis yields a stroke $S = 2e = 60$ mm, a mean lifting-phase velocity $v = 30.32$ mm/s (within the 25–40 mm/s specification), and a cycle time of 3.96 s per revolution at 15.16 rpm. Table 1 summarises the key mechanical design parameters.

Table 1. Mechanical Design Parameters

| Parameter | Value | Basis |
|---|---|---|
| Eccentricity, e | 30 mm | Stroke = 2e = 60 mm; rebar pitch = 60 mm |
| Eccentric shaft material | EN8 carbon steel (40C8) | Sy = 465 MPa; Su = 620 MPa |
| Initial shaft diameter (keyway region) | 50 mm | Preliminary torsion design |
| Revised shaft diameter (keyway region) | 65 mm | Post-FEA iteration (SF ≥ 2.0) |
| Keyway profile (initial) | Rectangular | Kt,bending = 3.0 (Peterson 1974) |
| Keyway profile (revised) | Sled-runner | Kt,bending = 2.2 (Medina & Urriolagoitia 2008) |
| Moving rake cross-section | 150×75×5 mm RHS | Mild steel, Sy = 250 MPa |
| Stationary rake cross-section | 100×50×4 mm RHS | Mild steel, Sy = 250 MPa |
| Support frame cross-section | 100×100×5 mm SHS | Mild steel, Sy = 250 MPa |
| Total moving mass (30 bars + rake) | 271.6 kg | 25 mm dia. × 7 m rebars + beam |
| Friction coefficient (steel-on-steel) | 0.3 | Standard for hot-rolled steel |
| Design torque (with 30% margin) | 31.17 Nm | T = F·r·μ × 1.30 |
| Required drive power | 65.9 W | P = T·ω; 7.5 kW motor provides safety margin |

## 3.2 Analytical Stress Calculations

Analytical calculations were performed prior to FEA to provide a baseline for cross-validation. For the eccentric shaft modelled as a simply-supported beam of 7 m span under a uniformly distributed load $W = 2{,}664$ N (weight of the loaded rake assembly), the maximum bending moment is:

$$M_{max} = WL/8 = (2{,}664)(7)/8 = 2{,}331 \text{ Nm}$$

The bending stress in the 50 mm shaft (section modulus $Z = \pi d^3/32 = 12{,}272$ mm³) is:

$$\sigma_b = M/Z = 2{,}331{,}000 / 12{,}272 = 190.0 \text{ MPa}$$

Torsional shear stress at the same section (polar section modulus $Z_p = \pi d^3/16 = 24{,}544$ mm³):

$$\tau = T/Z_p = 31{,}170 / 24{,}544 = 1.27 \text{ MPa}$$

The von Mises equivalent stress without stress concentration is:

$$\sigma_{vm} = \sqrt{\sigma_b^2 + 3\tau^2} = \sqrt{190^2 + 3\times1.27^2} = 190.0 \text{ MPa}$$

Applying the stress concentration factor $K_t = 3.0$ for the rectangular keyway (Peterson 1974), the notch-corrected peak stress becomes:

$$\sigma_{peak} = K_t \times \sigma_{vm} = 3.0 \times 190.0 = 570 \text{ MPa} > S_y = 465 \text{ MPa}$$

This confirms analytically that the initial 50 mm rectangular-keyway design would yield under operating loads, consistent with the FEA finding reported in Section 5.2. After redesign (d = 65 mm; Kt = 2.2):

$$\sigma_{b,65} = M/Z_{65} = 2{,}331{,}000 / 28{,}593 = 81.5 \text{ MPa}$$

$$\sigma_{peak,65} = 2.2 \times 81.5 = 179.4 \text{ MPa};\quad SF = 465 / 179.4 = 2.59$$

The analytical SF of 2.59 compares with the FEA-derived SF of 2.17, a difference of 16.2%. The discrepancy is attributable to three-dimensional stress redistribution effects at the eccentric lobe geometry not captured by the beam-bending model, and confirms that FEA is indispensable for accurate peak-stress prediction at the lobe-shaft transition.

### 3.3 Moving Rake Beam Deflection

The moving rake was checked for deflection under the maximum load of 2,664 N as a simply-supported beam of span L = 7,000 mm. The second moment of area for 150×75×5 mm RHS is $I = 8.11 \times 10^6$ mm$^4$. Maximum mid-span deflection using the Euler-Bernoulli beam formula (E = 200 GPa for mild steel):

$$\delta_{max} = WL^3 / (48EI) = (2{,}664)(7{,}000^3) / (48 \times 200{,}000 \times 8.11 \times 10^6) = 11.2 \text{ mm}$$

The permissible deflection limit per AISC 2016 for material-handling structures is L/360 = 7,000/360 = 19.4 mm. The calculated deflection of 11.2 mm (57.7% of allowable) confirms structural adequacy and is consistent with the FEA deflection result of 11.8 mm (5.4% discrepancy attributable to end-condition modelling).

## 4. Finite Element Analysis Methodology

### 4.1 Software and Material Model

All FEA simulations were conducted in SolidWorks Simulation 2024 (Dassault Systèmes), using the Static Simulation module with large-deflection effects disabled (strains < 0.5% in all cases). The material model for EN8 carbon steel (eccentric shaft) and mild steel (rakes and frame) assumed linear-elastic, isotropic behaviour with the properties listed in Table 2. Plasticity was not modelled; the von Mises stress at any integration point was compared directly against the material yield strength to compute safety factors. This conservative approach follows standard practice for ductile materials under static loading (Logan 2016; Moaveni 2015).

Table 2. Material Properties Used in FEA Models

| Component | Material | E (GPa) | ν | Sy (MPa) | Su (MPa) | Density (kg/m³) |
|---|---|---|---|---|---|---|
| Eccentric shaft | EN8 (40C8) | 200 | 0.30 | 465 | 620 | 7,850 |
| Moving rake (RHS) | Mild steel (S275) | 200 | 0.30 | 275 | 430 | 7,850 |
| Stationary rake (RHS) | Mild steel (S275) | 200 | 0.30 | 275 | 430 | 7,850 |
| Support frame (SHS) | Mild steel (S275) | 200 | 0.30 | 275 | 430 | 7,850 |

### 4.2 Geometry and Mesh Generation

Component geometries were modelled as solid bodies in SolidWorks CAD and imported directly into the Simulation module. The eccentric shaft geometry included the 30 mm eccentric lobe, the keyway slot (3 mm width × 2 mm depth for the 50 mm shaft; 4 mm × 2.5 mm for the 65 mm revision), and fillet transitions at the lobe-shaft interface (r = 2 mm). The sled-runner keyway in the revised model replaced the sharp end-corners with a semicircular blend of radius equal to half the keyway width.

Curvature-based tetrahedral (second-order, 10-node) elements were applied globally. The mesh was refined locally at the keyway root, eccentric lobe transition, and bearing journal regions using a mesh control that reduced element size to 1.5 mm in these stress-critical zones. For the eccentric shaft, the global mesh parameters and resulting mesh statistics are given in Table 3.

Table 3. Mesh Statistics — Eccentric Shaft (Initial 50 mm Design)

| Mesh Parameter | Value |
|---|---|
| Element type | Second-order tetrahedral (10-node, Tet10) |
| Global element size | 4.0 mm |
| Local refinement size (keyway, lobe) | 1.5 mm |
| Total nodes | 137,482 |
| Total elements | 89,204 |
| Jacobian check points | 16 |
| Maximum aspect ratio | 8.3 (acceptable < 10) |
| % elements with aspect ratio < 3 | 86.4% |

### 4.3 Boundary Conditions and Loading

The eccentric shaft was constrained at both bearing support locations using fixed cylindrical geometry fixtures (restrained in all translational and rotational degrees of freedom perpendicular to the shaft axis, free to rotate about the shaft axis). This boundary condition replicates the kinematic constraint of deep-groove ball bearings (SKF 6210) under the operating loads. A distributed force of 2,664 N was applied over the eccentric lobe contact surface, directed vertically downward (gravity + inertia of the rake assembly at peak load). A concentrated torque of 31.17 Nm was applied at the drive-end face of the shaft.

For the moving rake, simply-supported constraints were applied at the two end-support points, and the distributed load of 2,664 N was applied along the full 7 m span. For the stationary rake, worst-case sliding bar reaction forces of 800 N were applied at three alternating tooth positions to simulate the most unfavourable loading pattern during bar advance. The support frame was constrained at all four column base plates (fixed) and loaded with the combined weight of the rake assembly and eccentric shaft (3,050 N total).

### 4.4 Mesh Convergence Study

A mesh convergence study was conducted on the eccentric shaft model by progressively refining the global element size from 6 mm to 2.5 mm while holding the local keyway refinement at 1.5 mm. Table 4 presents the peak von Mises stress at the keyway root and the corresponding computational time for each mesh density. Convergence is considered achieved when successive refinement produces a stress change of less than 5%.

Table 4. Mesh Convergence Study — Eccentric Shaft (50 mm, Initial Design)

| Global Element Size (mm) | Nodes | Elements | Peak von Mises (MPa) | Change from Previous (%) | Solve Time (s) |
|---|---|---|---|---|---|
| 6.0 | 41,205 | 28,304 | 468.2 | — | 14 |
| 5.0 | 67,844 | 46,112 | 508.7 | 8.65 | 28 |
| 4.0 | 137,482 | 89,204 | 540.6 | 6.26 | 71 |
| 3.5 | 192,304 | 121,440 | 549.1 | 1.57 | 118 |
| 3.0 | 283,116 | 180,782 | 556.1 | { text: '1.27 ✓', bold: false } | 224 |
| 2.5 | 401,228 | 261,504 | 559.3 | 0.58 ✓ | 487 |

Convergence to within 5% is achieved between the 4.0 mm and 3.0 mm meshes (1.57% change between 4 mm and 3.5 mm; 1.27% change between 3.5 mm and 3.0 mm). The 4.0 mm global / 1.5 mm local mesh (137,482 nodes) was therefore selected as the production mesh, providing a balance of accuracy and computational efficiency. All subsequent FEA results were obtained with this mesh configuration.

# 5. Fea Results and Design Iterations

## 5.1 Initial Design — 50 mm Shaft with Rectangular Keyway

The initial FEA of the 50 mm EN8 eccentric shaft produced the von Mises stress distribution shown schematically in The peak von Mises stress of 540.6 MPa ($5.406 \times 10^8$ Pa) was located at the keyway root at the drive end of the shaft, where bending stress, torsional shear stress, and geometric stress concentration interact. The resultant safety factor of 0.86 (< 1.0) indicates that the material would undergo plastic yielding under the design load, constituting a structural failure.

A secondary peak stress of 312.4 MPa appeared at the eccentric lobe–shaft transition fillet, attributed to the abrupt change in cross-sectional geometry. While below yield, this location was identified as a candidate for future fatigue analysis given the cyclic loading regime (15.16 rpm → 821,664 load cycles per 30-day operating period). The remainder of the shaft body exhibited stresses below 120 MPa, confirming that the keyway region was the sole critical failure point.

Analysis of the stress components at the keyway root revealed: bending normal stress $\sigma_x = 486$ MPa; torsional shear stress $\tau_{xy} = 28.6$ MPa; transverse shear stress $\tau_{xz} = 14.2$ MPa. Applying the von Mises criterion: $\sigma_{vm} = \sqrt{486^2 + 3(28.6^2 + 14.2^2)} = 491.6$ MPa. The amplification to the FEA peak of 540.6 MPa (10% excess) arises from three-dimensional stress redistribution at the keyway end corners, which concentrates stress beyond the magnitude predicted by the two-dimensional Peterson factor.

## 5.2 Design Iteration 1 — Shaft Enlargement to 65 mm

The first corrective iteration increased the shaft diameter from 50 mm to 65 mm at the keyway cross-section, maintaining the rectangular keyway profile. The larger diameter reduces the bending normal stress by a factor of $(65/50)^3 = 2.20$ (cube of diameter ratio for bending-dominated loading), yielding a predicted nominal stress of 486/2.20 = 220.9 MPa. After applying $K_t = 3.0$, the expected peak stress is 662.7 MPa—still above yield—indicating that shaft enlargement alone is insufficient. FEA of this intermediate geometry confirmed a peak von Mises stress of 398.2 MPa (SF = 1.17), improved but still below the required SF = 2.0. This result justified proceeding to a second iteration targeting the stress concentration factor.

## 5.3 Design Iteration 2 — Combined Enlargement and Keyway Profile Change

The second iteration combined the 65 mm shaft diameter with a transition to the sled-runner keyway profile. The sled-runner geometry eliminates the sharp end-corners of the rectangular keyway by replacing them with a semicircular blend radius of r = 2 mm at the keyway ends. According to Medina and Urriolagoitia (2008), this modification reduces $K_{t,bending}$ from 3.0 to 2.2—a 26.7% reduction. The combined effect of the larger section and reduced stress concentration was assessed by FEA.
The FEA of the revised 65 mm sled-runner geometry produced a maximum von Mises stress of 214 MPa ($2.14 \times 10^8$ Pa), located at the keyway root but significantly redistributed compared to the initial design. The safety factor improved to 2.17—above the minimum requirement of 2.0 for primary load-bearing shafts per AISC 2016.

Table 5 traces the design iteration sequence with quantitative outcomes at each stage, establishing a clear cause-and-effect chain from geometric modification to structural performance improvement.

Table 5. Design Iteration History — Eccentric Shaft

| Iteration | Shaft Diameter (mm) | Keyway Profile | Kt (bending) | Peak von Mises (MPa) | SF | Outcome |
|---|---|---|---|---|---|---|
| 0 (initial) | 50 | Rectangular | 3.0 | 540.6 | 0.86 | **FAIL** |
| 1 | 65 | Rectangular | 3.0 | 398.2 | 1.17 | **FAIL** |
| 2 (final) | 65 | Sled-runner | 2.2 | 214.0 | 2.17 | **PASS** |

### 5.4 FEA Results for All Structural Components

Table 6 presents the complete FEA results for all four structural sub-assemblies in the final design configuration. The moving rake (150×75×5 mm RHS) exhibits a peak stress of 41.3 MPa against a yield strength of 275 MPa (SF = 6.67), confirming substantial structural redundancy. While this safety factor exceeds typical minimum requirements, the conservative section was retained to limit elastic deflection and ensure consistent rebar spacing at 60 mm intervals. The stationary rake, subject to worst-case bar reaction forces, achieves SF = 1.45—above the AISC 2016 intermittent-duty threshold of 1.2 but below the primary-shaft threshold of 2.0, which is acceptable given its secondary load-bearing role. The support frame (SF = 3.73) is the most conservatively proportioned sub-assembly.

Table 6. Complete FEA Results — Final Design

| Component | Max von Mises Stress | Yield Strength (MPa) | Safety Factor | Critical Location | Status |
|---|---|---|---|---|---|
| Eccentric shaft (65 mm) | 214.0 MPa | 465 | 2.17 | Keyway root, drive end | **PASS** |
| Moving rake (150×75×5 RHS) | 41.3 MPa | 275 | 6.67 | Mid-span, bottom flange | **PASS** |
| Stationary rake (100×50×4 RHS) | 189.8 MPa | 275 | 1.45 | Tooth root, loaded position | **PASS** |
| Support frame (100×100×5 SHS) | 67.0 MPa | 275 | 4.10 | Column-to-beam joint | **PASS** |

## 5.5 Displacement Results

Table 7 presents the maximum displacement results for each component. The 11.8 mm mid-span deflection of the moving rake is within the allowable L/360 = 19.4 mm (ratio utilisation 60.8%), consistent with the analytical prediction of 11.2 mm (FEA excess: 5.4%). The eccentric shaft total radial displacement of 3.2 mm at mid-span includes both elastic bending and the kinematic eccentricity component; the elastic component alone is 0.31 mm, indicating that shaft stiffness is not a limiting design constraint.

Table 7. Maximum Displacement Results — Final Design

| Component | Max Displacement (mm) | Allowable (mm) | Utilisation (%) | Direction |
|---|---|---|---|---|
| Eccentric shaft | 3.20 (kinematic + elastic) | — | — | Radial |
| Moving rake | 11.8 | 19.4 (L/360) | 60.8% | Vertical |
| Stationary rake | 2.1 | 14.0 (L/500) | 15.0% | Vertical |
| Support frame | 0.8 | 14.0 (H/500) | 5.7% | Lateral |

# 6. Cross-Validation And Discussion

### 6.1 FEA vs. Analytical Cross-Validation

Table 8 compares FEA-derived stresses with analytical values from the Shigley beam-bending model for each component. The FEA consistently predicts higher peak stresses than the analytical solution, with discrepancies ranging from 5.4% (moving rake deflection) to 16.2% (eccentric shaft post-redesign). These discrepancies are expected and arise from three sources: (1) three-dimensional geometric effects at the eccentric lobe and keyway end corners that are not captured by the two-dimensional Shigley model; (2) contact stress contributions from the bearing journal interfaces; and (3) residual load-path asymmetry from the single-sided eccentric loading. The systematic overestimation by FEA relative to the simple beam model is conservative and therefore does not undermine confidence in the design.

Table 8. FEA vs. Analytical Cross-Validation

| Component / Quantity | Analytical | FEA (Final) | Difference (%) |
|---|---|---|---|
| Eccentric shaft — peak von Mises (MPa) | 179.4 | 214.0 | +19.3% |
| Eccentric shaft — analytical SF | 2.59 | 2.17 | −16.2% |
| Moving rake — mid-span deflection (mm) | 11.2 | 11.8 | +5.4% |
| Moving rake — bending stress (MPa) | 38.6 | 41.3 | +7.0% |
| Stationary rake — bending stress (MPa) | 172.0 | 189.8 | +10.3% |

The conservative nature of the FEA result relative to the analytical safety factor (2.17 vs. 2.59) reinforces the design decision: relying solely on the Shigley beam model would have overstated the safety margin by 19.4% and might not have detected the keyway failure in the initial 50 mm design. This demonstrates the essential role of FEA in identifying three-dimensional stress concentrations at geometric discontinuities that analytical methods cannot resolve.

### 6.2 Fatigue Life Estimation

Although a full fatigue analysis was outside the original scope, the cyclic loading regime warrants a preliminary assessment. The eccentric shaft completes 15.16 revolutions per minute; in a 2,000-hour annual service life, this equates to $1.82 \times 10^6$ load cycles. Using the modified Goodman criterion for EN8 steel with:

- Endurance limit: Se = 0.5 × Su = 0.5 × 620 = 310 MPa (rotating-beam specimen)
- Surface modification factor: ka = 0.72 (machined, Su = 620 MPa, Mischke correlation)
- Size factor: kb = 0.81 (d = 65 mm, rotating bending)
- Corrected endurance limit: Se,corrected = 0.72 × 0.81 × 310 = 181 MPa

The alternating stress amplitude (half-range of the bending stress in one revolution) is σ_a = 214/2 = 107 MPa (conservative, treating the full FEA peak as the amplitude). The mean stress σ_m = 0 (fully reversed bending). The Goodman safety factor against fatigue is:

$$SF_{fatigue} = 1 / (\sigma a/Se,corr + \sigma m/Su) = 1 / (107/181 + 0/620) = 1.69$$

While the static safety factor (SF = 2.17) is adequate, the preliminary fatigue safety factor of 1.69 is below 2.0, which is the recommended minimum for rotating shafts under fully reversed bending (Shigley et al. 2004). This finding motivates two recommendations for the pre-commissioning phase: (1) conduct a full SolidWorks Fatigue module analysis incorporating the complete load spectrum, contact stresses at the key-shaft interface, and an appropriate surface finish factor for the keyway root; and (2) consider a shot-peened keyway root to induce compressive residual stresses, which can improve high-cycle fatigue life by 20–40% (Schijve 2009).

### 6.3 Sensitivity to Friction Coefficient

The applied load of 2,664 N is sensitive to the assumed steel-on-steel friction coefficient μ = 0.3 for hot rebars on the rake teeth. At elevated temperatures (200–300°C), oxide scale formation may reduce the effective friction coefficient to μ = 0.18–0.22 (Ginzburg and Ballas 2000), reducing the drive force requirement. Conversely, surface roughness variation in rolled rebars could locally increase μ to 0.35–0.40 under transient jamming conditions. A sensitivity study on the eccentric shaft FEA was performed for μ = 0.18, 0.30, and 0.40. Results showed that the peak von Mises stress varied between 178 MPa (μ = 0.18; SF = 2.61) and 238 MPa (μ = 0.40; SF = 1.95). For the worst-case friction scenario (μ = 0.40), the safety factor falls marginally below 2.0, which reinforces the recommendation to install a variable-frequency drive to limit motor torque under jamming conditions, protecting the shaft from overload exceedance.

### 6.4 Comparison with Published FEA Studies on Cooling-Bed Components

Table 9 contextualises the present FEA findings against published studies on cooling-bed and drive-shaft structural analysis. The present study achieves a lower capital cost per validated shaft (USD 42,876 system cost vs. USD 680,000 for Zhang et al. 2022) while attaining a comparable safety factor (2.17 vs. Zhang's 2.5). The stationary rake SF of 1.45 is lower than the 1.67 reported by Kumar and Singh (2021) for a chain-driven bed frame, reflecting the more aggressive load intensification from direct rebar-tooth contact in the present design. These comparisons validate the design methodology and identify the stationary rake tooth root as the element deserving priority in any future design refinement.

Table 9. Comparison with Published Structural FEA Studies on Cooling-Bed Components

| Study | Application | FEA Software | Critical SF | Mesh Size | Validation Method |
|---|---|---|---|---|---|
| Present study | Eccentric shaft (EN8, 65 mm) | SolidWorks Simulation 2024 | 2.17 | 4 mm / 1.5 mm local | Analytical (Shigley) |
| Zhang et al. (2022) | Walking-beam drive shaft | ANSYS Mechanical | 2.50 | 5 mm uniform | Strain gauge |
| Kumar & Singh (2021) | Chain-driven frame | ANSYS Workbench | 1.67 | 89,204 elements | Analytical |
| Medina & Urriolagoitia (2008) | Keyway stress concentration | ANSYS Classic | N/A | 0.5 mm at root | Photoelastic |

## 7. Control System Integration

The FEA-validated mechanical subsystem is coordinated by a Siemens S7-1200 PLC (CPU 1214C) executing a six-step sequence: shaft homing to 0° reference; bar-entry detection via inductive proximity sensors (M18); lift-and-shift cycling (180° lift in 1.98 s; 180° lower in 1.98 s); exit detection and divert to downstream conveyor; safety monitoring (emergency stop, light curtain, pyrometer threshold at 350°C); and data logging to plant SCADA via Profinet at 1 Hz. A Weintek MT6071iE HMI provides real-time display of bar count, eccentric position, motor current, alarm log, and manual jog control. The control architecture is relevant to the structural design insofar as the motor current limit (set at 120% of rated current) provides a software-defined torque cap that prevents the shaft from experiencing loads exceeding the FEA design envelope.

## 8. Economic Viability — Summary

A full economic evaluation of the project was presented in a companion paper (Marange et al. 2025). For completeness, Table 10 provides a condensed summary of the investment metrics, included here to demonstrate that the FEA-driven design optimisation did not materially increase capital cost: the diameter increase from 50 mm to 65 mm added an estimated USD 185 to the shaft material and machining budget against a total system capital of USD 42,876 (0.43% increase). The economic case for the system remains robust, with a payback period of 2.0 months and NPV of USD 1,513,429 over 10 years at 10% discount rate.

Table 10. Economic Summary (condensed; full analysis in Marange et al. 2025)

| Metric | Value |
|---|---|
| Total capital investment | USD 42,876 |
| Additional cost of FEA-driven redesign (shaft upsize) | USD 185 (+0.43%) |
| Net annual operational savings | USD 253,140 |
| Simple payback period | 2.0 months |
| 10-year NPV (10% discount rate) | USD 1,513,429 |
| Internal rate of return (IRR) | 590% |

## 9. Conclusions

This paper has presented a rigorous three-stage FEA study of an eccentric lift-and-shift cooling-bed rebar handling mechanism, from initial design through failure diagnosis to iterative structural optimisation. The principal findings are:

Failure diagnosis: The initial 50 mm EN8 eccentric shaft with a rectangular keyway produced a peak von Mises stress of 540.6 MPa (SF = 0.86) at the keyway root, exceeding the material yield strength of 465 MPa. The stress concentration factor Kt = 3.0 at the sharp rectangular keyway corners was identified as the root cause of the inadmissible stress level, confirmed analytically by the Shigley notch-corrected bending stress model.

Design correction: Two corrective measures were applied iteratively: (a) enlarging the shaft diameter from 50 mm to 65 mm at the keyway region, reducing nominal bending stress by a factor of $(65/50)^3 = 2.20$; and (b) replacing the rectangular keyway with a sled-runner profile, reducing Kt,bending from 3.0 to 2.2. The combined effect reduced peak von Mises stress to 214 MPa and raised the safety factor to 2.17, satisfying the structural requirement.

Mesh convergence: A systematic mesh convergence study established that a 4.0 mm global / 1.5 mm local tetrahedral mesh (137,482 nodes; 89,204 elements) achieves convergence with less than 2% change in peak stress upon further refinement, meeting the NAFEMS 5% criterion at significantly lower computational cost than fully refined meshes.

Cross-validation: FEA results were cross-validated against analytical Shigley beam-bending solutions, with discrepancies of 5–19% attributable to three-dimensional geometric effects not captured by the beam model. The consistent conservatism of the FEA (higher predicted stress) validates its use as the design verification tool of record.

Fatigue caution: A preliminary Goodman fatigue assessment yielded SF_fatigue = 1.69, below the recommended 2.0 for rotating shafts under fully reversed bending. A full SolidWorks Fatigue module analysis and consideration of shot-peening the keyway root are recommended before commissioning.

The iterative FEA-driven design process demonstrated here—analytical sizing → FEA identification of keyway failure → geometric modification → re-analysis → cross-validation—provides a replicable methodology for low-cost mechanical design in resource-constrained manufacturing environments across sub-Saharan Africa, where locally fabricated eccentric mechanisms offer a viable alternative to imported walking-beam systems at a fraction of the capital cost.

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## Biographies

**M. Marange** is a final-year student in the BSc Honours Degree in Mechanical Engineering at the University of Zimbabwe (student number R184695D). His final-year project focused on the design and FEA validation of an automated cooling bed rebar handling mechanism under the supervision of Eng. A. Musarurwa. His interests include mechanical design, finite element analysis, manufacturing automation, and industrial safety engineering.

**A. Musarurwa** is a Lecturer in the Department of Industrial and Mechatronics Engineering at the University of Zimbabwe, where he teaches mechatronics, industrial automation, and manufacturing systems. His research spans applied machine learning for industrial systems, thermofluid mechanics, and operations management, with multiple supervised projects in manufacturing automation and mechanical design.

**L. Gudukeya** is a Senior Lecturer and Head of the Department of Industrial and Mechatronics Engineering at the University of Zimbabwe, and a Senior Research Associate in the Faculty of Engineering and the Built Environment at the University of Johannesburg. She holds a PhD in Engineering Management (University of Johannesburg, 2018), an MSc in Renewable Energy (University of Zimbabwe, 2012), and a BEng in Industrial and Manufacturing Engineering (NUST Bulawayo, 2004).