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Comparative Numerical Modeling of Wear Evolution Using Archard-Based Hyperbolic and Parabolic Formulations

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Abstract

Composite materials offer customizability of properties, good specific strength and corrosion resistance. These make composites reliable for structural and wear-resistant applications. The wear of a material is sensitive to its properties in addition to the environment and operating conditions. The experimental evaluation of the wear of a material requires extensive experiments to determine the effect of all the parameters. Numerical modelling and simulation are effective tools to estimate the wear in the early stages of design decision-making. In the present study, a numerical model was developed using time dependent wear formulation derived from Archard's wear law for sliding contact. The wear depth was expressed as a function of time and spatial position along the contact, considering the non-uniform pressure conditions. Three numerical models were developed. The first model represents baseline wear accumulation governed by Archard's law. The second model introduces a first-order spatial derivative to account for directional wear transport, while the third model incorporates a second-order diffusion term to capture smoothing of wear gradients. All three models were discretized by the finite difference method and implemented using MATLAB. The first derivative model produced asymmetric wear profiles showing the influence of directional wear on material removal patterns, while the second derivative model generated smooth and stable wear distribution by reducing sharp gradients.

Keywords
Archard Wear Modelling, Nonlinear Wear Evaluation, Finite Difference Method, Hyperbolic and Parabolic Formulations, Pressure-Geometry Coupling

1. Introduction
Wear is a process of material removal from a contact in relative motion. The rate of wear depends on the material properties of the contacting surfaces, environmental effects, lubrication, contact geometry, contact pressure, temperature and presence of foreign bodies between the contacting surfaces etc. Studying the influence of all these parameters experimentally is costly and time consuming. The best way is to develop mathematical models using empirical and physical equations. Using contact mechanics and Archard wear equations is most predominantly used procedure to estimate the wear of sliding contacts (Sun et al. 2023).

The modelling of wear coefficient in Archard equation is the primary concern. Worn height stochastic process presented better results compared to Monte Carlo and Latin Hypercube models (Da Silva and Pintaude 2008). The deviations of experimental results from Archard equation were generally attributed to the surface hardening during the experimentation. The other parameters like thermal changes due to friction heating during the experiment have negligible effects on the wear rate changes (Rupert and Schuh 2010). Simple Archard model was implemented on wear of plough blades (Zhang et al. 2024), screw conveying (Yang et al. 2025), wheel wear (Zeng et al. 2025), metal hose (Lu et al. 2025), rolling contacts (Liu et al. 2022), hip implants (Mattei and Di Puccio 2023) and impeller-tumbler ore abrasiveness (Tino et al. 2026). Most of these studies added further models like contact mechanics, finite element methods, discrete element methods, dynamic wear models etc, to improve the accuracy of estimation.

Nonlinear contact models along with finite element methods were attempted to estimate the wear to a certain degree of accuracy. Introduction of piecewise rigid body motions, natural tangential stiffness and experimental wear coefficients and friction coefficients helped in estimating the wear rate with reasonable accuracy (Telliskivi 2004). The estimation of wear using Archard's law fails if there is a presence of wear mechanisms other than plastic deformation (Hu et al. 2022), failure of estimating accurate normal contact load and contact duration due to the applied load and sliding distance. At asperity level, Archard law holds good with linear wear (Aghababaei and Zhao 2021). The surface roughness, asperity height, slope, contact size (Zhu and Shipway 2021) and hardness of both the surfaces influence the wear, Archard wear can be modified to include these so the accuracy of estimation improves (Tabrizi et al. 2021). The validity of Archard law was studied using large datasets of experimental works and deduced that the linear relationship between wear and applied loads is valid for maximum initial pressure greater than 1.5 GPa (Reichelt and Cappella 2021). The wear coefficient is not constant for all the initial pressures and the transition between constant and non-constant depends on the initial contact pressure.

This indicates that the wear estimation from Archard law can be improved by introducing addition terms to include the effect of the wear transport, debris ejection, hardness changes, friction, surface asperities etc. The objective of this work is to develop and compare several Archard-based numerical formulations incorporating pressure feedback, transport, and diffusion mechanisms for studying spatial wear evolution in sliding contacts. In the present study, the contact pressure was modelled using parabolic pressure distribution curve. The baseline wear performance was estimated using simple Archard wear equation. The effect of the addition of wear transport and wear diffusion terms to the Archard equation was reported. Finite difference method was utilized to solve these equations.

2. Mathematical Formulation

The wear rate is estimated using the Archard wear law. It is defined for the materials in relative motion that are in direct contact with each other. As the samples are axisymmetric, the surface wear can be modeled using single dimension. The wear is dependent on the contact pressure, which is defined by equation 1.

$$\boldsymbol{P(x) = P_o\left(1 - \frac{x}{L/2}\right)^2} \quad (1)$$

Where, P is the pressure distribution, P_o is the maximum pressure on the sample, L is surface length of the sample, and x is the distance of a given point on the sample from center of the sample. The range of x is defined as (in equation 2)

$$x \in \left[-\frac{L}{2}, \frac{L}{2}\right] \quad (2)$$

The wear depth estimated in this study are not exact but are obtained from the assumptions of the Archard wear equation. The base line for the wear depth is set with simple wear equation shown in equation 3 and further terms are added the change in wear depth was analyzed.

$$\frac{\partial h(x,t)}{\partial t} = kP(x)v\,\phi \quad (3)$$

Where k is the wear coefficient, its value is around 1.7×10^{-5} for steels and PTFE based materials. The sliding velocity of the sample v is 2 m/s for this study. The equations were solved for the sliding distance of 2000m, and the surface profile was plotted for every 400m of sliding. The ϕ is the composite factor which is dependent on the reinforcement volume fraction v_f (equation 4).

$$\phi = 1 + \alpha \times v_f \tag{4}$$

The α is sensitivity factor. The influence of variation of reinforcement volume fraction and the sensitivity of composite can be studied by varying these parameters but for the present study the effect of reinforcement is limited to 1%. The value of α is 0.01 and v_f is 0.1.

This model is a source driven equation, which considers only the source load, sliding velocity, composite modification factor and wear coefficient. All the terms in the equation are constant except for the pressure distribution. The wear rate equation shown in equation 2 is proportional to the pressure applied at the contact. So, the wear rate takes the shape of the pressure distribution. The position with highest pressure will have higher wear rate and the contact discontinuity occur at this place. To avoid this, the pressure distribution correction term is added after every step of wear (equation 5).

$$P(x) = P_o\left(1 - \frac{x}{L/2}\right)^2\left(1 - \beta\frac{h_i^n}{h_{max}}\right) \tag{5}$$

The pressure sensitivity β is taken as 0.8. The low value of β reduces the sensitivity and higher value overcorrects the effect of pressure. The effect of pressure sensitivity can also be studied in future. Adding pressure sensitivity to the equation 3 introduces nonlinear saturation effect to the equation.
The wear transport term is added to equation 3 which introduces advective transfer of wear (equation 6). The term c is wear transport coefficient.

$$\frac{\partial h}{\partial t} = kP(x)v\phi - c\frac{\partial h}{\partial x} \tag{6}$$

For the third model, wear diffusion term is added which helps smoothen the wear curves (equation 7). The term D is wear diffusion coefficient.

$$\frac{\partial h}{\partial t} = kP(x)v\phi + D\frac{\partial^2 h}{\partial x^2} \tag{7}$$

The values of wear transport coefficient and wear diffusion coefficient are obtained from the stability conditions. The wear transport equation is a hyperbolic equation, classical Courant-Friedrichs-Lewy stability condition is used to determine the wear transport coefficient. The maximum value of w is considered in this study to keep the sensitivity to a maximum value.

$$C = \frac{w\Delta t}{\Delta x} \leq 1 \Rightarrow w \leq \frac{\Delta x}{\Delta t} \tag{8}$$

The wear diffusion equation is a parabolic equation. Explicit diffusion stability criterion is used to obtain the wear diffusion coefficient. The largest possible value of c is considered to keep the sensitivity to a maximum value.

$$r = \frac{D\Delta t}{\Delta x^2} \leq \frac{1}{2} \Rightarrow D \leq \frac{\Delta x^2}{2\Delta t} \tag{9}$$

The four models are solved using finite difference methods. The finite difference formulations of all the models are shown below.

$$h_i^{n+1} = h_i^n + \Delta t(kp_i v\phi) \tag{10}$$
$$h_i^{n+1} = h_i^n + \Delta t\left(kp_i v\phi - c\frac{h_i^n - h_{i-1}^n}{\Delta x}\right) \tag{11}$$
$$h_i^{n+1} = h_i^n + \Delta t\left(kp_i v\phi + D\frac{h_{i+1}^n - 2h_i^n + h_{i-1}^n}{\Delta x^2}\right) \tag{12}$$

To solve the equations, zero initial wear and zero gradient at the boundaries are considered. The initial and the boundary conditions can be written as

$$h(x,0) = 0 \tag{13}$$
$$\frac{\partial h}{\partial x} = 0 \quad \text{at } x = \pm L/2 \tag{14}$$

The objective of this study is methodological comparison rather than experimental calibration of wear coefficients.

3. Results and Discussion

The pressure distribution at the contact zone was modelled as a parabolic curve with maximum pressure at the center of contact and minimum pressure at the outer edge of contact. The initial maximum pressure is assumed to be 10 MPa. The pressure distribution curve is represented in Figure 1.

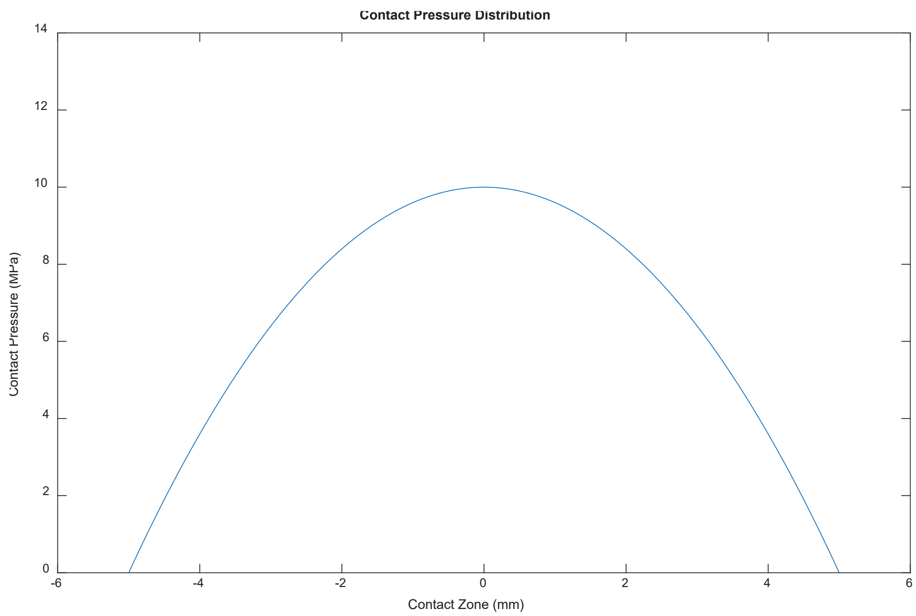

Figure 1. Distribution of contact pressure on a sample with maximum pressure of 10 MPa

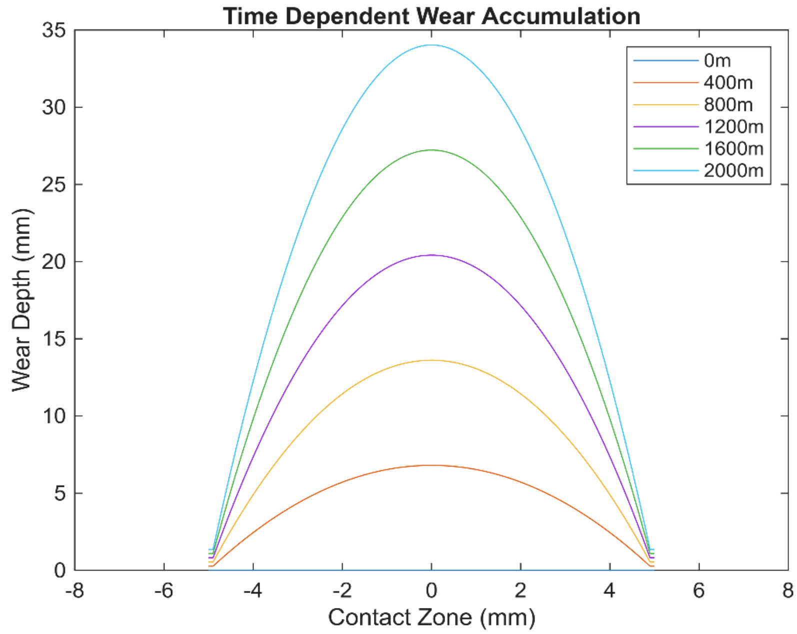

Figure 2. Time dependent wear accumulation up to 2000m of sliding

The wear depth of the sample under sliding motion was estimated with simple Archard wear equation. All the terms of Archard equation except for the contact pressure are constant (Figure 2). The worn surface looks like the pressure distribution curve as shown in Figure 2. It also shows the change of wear with the sliding distance. The increase in sliding distance increases the wear linearly as per Archard due to the non-availability of the sliding distance term in the equation. In the simple equation, wear just accumulates with time. The wear depth was plotted after every 400m sliding distance until 2000m. The wear rate is maximum at maximum contact pressure but as the surface wears out, the contact area reduces and the pressure distributes to a different location. With the change in the pressure distribution, the maximum wear position should change. To include this, the pressure correction term was included in the Archard equation as shown in equation 5. This reduced the wear concentration at a single point. The wear rate of the sample with pressure correction term reduced at the center of the sample but the wear at the edges increased compared to the simple equation. The wear rate of the sample with the pressure correction term was shown in Figure 3. The wear rate of the sample after every 400m sliding distance was plotted in Figure 3 until the sliding distance of 2000m. The wear rate is still followed by same pattern after every iteration of sliding distance. Also, the wear rate is still maximum at the center of the sample.

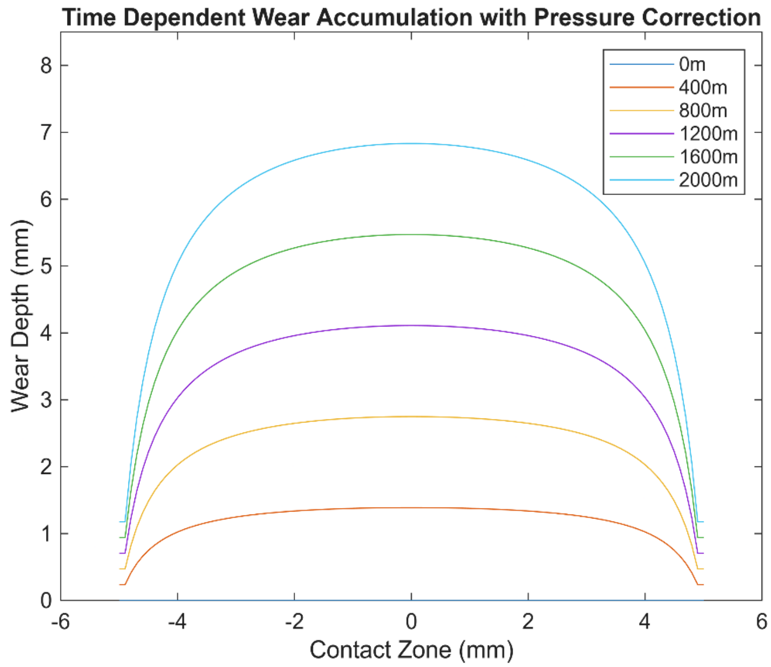

Figure 3. Time dependent wear accumulation with corrected pressure distribution after every step

To improve the estimation of wear rate further, wear transport term is included in the Archard equation. The wear transport coefficient was not optimized in this study. The maximum possible wear transport coefficient is considered for this study. This ensures that the model is stable as well as the effect of the wear transport is maximum. The effect of varying the wear transport coefficients can be further studied in future. With the addition of wear transport terms, the material flows from the initial contact point to the final contact point as shown in Fig. 4. Hence the plots show highest wear at the initial contact point. Also, the maximum wear is far less than the previous model as the material is not removed completely (Figure 3).

Addition of wear diffusion term to the Archard equation was also attempted. The wear diffusion term is a second order term which smoothens the curve but does not include the material flow. The wear curves with the wear diffusion term can be seen in Figure 5. These curves are smoother than the curves shown in Figure 3, but the trend is like them. The maximum wear rate with the diffusion curve is nearly equal to the maximum wear rate obtained by the pressure correction equation as shown in Figure 6. The simple Archard equation show the largest value of maximum wear rate while the material transfer model show the smallest value of maximum wear rate (Figure 4).

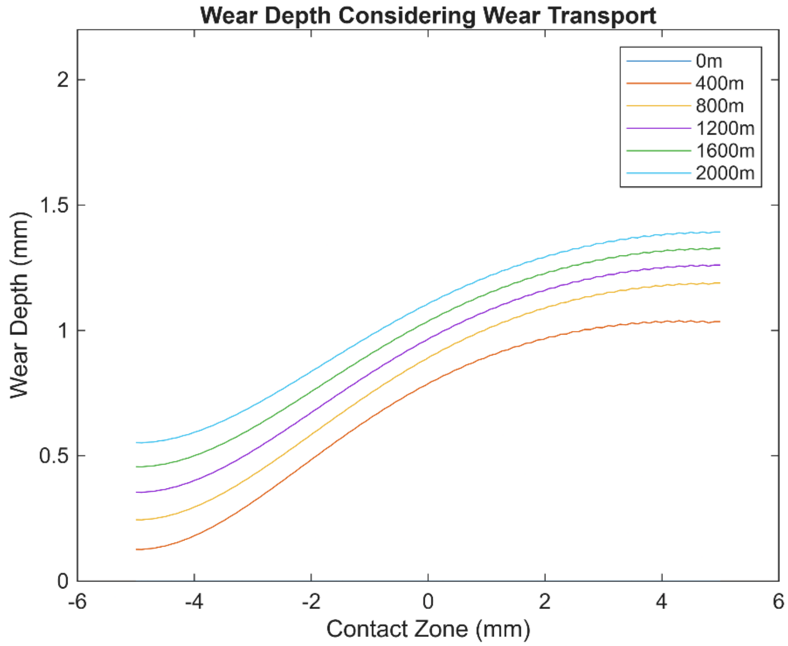

Figure 4. Wear depth of a sample considering wear transport up to a sliding distance of 2000m

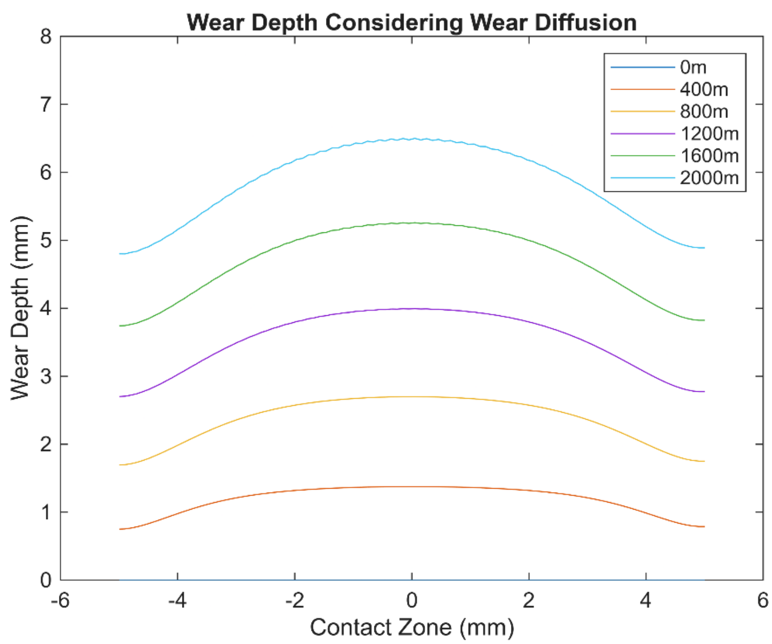

Figure 5. Wear depth of a sample considering wear diffusion up to a sliding distance of 2000m

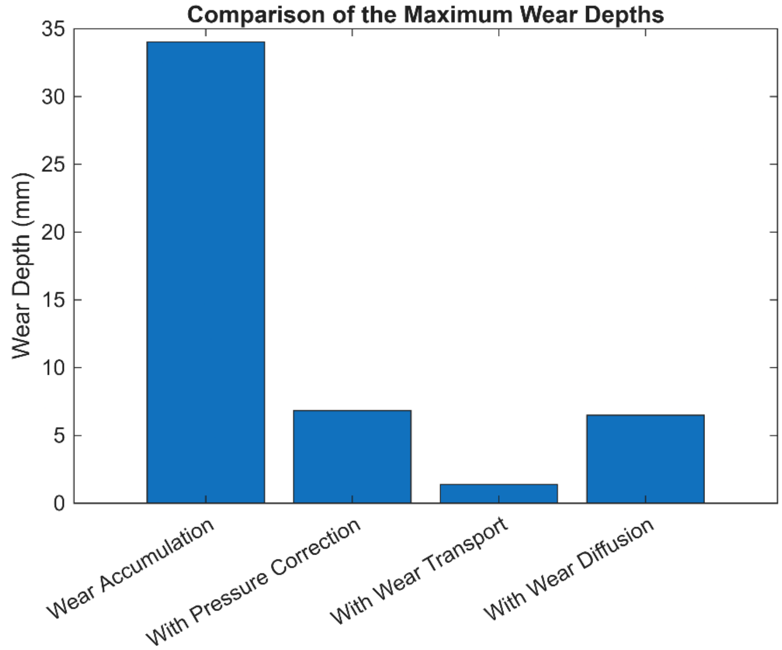

Figure 6. Comparison of maximum wear depth on sample for all four models after sliding for 2000m

4. Conclusions

The conclusions of the present study are

- Simple Archard equation may not be sufficient to estimate the wear rate at all conditions.
- The addition of wear transport and wear diffusion terms smoothens the curves.
- The comparative study demonstrates that inclusion of transport and diffusion mechanisms significantly influences spatial wear evolution and peak wear magnitude.
- Considering the maximum possible wear transport and wear diffusion coefficients reduced the maximum wear rate considerably as the effect of these terms are the highest.
- The pressure sensitivity, coefficients of wear transport and wear diffusion may also play a role in correct estimation of wear rate which can be studied in future.

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