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A Policy-Based Rough Optimization with Large Neighborhood Search for Dual-Resource Flexible Job Shop Scheduling Under Time-of-Use Electricity Tariffs

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Abstract

Manufacturing systems face growing pressure to improve operational efficiency while controlling electricity costs under Time-of-Use (TOU) tariffs, where production timing directly affects the electricity bill. This challenge is especially acute in flexible job shops, where each operation must be assigned not only to an eligible machine but also to a qualified worker. This paper studies the dual-resource flexible job shop scheduling problem under TOU tariffs (DRC-FJSP-TOU), which jointly determines machine assignments, worker assignments, and operation start times. A mathematical formulation is developed to minimize electricity cost through both energy and demand charges. To address the problem's computational difficulty, a policy-based rough optimization with large neighborhood search (Pro-LNS) framework is proposed. The method combines a PPO-based constructive policy with an adaptive large neighborhood search phase, and the resulting solution is used to warm-start an exact mixed-integer linear programming (MILP) model. Computational experiments on 11 benchmark instances under a matched 3600-second time budget show that the standalone MILP reaches optimality only for the smallest instance, whereas the Pro-LNS-assisted MILP consistently achieves lower objective values and tighter optimality gaps, reducing final gaps to below 8% across all nontrivial instances. These results show that the proposed framework is effective for electricity-cost-aware dual-resource scheduling under TOU tariffs.

Keywords
Dual-Resource Flexible Job Shop Scheduling, Time-of-Use Electricity Tariffs, Mixed-Integer Linear Programming, Reinforcement Learning, Large Neighborhood Search

1. Introduction
Manufacturers today are being pushed to improve cost efficiency and energy performance at the same time, yet many plants cannot respond through immediate capital investment alone. In such a setting, production scheduling becomes one of the few levers that can be changed quickly, repeatedly, and at relatively low cost. That lever becomes even more consequential under Time-of-Use (TOU) electricity tariffs, because two schedules that complete the same jobs can still generate very different electricity bills simply because processing occurs in different tariff windows (Ho et al. 2021; Catanzaro et al. 2023).

Historically, scheduling and electricity pricing have not been integrated neatly. Most scheduling models were developed around machines, with the main effort devoted to assigning operations to eligible machines and sequencing them efficiently. Labor was often simplified, abstracted, or assumed available when needed. Real factories do not operate so cleanly. In many shop floors, a machine assignment is meaningful only if an appropriately qualified worker is also available to execute it, which makes machine-only scheduling an incomplete representation of actual production feasibility (Homayouni and Fontes 2021; Agrawal et al. 2023).

The problem addressed in this paper arises by adding these realities one at a time. The flexible job shop scheduling problem (FJSP) allows each operation to be processed on one among several alternative machines, thereby capturing routing flexibility in modern production systems. A dual-resource constraint (DRC) extension then requires each operation to be assigned not only to a feasible machine but also to a qualified worker. When Time-of-Use (TOU) pricing is added, the cost of processing becomes start-time dependent because the planning horizon is divided into tariff periods with different electricity prices. The resulting problem is the dual-resource flexible job shop scheduling problem under Time-of-Use electricity tariffs (DRC-FJSP-TOU), in which machine choice, worker choice, and timing must be determined jointly rather than separately (Meng et al. 2019; Jiang and Wang 2020).

This combined perspective is important for more than one reason. First, TOU-aware scheduling can materially reduce production cost by shifting load away from expensive periods and by accounting more carefully for energy consumption. Second, it provides better decision support because it reflects the fact that execution quality depends not only on machine availability but also on worker flexibility and skill compatibility. Third, it offers a practical path toward more energy-conscious production without requiring immediate changes to the physical asset base, which is especially valuable when operational improvement must come from better planning rather than new equipment (Guo et al. 2022; Geng et al. 2020).

Despite this practical importance, the problem is computationally difficult. The FJSP is NP-hard, and the addition of dual resources and tariff-sensitive timing enlarges the decision space even further. Each operation must be matched with an eligible machine, a qualified worker, and a start time while satisfying precedence relations, machine-capacity constraints, worker-capacity constraints, and time-dependent electricity-cost effects. This tight coupling makes exact formulations powerful for problem representation and benchmarking, but increasingly difficult to solve as instance size grows (Boyer et al. 2021).

This tension between modeling fidelity and computational tractability motivates the present study. In this paper, electricity cost is optimized directly, while machine and worker requirements are enforced as hard feasibility conditions throughout the schedule. To support that goal, a mathematical formulation is used to represent the problem rigorously, and a policy-based rough optimization with large neighborhood search (Pro-LNS) is proposed to handle the search process more effectively. The policy is used to construct a rough feasible schedule quickly, after which large neighborhood search selectively destroys and repairs portions of the solution so that costly regions can be improved more systematically.

Accordingly, this study contributes in two ways.

1. Problem formulation: A mathematical model is developed for the DRC-FJSP-TOU. The formulation assigns each operation to exactly one eligible machine, one qualified worker, and one start time; enforces job

precedence together with machine-capacity and worker-capacity constraints; and minimizes total electricity cost through both Time-of-Use energy charges and demand charges, represented by separate peak-load variables for on-peak and off/super-off-peak periods.

2. Solution and evidence: A policy-based rough optimization with large neighborhood search (Pro-LNS) is proposed and evaluated on 11 benchmark instances. For each instance, two solution pipelines are examined under a matched time budget. In the first, the mixed-integer linear program (MILP) is solved directly for up to 3600 seconds. In the second, Pro-LNS is first used to generate a high-quality feasible schedule, and that schedule is then supplied as a warm-start incumbent to the MILP, with the combined Pro-LNS runtime and warm-started MILP runtime constrained to the same 3600-second limit. This design allows a fair comparison between standalone exact optimization and Pro-LNS-assisted exact optimization. The computational study reports objective values, CPU runtimes, and optimality gaps, and shows that Pro-LNS consistently provides strong incumbents that materially improve the quality of the warm-started MILP solutions and substantially reduce final optimality gaps relative to solving the MILP alone.

The remainder of this paper is organized as follows. Section 2 reviews the literature on flexible job shop scheduling, dual-resource-constrained scheduling, and energy-aware scheduling under Time-of-Use tariffs. Section 3 presents the mathematical formulation and the proposed Pro-LNS methodology. Section 4 reports the computational results and discusses the performance of the proposed approach. Section 5 concludes the paper and outlines future research directions.

2. Literature Review

The flexible job shop scheduling problem (FJSP) has remained important because it captures a defining feature of modern manufacturing systems: routing flexibility. Unlike the classical job shop, where each operation is tied to a predetermined machine, the FJSP allows an operation to be processed on one among several alternatives, which makes the model more realistic but also substantially enlarges the decision space through the joint interaction of routing and sequencing decisions (Özgüven et al. 2010; Wu and Wu 2017). As this literature matured, it became increasingly clear that machine flexibility alone was not enough to represent actual production environments. In many workshops, an operation is executable only when a suitable worker is available in addition to a suitable machine, which led to dual-resource constrained flexible job shop models in which machine assignment and worker assignment must be decided together rather than sequentially (Lei and Guo 2014; Zheng and Wang 2016). Later studies further enriched this perspective by incorporating worker learning, worker-dependent loading and unloading effects, multi-skilled operators, and sequence-dependent setups, all of which reinforced the idea that human resources are not peripheral to scheduling feasibility but central to it (Wu et al. 2018; Zhang et al. 2021). As a result, the literature now treats worker-aware and dual-resource formulations as a necessary step toward realism, even though they also intensify the combinatorial difficulty of the underlying problem (Barak et al. 2024; Wang et al. 2025).

A parallel stream of research emerged from the growing importance of energy-aware manufacturing. Early work in this direction focused on reducing energy consumption by modeling machine states such as processing, idle, and shutdown modes, thereby showing that scheduling affects not only completion time but also how efficiently resources consume power over the production horizon (Meng et al. 2019; Meng et al. 2022). From there, the literature moved toward electricity pricing structures that vary over time. Once Time-of-Use (TOU) tariffs are introduced, start time becomes an economic decision variable, because the same operation may incur different electricity costs depending on the tariff window in which it is processed (Moon and Park 2014; Jiang and Wang 2020). This shift changed the role of scheduling in a fundamental way: the problem was no longer only to assign and order operations efficiently, but also to place load strategically across the tariff horizon. Studies on TOU-aware flexible job shops showed that meaningful cost savings can be achieved through timing adjustments alone, even when makespan is held fixed or nearly fixed, which made tariff-aware scheduling especially attractive for firms seeking operational improvement without immediate capital investment (Park and Ham 2022; Ho et al. 2021). At the same time, green and multi-resource extensions began to connect electricity pricing with labor, carbon, and service considerations, suggesting that cost, sustainability, and workforce realism are increasingly converging within the same scheduling framework (Gong et al. 2019; Jia et al. 2024).

As the modeling side became richer, exact optimization emerged as the main tool for formal problem representation. Mixed-integer linear programming, integer programming, and constraint programming models were used to encode flexible routing, worker assignment, energy states, transport effects, and tariff-sensitive timing in a mathematically precise way (Agrawal et al. 2023; Park and Ham 2022). These models made an essential contribution to the literature because they clarified the structure of the problem and provided exact benchmarks for smaller instances. Exact formulations also made it possible to isolate the value of additional realism, whether in the form of worker flexibility, richer energy accounting, or time-varying electricity prices, without ambiguity in the solution process (Meng et al. 2019; Ham et al. 2021). Yet the same literature also showed a recurring limit: once several realistic features are modeled jointly, exact methods rapidly become difficult to scale. Even without combining dual resources and Time-of-Use tariffs in a single formulation, generalized FJSP variants and energy-aware extensions often outgrow what mixed-integer linear programming (MILP) or constraint programming (CP) can solve comfortably at larger sizes, which is why exact models are frequently used for benchmarking or small-instance validation rather than as the sole practical optimizer (Boyer et al. 2021; Meng et al. 2020).

This computational barrier pushed the field toward hybrid and metaheuristic methods. Rather than relying on exhaustive enumeration, these approaches seek high-quality schedules by combining global exploration with local improvement, often in ways that retain problem structure while remaining scalable. Within this stream, neighborhood-based improvement has become one of the most persistent design patterns. Variable neighborhood search, generalized variable neighborhood search, and related local-improvement frameworks have been used successfully in dual-resource, energy-aware, and distributed flexible job shop settings, showing that structured neighborhood moves can substantially strengthen solution quality when exact search becomes impractical (Lei and Guo 2014; Meng et al. 2019). The same pattern appears in matheuristics, where a metaheuristic first generates a promising incumbent and an exact or constraint-based component is then used to repair, intensify, or refine the solution. Such pipelines have been shown to balance quality and runtime effectively in complex flexible job shop scheduling problem (FJSP) variants involving lot streaming, reconfigurable machines, and distributed scheduling, which makes them especially relevant when a high-fidelity exact model exists but cannot be relied upon alone at larger scales (Fan et al. 2023; Meng et al. 2024). More recent work has gone even further by embedding adaptive large-neighborhood ideas inside CP-based frameworks, again reinforcing the view that neighborhood design is not a secondary enhancement but one of the main engines of scalable performance in modern scheduling research (Kasapidis et al. 2025; Sun et al. 2023).

More recently, literature has begun to replace handcrafted decision rules with learned policies. Early reinforcement learning approaches typically operated over rule sets, selecting among dispatching rules in response to changing shop states rather than committing to a single fixed heuristic (Luo 2020; Han and Yang 2020). Subsequent deep reinforcement learning moved beyond rule selection and toward end-to-end constructive scheduling, where a learned policy directly generates operation-machine assignments one decision at a time. This shift was strengthened by graph neural networks, attention mechanisms, and multi-action policy structures that allowed richer state representations and more expressive scheduling decisions in flexible job shops (Song et al. 2023; Wang et al. 2024). A second line of work used learning not to construct the schedule directly, but to guide the search process itself. In these studies, reinforcement learning selected neighborhoods, shaking sequences, local search operators, or other search components inside broader metaheuristic frameworks, thereby combining learned guidance with the intensification power of neighborhood-based search (Chang et al. 2025; Zhang et al. 2024). Together, these works suggest a clear progression in the field: from fixed heuristics to adaptive rule selection, to learned constructive policies, and finally to hybrid systems in which learning shapes how the search moves through the solution space.

Despite this progress, the intersection most relevant to the present paper remains underexplored. Dual-resource scheduling has been studied with energy awareness, but usually in terms of total energy consumption rather than electricity cost under tariff structure (Meng et al. 2019; Zhang et al. 2024). Time-of-Use-aware scheduling has shown that start-time decisions materially affect electricity cost, but most of that literature remains centered on machine-based or single-resource settings rather than worker-constrained flexible job shops (Park and Ham 2022; Jiang and Wang 2020). Learning-based scheduling has demonstrated that policies can either construct strong schedules or guide neighborhood search effectively, yet these methods rarely incorporate Time-of-Use-based electricity cost as the central objective and do not typically integrate with an exact warm-start pipeline for final refinement (Chang et al. 2025; Song et al. 2023). The resulting gap is therefore not simply the absence of one more algorithm. It marks the point at which several mature research directions naturally converge: dual-resource scheduling, tariff-aware cost optimization, policy-guided decision making, neighborhood-based improvement, and exact-model benchmarking. Bringing these elements together is not only a logical continuation of the literature, but also a necessary step toward scheduling methods that are both computationally effective and operationally credible. This convergence motivates the present study, in which the dual-resource flexible job shop scheduling problem under Time-of-Use electricity tariffs (DRC-FJSP-TOU) is formulated as a mathematical model for electricity-cost minimization and addressed computationally through a policy-based rough optimization with large neighborhood search (Pro-LNS) within a matched-time benchmarking framework against exact optimization.

3. Methods

This section presents the methodology adopted in this study in three parts. Section 3.1 develops the mathematical formulation of the dual-resource flexible job shop scheduling problem under Time-of-Use electricity tariffs (DRC-FJSP-TOU), including the decision variables, objective function, and operational constraints. Section 3.2 then describes the proposed policy-based rough optimization with large neighborhood search (Pro-LNS) framework and explains the schedule construction and neighborhood-based improvement paradigm. Finally, Section 3.3 outlines the experimental design and computational setup used to evaluate the proposed approach, including the benchmark instances, comparison structure, and performance measures.

3.1 Mathematical Formulation of the DRC-FJSP-TOU

The dual-resource flexible job shop scheduling problem under Time-of-Use electricity tariffs is formulated as a mathematical model. Each operation must be assigned to exactly one eligible machine, exactly one qualified worker, and exactly one start time. The formulation enforces technological precedence, machine-capacity, and worker-capacity constraints while minimizing total electricity cost. The cost consists of two components: an energy charge determined by Time-of-Use tariff rates and a demand charge determined by peak coincident load during on-peak and off/super-off-peak periods. The notation used in the model is summarized in Table 1.

Table 1. Sets, indices, parameters, and decision variables for the DRC-FJSP-TOU model

Symbol	Description
$J = 0,1,\dots,n-1$	Set of jobs
$O_j = 0,1,\dots,n_j-1$	Set of operations of job j
$M = 0,1,\dots,\lvert M\rvert-1$	Set of machines
$W = 0,1,\dots,\lvert W\rvert-1$	Set of workers
$T = 0,1,\dots,H-1$	Set of discrete time periods
$A_{jo} \subseteq M \times \mathbb{Z}_+$	Feasible machine-processing-time pairs for operation (j,o)
F_{jo}	Feasible assignment set for operation (j,o), defined as $F_{jo} = (m,\ell,s,p)\colon (m,p) \in A_{jo},\ \ell \in W,\ a_{\ell m} = 1,\ s \in 0,\dots,H-p$
$a_{\ell m} \in 0,1$	Equals 1 if worker ℓ is qualified to operate machine m, and 0 otherwise
$q_m \in \mathbb{Z}_+$	Power consumption of machine m in kW
H	Upper bound on the planning horizon
r^{on}	On-peak energy price
r^{off}	Off-peak energy price
r^{super}	Super-off-peak energy price
α^{on}	On-peak demand charge rate
$\alpha^{offsuper}$	Off/super-off-peak demand charge rate
$hour(t)$	Hour corresponding to time period t, defined as $t\ mod\ 24$
$type(t)$	Tariff category at time t: super, on, or off
r_t	Energy rate at time period t
c_{mps}	Energy-cost coefficient for assigning an operation to machine m with duration p starting at time s, defined as $c_{mps} = \sum_{\tau=s}^{s+p-1} q_m r_\tau$
$x_{jom\ell s} \in 0,1$	Equals 1 if operation (j,o) starts at time s on machine m with worker ℓ, and 0 otherwise

Symbol	Description
$P^{on} \geq 0$	Maximum coincident load during on-peak periods
$P^{offsuper} \geq 0$	Maximum coincident load during off-peak and super-off-peak periods

The Time-of-Use tariff structure is defined by the hour of the day associated with each time period. Let

$$\boldsymbol{hour(t) = t\ mod 24.}$$

Then the tariff type is given by

$$\boldsymbol{type(t)} = \begin{cases} \boldsymbol{super}, & \boldsymbol{0 \leq hour(t) < 6}, \\ \boldsymbol{on}, & \boldsymbol{15 \leq hour(t) < 19}, \\ \boldsymbol{off}, & \text{otherwise}. \end{cases}$$

Accordingly, the energy rate at time period $\boldsymbol{t}$ is

$$\boldsymbol{r_t} = \begin{cases} \boldsymbol{r^{super}}, & \boldsymbol{type(t) = super}, \\ \boldsymbol{r^{on}}, & \boldsymbol{type(t) = on}, \\ \boldsymbol{r^{off}}, & \boldsymbol{type(t) = off}. \end{cases}$$

Thus, for any feasible assignment $\boldsymbol{(m, \ell, s, p) \in F_{jo}}$, the associated energy cost is determined by the machine power, processing duration, and tariff periods spanned by the operation start time. Using the notation in Table 1, the complete model is written as follows:

$$\min Z = \sum_{j \in J} \sum_{o \in O_j} \sum_{(m,\ell,s,p) \in F_{jo}} c_{mps} x_{jom\ell s} + \alpha^{on} P^{on} + \alpha^{offsuper} P^{offsuper} \tag{1}$$

Subject to

$$\sum_{(m,\ell,s,p) \in F_{jo}} x_{jom\ell s} = 1, \qquad \forall j \in J,\ \forall o \in O_j \tag{2}$$

$$\sum_{(m,\ell,s,p) \in F_{j,o+1}} s\, x_{j,o+1,m\ell s} \geq \sum_{(m,\ell,s,p) \in F_{jo}} (s+p) x_{jom\ell s}, \qquad \forall j \in J,\ \forall o = 0, \ldots, n_j - 2 \tag{3}$$

$$\sum_{j \in J} \sum_{o \in O_j} \sum_{\substack{(m',\ell,s,p) \in F_{jo}: \\ m'=m,\ s \leq t < s+p}} x_{jom'\ell s} \leq 1, \qquad \forall m \in M,\ \forall t \in T \tag{4}$$

$$\sum_{j \in J} \sum_{o \in O_j} \sum_{\substack{(m,\ell',s,p) \in F_{jo}: \\ \ell'=\ell,\ s \leq t < s+p}} x_{jom\ell' s} \leq 1, \qquad \forall \ell \in W,\ \forall t \in T \tag{5}$$

$$\sum_{j \in J} \sum_{o \in O_j} \sum_{\substack{(m,\ell,s,p) \in F_{jo}: \\ s \leq t < s+p}} q_m x_{jom\ell s} \leq P^{on}, \qquad \forall t \in T \text{ such that } type(t) = on \tag{6}$$

$$\sum_{j \in J} \sum_{o \in O_j} \sum_{\substack{(m,\ell,s,p) \in F_{jo}: \\ s \leq t < s+p}} q_m x_{jom\ell s} \leq P^{offsuper}, \qquad \forall t \in T \text{ such that } type(t) \in \{off, super\} \tag{7}$$

$$x_{jom\ell s} \in \{0, 1\}, \qquad \forall j \in J,\ \forall o \in O_j,\ \forall (m, \ell, s, p) \in F_{jo} \tag{8}$$

$$P^{on} \geq 0, \qquad P^{offsuper} \geq 0 \tag{9}$$

The objective function in Equation (1) minimizes total electricity cost. The first term represents the Time-of-Use energy charge accumulated across all selected assignments, and the remaining terms represent demand charges based on the maximum coincident load during on-peak and off/super-off-peak periods. Equation (2) ensures that every operation is assigned exactly once. Equation (3) enforces precedence between consecutive operations of the same job. Equations (4) and (5) impose machine-capacity and worker-capacity limits, respectively, so that neither resource can process more than one operation at the same time. Equations (6) and (7) define the demand-peak structure by limiting the total load in each tariff category through two peak variables. Equations (8) and (9) specify the binary and nonnegativity requirements of the decision variables. Worker qualification and machine feasibility are enforced implicitly through the construction of the feasible assignment sets, since binary variables are created only for eligible machine-worker-start-time combinations. The formulation minimizes electricity cost only, while makespan is computed after solving from the selected schedule rather than being included in the objective.

3.2 Policy-based Rough Optimization with Large Neighborhood Search (Pro-LNS)

To solve the DRC-FJSP-TOU defined in Section 3.1, a Policy-based Rough Optimization with Large Neighborhood Search (Pro-LNS) framework is developed. The framework (as shown in Figure 1) combines a constructive reinforcement learning (RL) procedure, implemented through a Proximal Policy Optimization (PPO) algorithm, with an adaptive large neighborhood search (LNS) improvement phase. Both phases operate directly on the same decision structure defined by the feasible assignment sets F_{jo}, and all generated schedules strictly satisfy the precedence, machine-capacity, worker-capacity, and feasibility conditions imposed by Equations (2)–(9).

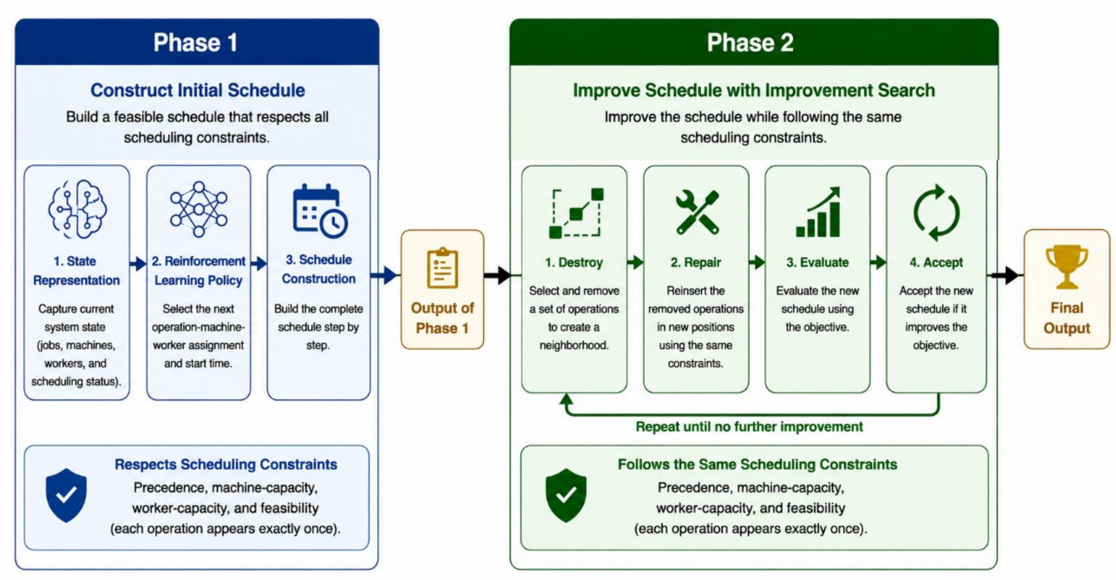

Figure 1. Overview of the Pro-LNS framework.

The first phase constructs a complete feasible schedule by sequentially selecting assignments $(m, \ell, s, p) \in F_{jo}$ for each operation. The second phase improves the constructed schedule by iteratively removing and reinserting a subset of operations, where reinsertion decisions are evaluated using the same electricity-cost objective defined in Equation (1).

3.2.1 Phase I: MDP-based Reinforcement Learning

The DRC-FJSP-TOU is modeled as a finite-horizon Markov decision process (MDP):

$$(\mathcal{S}, \mathcal{A}, \mathcal{T}, r, \overline{H}), \overline{H} = \sum_{j \in J} |O_j|,$$

Where $\overline{H}$ is the total number of operations. At each decision step, exactly one operation is assigned using a feasible tuple from F_{jo}, and the process terminates once all operations are scheduled.

State Space: At step t, the state $s_t$captures the current partial schedule through:

- Ready-operation indicators:

- $f_{t,(j,o)} = \begin{cases} 1, & \text{if operation } (j,o) \text{ is ready,} \\ 0, & \text{otherwise,} \end{cases}$

- Machine availability: the earliest available time of each machine $m \in M$,
- Worker availability: the earliest available time of each worker $\ell \in W$,
- Load profile: the current power consumption at each time period $t \in T$, computed using the same structure as in Equations (6) and (7),
- Peak loads: the current values of P^{on} and $P^{offsuper}$.

These components contain the information required to evaluate feasibility and electricity cost.

Action space: At state s_t, the action corresponds to selecting a feasible assignment

$$a_t = (j, o, m, \ell, s, p), (m, \ell, s, p) \in F_{jo},$$

for some operation (j, o) with $f_{t,(j,o)} = 1$. This ensures that each decision directly corresponds to selecting one binary variable $x_{jom\ell s}$ in Equation (8). All feasibility conditions, including machine eligibility, worker qualification, and time feasibility, are enforced through the definition of F_{jo}.

Transition function: When action $a_t = (j, o, m, \ell, s, p)$ is selected, operation (j, o) is scheduled at time s with completion time $C_{j,o} = s + p$. The partial schedule is updated by fixing $x_{jom\ell s} = 1$, and the state variables are updated accordingly:

- Machine m and worker ℓ become occupied over $[s, s + p - 1]$,
- Availability times of m and l are updated,
- The load profile is updated by adding q_m over $[s, s + p - 1]$,
- The peak-load variables P^{on} and $P^{offsuper}$ are updated.

Thus, every transition maintains feasibility with respect to Equations (3)–(7).
Reward function: Let Z_t denote the electricity cost of the partial schedule after t assignments, computed using the objective structure in Equation (1). The immediate reward is defined as

$$r(s_t, a_t) = -(Z_{t+1} - Z_t),$$

which corresponds to the negative marginal increase in total electricity cost. For action $a_t = (j, o, m, \ell, s, p)$, this can be written as

$$r(s_t, a_t) = -\left[c_{mps} + \alpha^{on} \Delta P^{on} + \alpha^{offsuper} \Delta P^{offsuper}\right],$$

Where ΔP^{on} and $\Delta P^{offsuper}$ are the increases in the corresponding peak loads caused by the assignment.

Learning objective: The policy is trained to maximize expected cumulative reward

$$J(\theta) = \mathbb{E}[\sum_{t=0}^{\overline{H}-1} r(s_t, a_t)],$$

which is equivalent to minimizing the total electricity cost in Equation (1).

The PPO construction module converts the MDP formulation into a trainable scheduling policy that generates a complete feasible schedule σ before LNS refinement. Neural padding is employed to ensure compatibility across instances of different sizes, allowing all benchmark instances to be processed by the same neural architecture. The policy and value functions are represented by a Multi-Layer Perceptron with two hidden layers of 256 neurons each and Rectified Linear Unit (ReLU) activation. PPO training uses the clipped surrogate objective with a mean-squared-error value loss, entropy regularization, and per-batch advantage normalization. The learning rate is set to 5×10^{-5}, with a mini-batch size of 64, 10 training epochs per update, maximum gradient norm of 0.5, entropy coefficient of 0.001, clipping range of 0.2, discount factor $\gamma = 0.99$, generalized advantage estimation parameter $\lambda = 0.95$, and value-function coefficient of 0.5. The policy is trained offline for 500,000 timesteps using eight parallel vectorized environments on 20 synthetically generated large-scale training instances, each with 90–110 jobs, 54–66 machines, five operations per job, 3–62 eligible machines per operation, and processing times of 5–35 minutes; the trained policy is then fixed for all benchmark evaluations without further fine-tuning. As shown in

Figure 2, the moving-average episodic return improves and stabilizes toward the end of training, indicating that the trained policy is suitable for generating the feasible initial schedule σ used as the starting point for the adaptive LNS phase.

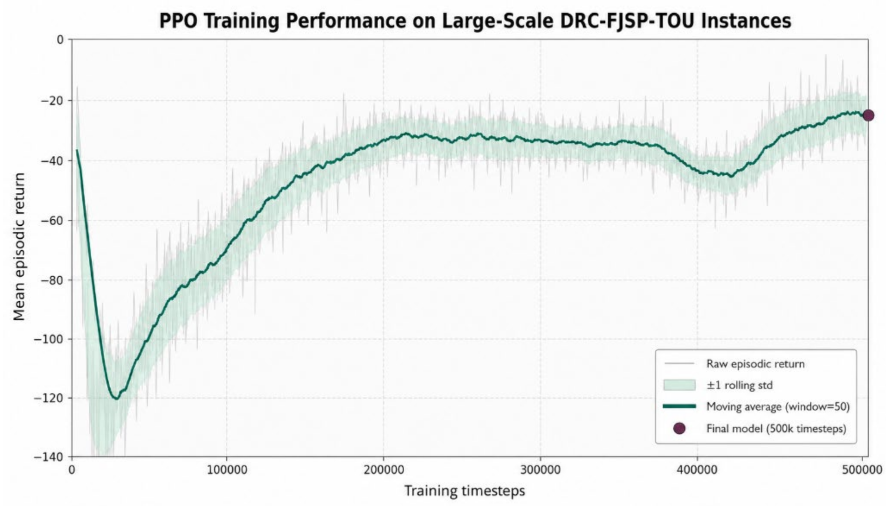

Figure 2. training performance on large-scale DRC-FJSP-TOU instances over 500,000 timesteps. The gray trace represents raw episodic returns, the solid green line represents the moving-average return with a window size of 50, the light-green shaded region represents $\pm\mathbf{1}$ rolling standard deviation, and the purple marker denotes the final trained policy used for subsequent schedule construction.

3.2.2 Phase II: Adaptive Large Neighborhood Search

Given a feasible schedule σ, define its objective value as

$$J(\sigma) = \sum_{j\in J}\sum_{o\in O_j}\sum_{(m,\ell,s,p)\in F_{jo}} c_{mps}x_{jom\ell s} + \alpha^{on}P^{on} + \alpha^{offsuper}P^{offsuper}.$$

This is identical to Equation (1), ensuring consistency between the exact formulation and the improvement procedure.

Adaptive removal: At iteration t, a subset of k_t operations is removed from the current schedule. For each operation (j,o), let its assigned tuple be (m,ℓ,s,p). Its removal score is defined as

$$\text{score}_{j,o} = c_{mps} + \alpha^{on}\Delta P_{j,o}^{on} + \alpha^{offsuper}\Delta P_{j,o}^{offsuper},$$

which represents its contribution to the objective. The k_t operations with the largest scores are removed. The neighborhood size k_t is adaptively adjusted based on whether improvement is achieved.

Greedy reinsertion: Removed operations are reinserted sequentially. For each operation (j,o), all feasible assignments

$$(m,\ell,s,p) \in F_{jo}$$

that satisfy precedence, machine-capacity, and worker-capacity constraints in the partial schedule are evaluated. The selected assignment minimizes

$$\Delta J(m,\ell,s,p) = c_{mps} + \alpha^{on}\Delta P^{on} + \alpha^{offsuper}\Delta P^{offsuper}.$$

Acceptance criterion: Let σ^t and σ^{t+1} denote the schedules before and after reinsertion. The new solution is accepted if

$$J\sigma^{t+1} < J\sigma^t$$

and the neighborhood size is reduced; otherwise, it is increased.

Termination: The remove-reinsert-accept cycle continues until convergence is achieved, defined as the absence of improvement in the objective over a sequence of iterations. Specifically, the search terminates if no improvement occurs for a dynamically adjusted number of iterations proportional to the number of operations in the schedule. The final solution remains feasible and improves the objective defined in Equation (1).

3.3 Experimental Design and Computational Setup

The benchmark instances are taken from the Kacem and Barnes flexible job shop datasets and are extended here to DRC-FJSP-TOU instances by adding worker-related and energy-related parameters (Kacem et al. 2002; Chambers and Barnes 1996). For each instance, the number of workers is set equal to the number of machines. The worker qualification matrix is generated machine by machine using the frequency with which each machine appears in the original benchmark routing data. For each machine m, the frequency f_m is computed as the number of times it appears across all operation alternatives, and a normalized value $\tilde{f}_m = (f_m - f_{min})/(f_{max} - f_{min})$ is used to define a qualification ratio $\rho_m = \rho_{min} + \tilde{f}_m(\rho_{max} - \rho_{min})$, with $\rho_{min} = 0.35$ and $\rho_{max} = 0.75$. The number of qualified workers is then set to $k_m = max\{2, min(|W|, \text{round}(|W|\rho_m))\}$, and k_m workers are sampled uniformly without replacement and assigned to machine m. Machine power values are generated using the same normalized frequency, with $q_m = \text{round}(p_{min} + \tilde{f}_m(p_{max} - p_{min}))$ and $p_{min} = 3, p_{max} = 9$ kW. The Time-of-Use tariff structure is based on the Evergy Kansas Central Industrial and Large Power Time of Use schedule, with super-off-peak periods from 12am to 6am, on-peak periods from 3pm to 7pm, and off-peak periods otherwise (Evergy Kansas Central 2025). These periods are parameterized in the model using representative energy rates and corresponding demand-charge components that preserve the relative cost differences across tariff categories.

Following the construction of the benchmark instances, the computational study examines the effectiveness of the proposed policy-based rough optimization with large neighborhood search (Pro-LNS) framework both as a standalone solution method and as a warm-start mechanism for exact optimization. All experiments run on the Google Colab platform under a standard CPU runtime, using a session configured with an Intel(R) Xeon(R) CPU @ 2.20GHz and approximately 12 GB of RAM.

For each instance, two solution pipelines run under a fixed time budget. In the first pipeline, the mathematical formulation described in Section 3.1 enters Gurobi as a mixed-integer linear programming (MILP) model and receives a maximum runtime of 3600 seconds. In the second pipeline, Pro-LNS first constructs a feasible schedule, after which the resulting solution enters the Gurobi MILP solver as a warm-start incumbent. The combined runtime of the Pro-LNS phase and the subsequent MILP optimization remains capped at the same 3600-second limit. This matched-time design maintains identical computational effort across both pipelines and supports a fair comparison between standalone exact optimization and Pro-LNS-assisted optimization.

4. Results

To evaluate the effectiveness of the proposed policy-based rough optimization with large neighborhood search (Pro-LNS) framework, computational experiments are conducted on the benchmark instances described in Section 3.3 under a fixed time budget of 3600 seconds. The performance of the standalone mixed-integer linear programming (MILP) is compared against the Pro-LNS-assisted MILP, where Pro-LNS provides a warm-start incumbent to the solver.

The results of this comparison, including objective values, computational times, and optimality gaps, are summarized in Table 2.

Table 2. Computational Results

Instance	Standalone MILP Objective ($)	Standalone MILP CPU (s)	Standalone MILP Optimality Gap (%)	Pro-LNS-Assisted MILP Objective ($)	Pro-LNS CPU (s)	Pro-LNS-Assisted MILP CPU (s)	Pro-LNS-Assisted MILP Optimality Gap (%)
k1	$47.20	552	0	$47.20	6.12	392	0
k2	$125.00	3600	79.28	$71.90	6.78	3600	3.12
k3	$192.20	3600	72.12	$115.90	7.23	3600	3.78
k4	$347.30	3600	85.35	$195.70	8.05	3600	4.45
mt10c1	$1,118.60	3600	20.13	$969.50	7.56	3600	4.12
mt10x	$1,120.00	3600	21.21	$969.20	7.89	3600	4.89
mt10cc	$1,120.50	3600	21.71	$969.80	8.45	3600	5.34
mt10xx	$1,124.70	3600	22.68	$969.80	8.78	3600	5.78
mt10xy	$1,121.00	3600	20.99	$984.30	9.12	3600	6.23
mt10xxx	$1,141.10	3600	25.43	$970.40	9.56	3600	6.67
mt10xyz	$1,142.40	3600	24.44	$983.30	9.98	3600	7.12

As shown in Table 2, the standalone MILP approach reaches optimality only for the smallest instance, k1, where both methods produce the same objective value and zero optimality gap. For all larger instances, however, the standalone MILP encounters substantial difficulty within the 3600-second limit. This pattern appears most clearly in the Kacem instances k2 to k4, where the optimality gap ranges from 72.12% to 85.35%, and in the mt10 family, where the corresponding gaps remain between 20.13% and 25.43%. These results indicate that as problem size and flexibility increase, the exact formulation struggles to identify high-quality incumbents and to close the bound effectively within the allotted runtime.

By contrast, the Pro-LNS-assisted MILP consistently improves solution quality under the same matched time budget. For k2 to k4, the Pro-LNS-assisted MILP reduces the optimality gap to a range of 3.12% to 4.45%, while for the mt10 instances the gap falls to between 4.12% and 7.12%. The objective values also improve substantially across all nontrivial instances, with especially pronounced reductions for k2, k3, and k4, and consistent improvements throughout the mt10 set. These gains are achieved with only a small additional Pro-LNS runtime of approximately 6 to 10 seconds, which represents a negligible portion of the total 3600-second budget. Overall, the results demonstrate that the Pro-LNS-assisted MILP provides strong initial incumbents that significantly enhance solver performance, yielding lower electricity-cost solutions and substantially tighter optimality gaps than the standalone MILP.

This improvement mechanism can be illustrated using instance k2. The standalone MILP yields a total electricity cost of $125.00, with a substantial share attributed to on-peak demand charges ($76.00), and produces a makespan of 57. In contrast, the Pro-LNS-assisted MILP reduces the total electricity cost to $71.90 while also lowering the makespan to 36. The key difference lies in the temporal distribution of processing. In the standalone MILP schedule, part of the workload occurs during on-peak periods, resulting in an on-peak demand of 3.76 kW and the associated demand charges. By comparison, the Pro-LNS-assisted MILP avoids on-peak usage entirely, reducing on-peak demand to zero and eliminating that cost component. Processing instead shifts to off/super-off-peak periods, where a higher peak demand of 18.00 kW is incurred under lower-cost tariffs. This behavior indicates that the Pro-LNS-assisted MILP systematically shifts workload toward lower-cost periods while maintaining effective coordination across machines and workers, thereby reducing overall electricity cost.

5. Conclusion

This study develops a mathematical formulation for the dual-resource flexible job shop scheduling problem under Time-of-Use electricity tariffs (DRC-FJSP-TOU), in which each operation must be assigned to an eligible machine, a qualified worker, and a start time while satisfying precedence, machine-capacity, and worker-capacity constraints. The formulation captures total electricity cost through both Time-of-Use energy charges and demand charges, thereby providing a rigorous representation of the scheduling environment considered in this paper. To solve the problem more effectively, a policy-based rough optimization with large neighborhood search (Pro-LNS) framework is proposed, combining a PPO-based constructive policy with an adaptive LNS improvement phase and a warm-start integration with the exact mixed-integer linear programming (MILP) model.

The computational results on 11 benchmark instances show that this hybrid design materially improves performance relative to the standalone MILP under the same 3600-second time budget. While the standalone MILP reaches optimality only for the smallest instance and exhibits large optimality gaps on the remaining problems, the Pro-LNS-assisted MILP consistently delivers lower objective values and substantially tighter final gaps, reducing them to below 8% across all tested instances. More broadly, the results suggest that these gains arise not only from stronger initial incumbents, but also from a more effective temporal organization of processing that better exploits lower-cost tariff periods while maintaining coordination across machines and workers. Overall, the findings support the proposed framework as a practical and computationally effective approach for energy-aware dual-resource scheduling under Time-of-Use tariffs.

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